g cm$^{-2}$; the value of $L$ is found to be 145 g cm$^{-2}$ for Buenos Aires, and 149 g cm$^{-2}$ for Asunción.

Finally, Fig. 4 shows the curves obtained by Simpson in the northern hemisphere for the variation of the absorption mean free path as a function of the atmospheric thickness, with the geomagnetic latitude $\lambda$ as a parameter; we have there introduced our own values for $\lambda=23.3°$S (Buenos Aires) and $\lambda=15°$S (Asunción). They show a good agreement within our experimental errors.

Our results are thus consistent with Simpson's suggestions[2,3] that the variation of the absorption mean free path with geomagnetic latitude and with altitude must be a consequence of the average energy of the primary nucleons as they start the nucleonic shower and of the degradation of the energy through collision and nuclear interaction phenomena along the penetration of the nucleons into the depths of the atmosphere.

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# Estimate of the Nucleon Mass Difference from Hofstadter's New Form Factors

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It is shown that the set of isotopic form factors recently proposed by Hofstadter and Herman seems favorable for an explanation of the observed nucleon mass difference in terms of the electromagnetic self-energy, if an extended core radius is suitably introduced.

An attempt has been made[1] to explain the observed nucleon mass difference, $M_n-M_p=1.29$ Mev, by assuming trial form factors which deviate from Hofstadter's old form factors determined by the electron scattering experiments at rather small values of the momentum transfer $q$. Meanwhile, according to the new experimental data[2,3] on the electromagnetic structure of the proton and neutron, Hofstadter and Herman[4] have recently presented a tempting unified interpretation of the nucleon form factors. They have found the four isotopic form factors of the Clementel and Villi (C–V) form.[5] Although the C–V form is interesting from the viewpoint of the dispersion relations, their form factors do not give a convergent result for the nucleon mass difference.[1] It should be noted, however, that the signs of the core terms in the Hofstadter new isotopic form factors just satisfy the previously conjectured condition under which the nucleon mass difference can be explained.[6] In the present report it is shown that, if a suitably extended core is assumed with its radius smaller than the nucleon Compton wavelength, then the Hofstadter new form factors, thus modified, can reasonably explain the observed mass difference.

For simplicity a common parameter $r_c$ is introduced into the core terms of Hofstadter's new isotopic form factors as follows:

$$F_{1S}=\frac{0.44}{1+r_c^2q^2}+\frac{0.56}{1+0.214q^2}, \tag{1}$$

$$F_{2S}=\frac{4.0}{1+r_c^2q^2}-\frac{3.0}{1+0.214q^2}, \tag{2}$$

$$F_{1V}=F_{2V}=-\frac{0.20}{1+r_c^2q^2}+\frac{1.20}{1+0.10q^2}. \tag{3}$$

As has been estimated in reference 1, the strong interaction correction to the nucleon mass difference seems to be less than 0.2 Mev, and so the major contribution may be regarded as coming from the direct electromagnetic effect. The $e^2$-order self-energy thus gives the

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[6] See (3.8′) of reference 1. Note that the normalizing constants of the isotopic form factors are differently defined in references 1 and 4. In the present report, the choice of Hofstadter's normalization in reference 4 is used.

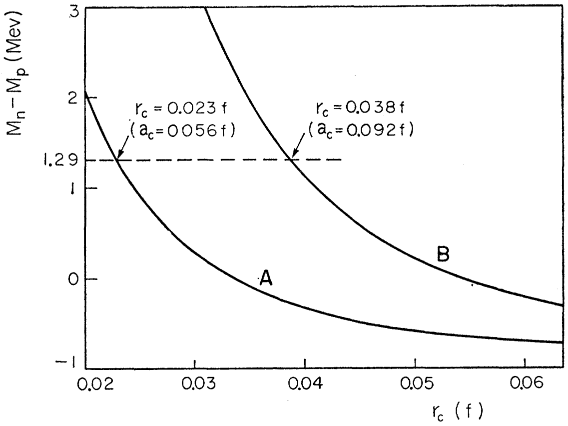

FIG. 1. The calculated nucleon mass difference as a function of the core parameter $r_c$. (For $r_c$=0.023f, $a_c$ should read 0.056f.)

