PHYSICAL REVIEW VOLUME 124, NUMBER 4 NOVEMBER 15, 1961

Temperature Dependence of Electron Emission in the Field Emission Region

RALPH KLEIN AND LEWIS B. LEDER
National Bureau of Standards, Washington, D. C.
(Received June 29, 1961)

The variation in field emission current as a function of the temperature of the emitter has been measured for tungsten over the range from 4.2°K to 400°K. The expected form of the temperature dependence of the emission current confirms the relationship previously postulated on theoretical grounds. This is $j(T)=j(0)(KT/\sin KT)$, where K is a constant depending on the work function and the field at the surface of the emitter. A current-increment method, in which the increase of current is plotted against the square of the temperature increase at constant field, gives a straight line whose slope permits surface field calculations more precise than heretofore possible by electron microscope measurement of tip radius.

I. INTRODUCTION

IN electron emission from metals it has been convenient to consider three regions. These are the field emission, the thermionic emission, and the transition region. They have been considered theoretically by Murphy and Good.[1] The literature on thermionic emission, both theoretical and experimental, is extensive.[2] The transition region, that is, where both field emission and thermal emission contribute significantly to the total emission, has been called T-F emission by Dolan and Dyke.[3] Dyke and co-workers[4] have provided experimental studies in which the theoretical expectations have been confirmed. The region of field emission, that is, where the emitted electrons originate largely from below the Fermi level, has not been investigated in any detail in its relationship to temperature. The present experiments are concerned with the temperature dependence of emission at temperatures below 400°K.

II. THEORETICAL

The number of electrons within an energy interval dW being emitted per second per unit area under field emission conditions is[5]

$$P(W)dW=\frac{4\pi mkT}{h^3}\exp\left[-\frac{4(2m|W|^3)^{\frac{1}{2}}}{3\hbar eF}v\left(\frac{(e^3F)^{\frac{1}{2}}}{|W|}\right)\right]\times\ln(1+e^{-(W-\zeta)/kT})dW, \quad (1)$$

where W is the energy with respect to free space, ζ is the Fermi level, and $v[(e^3F)^{\frac{1}{2}}/|W|]$ is a tabulated elliptic function.[6] Expression (1) is obtained from a WKB approximation of barrier penetration probability combined with the Fermi-Dirac distribution. Expansion around $W=\zeta$ and an integration over all energies gives the emission current density at temperature T

$$j(T)=\left(\frac{4\pi mekTd}{h^3}\right)e^{-c}\int_0^\infty\frac{\xi^{kT/d-1}}{1+\xi}d\xi, \quad (2)$$

where ξ is a function equal to $e^{(W-\zeta)/kT}$,

$$d=\frac{\hbar eF}{2(2m\varphi)^{\frac{1}{2}}t(y)}, \quad (3)$$

$$c=\frac{4(2m\phi^3)^{\frac{1}{2}}}{3\hbar eF}v(y), \quad (4)$$

$t(y)=v(y)-\frac{2}{3}y$ and ϕ is the work function. Both $v(y)$ and $t(y)$ are very close to one. If kT/d is less than one, so that

$$F/\varphi^{\frac{1}{2}}t(y)>2^{\frac{3}{2}}m^{\frac{1}{2}}kT/\hbar e, \quad (5)$$

then

$$j(T)=j(0)KT/\sin KT \quad (6)$$

(where $K=\pi k/d$) which gives the temperature effect on the emission current density in the field emission region.[1] If T is less than $3.15\times10^{-5}\ F/\phi^{\frac{1}{2}}$, then $KT/\sin KT$ may be approximated by $1+\frac{1}{6}K^2T^2$ within 1%. The error decreases rapidly with decreasing temperature. Then

$$j(T)\cong j(0)[1+\tfrac{1}{6}K^2T^2]. \quad (7)$$

The total current, which is measured in this experiment, is

$$i(T)=\int j(T)dA, \quad (8)$$

where the integral is taken over the emitting area. The current density is not uniform over the surface because of the variation in work function and field. $i(0)$ in terms of an average work function has been discussed previously.[7] Therefore,

$$i(T)=i(0)[1+\tfrac{1}{6}K^2T^2], \quad (9)$$

where the i's refer to the total emission and K contains the average work function. According to the mode of measurement

$$\Delta i=i(T)-i(4.2), \quad (10)$$

[1] E. L. Murphy and R. H. Good, Jr., Phys. Rev. **102**, 1464 (1956).
[2] See for example W. B. Nottingham, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21.
[3] W. W. Dolan and W. P. Dyke, Phys. Rev. **95**, 327 (1954).
[4] W. P. Dyke *et al.* Phys. Rev. **99**, 1192 (1955).
[5] R. H. Good, Jr., and E. W. Muller, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, p. 191.
[6] $v(y)$ is tabulated by R. E. Burgess, H. Kroemer, and J. M. Houston, Phys. Rev. **90**, 515 (1953).
[7] R. Klein, J. Chem. Phys. **21**, 1177 (1953).

so that

$$\frac{\Delta i}{i(4.2)}=\frac{\frac{1}{6}K^2[T^2-(4.2)^2]}{1+\frac{1}{6}K^2(4.2)^2}\cong\tfrac{1}{6}K^2[T^2-(4.2)^2], \quad (11)$$

and Δi is linear with the square of the temperature. $K=\pi k/\bar{d}=2.77\times10^4\bar{\phi}^{\frac{1}{2}}/\bar{F}$, where the bar indicates an average value.

III. EXPERIMENTAL

The field emission tube is shown in Fig. 1. The emitter, a 4-mil tungsten wire, was spot welded to a tantalum loop, which, as shown on the figure, was supported by four leads. A tantalum loop was used to avoid the peculiar current-temperature characteristics of a tungsten loop with its ends at 4.2°K. These characteristics have been discussed previously[8] with respect to heating by constant current devices. These leads formed a four-terminal network for measuring the resistance of the loop. The two outer leads (1 and 4) carried the current, which was measured with a precision dc ammeter, and the potential drop was measured across the two inner leads (2 and 3) with a sensitive suspension galvanometer. The assembly was electropolished and the emitting point was formed. The tube was baked, under vacuum, for 12 hr at 450°C. The loop was outgassed by resistive heating, and a tantalum getter was repeatedly flashed until a pressure of 2×10^{-10} mm Hg was attained. After the tube was sealed off from the vacuum system the emission pattern was examined to ensure the absence of extraneous emitting sources.

The heating and measuring circuit is shown in Fig. 2. The entire circuit above the electrometer marked K was carefully shielded and the storage battery, resistors, and switch were enclosed in a grounded case. Shielded wires were brought into the liquid helium Dewar to the field emission tube, and the tube itself was wrapped with aluminum foil connected to the cable shield. Shielding of the tube was necessary because bubbling of the liquid nitrogen in the outer Dewar produced noise signals. A temperature-resistance relationship for the loop was obtained by calibration at several fixed temperature points.

After the addition of the liquid helium to the Dewar containing the field emission tube, the emitter tip was flashed. The temperature of the loop and emitter was set by adjusting the current through the loop with a variable resistor in series with the storage battery.

The initial current of 2.6×10^{-10} amp was set with the emitter at 4.2°K. This current was bucked out to zero on the electrometer, as shown in Fig. 2, with fixed battery voltage and a precision potentiometer used as a voltage source. Heating of the loop caused a change in emission current. This was read on the electrometer directly, the shunt of the electrometer being kept fixed.

[8] R. Klein and J. Arol Simpson, Rev. Sci. Instr. 29, 770 (1958).

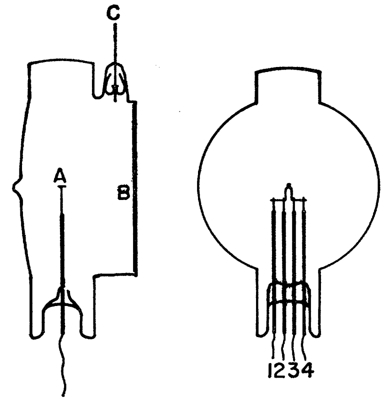

FIG. 1. Field emission tube. A is the emitter tip, B is the phosphor screen deposited on a tin oxide coated surface, and C is the high voltage lead-in. The leads *1,2,3,4* form a four-terminal network for resistance measurement.