neutron-proton mass difference in the form

$$M_n - M_p = \delta M_n - \delta M_p = (e^2/\pi)M \int d^4q [-F_{1S}F_{1V}I_{11}(q) + (0.0602F_{1V}F_{2S} - 1.853F_{1S}F_{2V})I_{12}(q) + 0.4461F_{2S}F_{2V}I_{22}(q)], \quad (4)$$

where $I_{11}(q)$, $I_{12}(q)$, and $I_{22}(q)$ denote the Dirac-Dirac, Dirac-Pauli, and Pauli-Pauli self-interactions, respectively. Without the form factors, the integral of $I_{11}$ diverges logarithmically and those of $I_{12}$ and $I_{22}$ diverge quadratically. Inserting the form factors (1)–(3) into Eq. (4), one obtains the mass difference as a function of the core parameter $r_c$.

As the experimental form factors have been found referring only to the space-like $q^2$, it is not clear whether the expressions (1)–(3) can be used for the time-like $q^2$. For this reason the integration of (4) is made in two different ways.

(A) The form factors are used for the time-like $q^2$ as well as for the space-like $q^2$. The usual Feynman cutoff technique can be used.

(B) The integration over $q_4$ is first carried out without the form factors. Then the integration over $\mathbf{q}$ is performed with the form factors, in which $q^2$ is replaced by $\mathbf{q}^2$.

Figure 1 shows the calculated mass difference versus the core parameter $r_c$ in these two cases. Both cases give the similar qualitative tendency, but the observed mass difference is reproduced at somewhat different values of $r_c$. The corresponding root-mean-square radius of the core, $a_c = (6)^{\frac{1}{2}} r_c$, is less than 0.1 fermi, as is listed in Table I. These numerical values seem quite reasonable and do not influence the recent analyses by Hofstadter and others based on the experiments below $q^2 \lesssim 30$ f$^{-2}$. Table I lists also the individual mass shifts $\delta M_n$ and $\delta M_p$ in each case.

TABLE I. Calculated results.

| Case | $r_c$ (fermi) | $a_c$ | $\delta M_n$ (Mev) | $\delta M_p$ | $\Delta M_{11}$ | $\Delta M_{12}$ (Mev) | $\Delta M_{22}$ |
|---|---|---|---|---|---|---|---|
| (A) | 0.023 | 0.056 | −4.02 | −5.31 | −0.78 | −13.20 | 15.27 |
| (B) | 0.038 | 0.092 | −1.83 | −3.12 | −1.07 | −7.59 | 9.95 |

The contributions to $M_n - M_p$ from $I_{11}$, $I_{12}$, and $I_{22}$, which are denoted as $\Delta M_{11}$, $\Delta M_{12}$, and $\Delta M_{22}$, in Table I, indicate that the nucleon mass difference is explained as a result of the predominance of $\Delta M_{12} + \Delta M_{22}$ due to the magnetic moment self-energy over $\Delta M_{11}$ due to the charge self-energy, as was proposed in the earlier report by the author.[7] In the earlier works,[8] it was tried to explain the mass difference by making $\Delta M_{12}$ positive, because $|\Delta M_{22}|$ was considered to be very small. On the other hand, Cini and others[9] tried to make $\Delta M_{11}$ positive and large by assuming a strong charge concentration for the neutron, because the use of Hofstadter's old exponential form factor gave a negative $\Delta M_{12}$. In contrast with these works so far, the present result points out an excess of positive $\Delta M_{22}$ over the absolute value of negative $\Delta M_{12}$.

Since this kind of calculation contains a small difference of rather large numbers, the numerical details should not be taken very seriously. It may be concluded, however, that the set of Hofstadter's new form factors is quite favorable to produce the observed nucleon mass difference, if a reasonable assumption is made for the extended core radius.

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