Since thermal expansion of the emitter as the temperature is increased causes the field to decrease (increase of emitter radius with fixed voltage) it was necessary to make corrections to obtain the current change under constant field conditions. The correction is easily made since for constant field $\Delta r/r=\Delta V/V=\int_0^T\alpha dt$, where α is the linear coefficient of expansion, r is the radius of the emitter, Δr is the change in radius, V is the applied voltage to the emitter, and ΔV is the voltage change.[9] A calibration of the emitter was made by measuring a Δi_c corresponding to a ΔV. For ΔV small relative to V, Δi_c is proportional to ΔV. The $\Delta r/r$ for each measured temperature was calculated and from this the value of Δi_c to be added to the measured current change was determined.

The thermal expansion also increases the area which

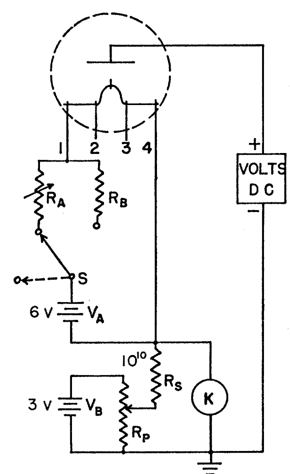

FIG. 2. Circuit diagram for current increment measurements. V_A is a storage battery for the heating current, R_A is a variable resistor, R_B is a fixed resistor, V_B is the bucking voltage source, R_P is a precision potentiometer, K is an electrometer voltmeter, and R_P is the voltmeter shunt.

[9] This was kindly pointed out to us by Dr. R. D. Young of Pennsylvania State University (now at the National Bureau of Standards). It is evident that the expression $\Delta v/v=\Delta r/r$ is strictly true only if the entire configuration involving point, tube, and circuit are subjected to the same scale change. However, the radius of the point is sufficiently small with respect to tip-to-screen distance (of the order of 1 to 10^5), that the small perturbation introduced by the difference in scale change between tip radius and tip-to-screen distance (only the tip is heated) can be ignored.

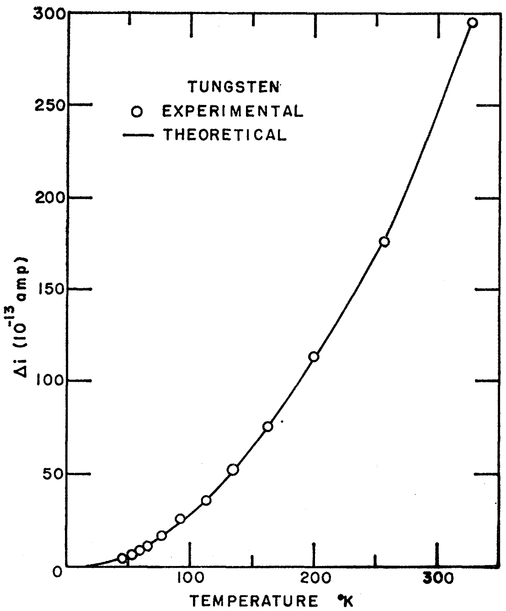

FIG. 3. Change in emission current at constant field as a function of temperature. The solid curve is the theoretical temperature dependence.

correspondingly increases the emission current. This change is such that $\Delta A/A=\Delta i_c'/i$. For increased temperature this correction would be subtracted from Δi, so that $\Delta i=\Delta i_m+\Delta i_c-\Delta i_c'$. The correction due to the area change, however, is small enough to be ignored even at the highest temperatures used in this experiment.

IV. RESULTS

An emission current of 2.6×10^{-10} amp was established at 4.2°K. This required a voltage of 4060 v between the emitter and anode which remained fixed during the entire series of measurements. The change in emission current was determined several times at a given temperature by switching back and forth between that temperature and 4.2°K. The Δi used was an average of these measurements. The noise level was approximately 1×10^{-13} amp, and a current change of one and one-half this amount, corresponding to a change of about one part in 2000 in the initial current, could be easily measured.

We have plotted the change in current, corrected for thermal expansion, as a function of temperature for the tungsten emitter in Fig. 3. These same data are plotted in Fig. 4 as a function of $(T^2-4.2^2)$. No correction has been made for the change in work function with temperature because of lack of data and the uncertainty existing as to the magnitude of the temperature variation of the work function in the range of 4.2 to 400°K considered here. The slopes of the Fowler-Nordheim plots at 4.2°K and 300°K for the tungsten emitter were found to be the same within the precision of these measurements.

V. DETERMINATION OF THE FIELD AT THE EMITTER SURFACE

The slope of the line of Fig. 4 can be written, using Eq. (11), as[5]

$$\Delta i/[T^2-(4.2)^2]=i(4.2)(\pi k)^2/6d^2 = 1.28\times10^8 i(4.2)\bar{\phi}/\bar{F}^2. \quad (12)$$

Substitution of an average value of the work function permits the evaluation of the field at the surface of the emitter, since $t(y)$ in the expression for d [Eq. (4)] differs very little from one. An iterative procedure would provide for greater consistency, but this is hardly justified. The value of the field is not constant over the surface of the emitter, but drops off with angular distance from the apex; this effect becomes greater when the tip shape becomes less spherical and more hyperbolic. The field calculated by the current-increment method (we shall refer to this as CIM) described above refers to an average field, but an average strongly weighted in favor of the high field areas.

For tungsten at 4060 v the slope gave a field of $(2.34\pm0.03)\times10^7$ v/cm. An electron microscope picture[10] of the tungsten tip at a magnification of approximately 13 500 is shown in Fig. 5. The radius was measured as 3000±100 A. Drechsler and Henkel[11] give the following formula for computing the field strength

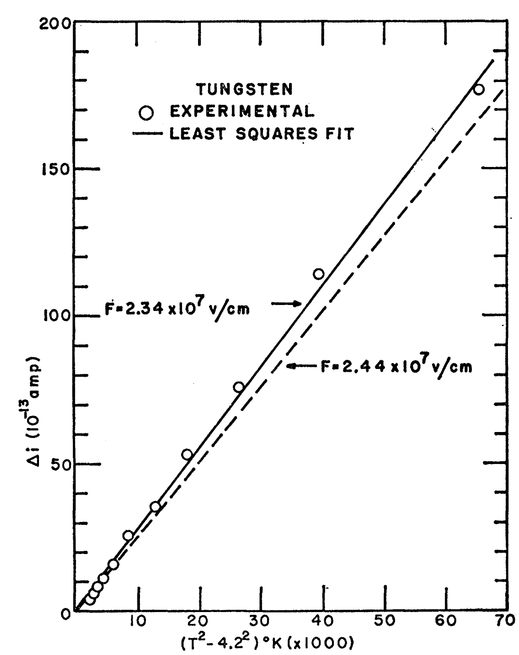

FIG. 4. Change in emission current at constant field as a function of the square of the temperature. The dashed curve is included to illustrate the sensitivity of the CIM method to small field differences.

---

[10] We are indebted to Mr. D. Ballard of the Metallurgy Division, National Bureau of Standards for taking these pictures.

[11] M. Drechsler and E. Henkel, Z. angew. Physik 6, 341 (1954).

at the tip:

$$F=\frac{V}{r}\left[\alpha+\frac{2(1-\alpha)}{\ln(4l/r)}\right], \tag{14}$$

where V is the applied voltage, r is the tip radius, l is the distance between tip and screen, and α is a form factor given in their Fig. 2. By comparison with their figure a value of $\alpha=0.05$ was found, and for $V=4060$ v, $r=3\times10^{-5}$ cm, and $l=2.5$ cm, a value of $F=2.7\times10^{7}$ v/cm was calculated. Considering the uncertainty in the measurements (magnification, tip profile, radius of curvature at the apex, tip to screen distance, etc.) the agreement with the CIM value of 2.34×10^{7} v/cm is remarkably good.

The CIM method is quite sensitive to small field changes. In Fig. 3 the solid line was plotted using the CIM value of field. The closeness of fit with the measured points is excellent. The dashed curve in Fig. 4 has been plotted for tungsten using a field of 2.44×10^{7} v/cm to illustrate the sensitivity of the method. It is estimated that a change of 0.03×10^{7} v/cm could easily be distinguished.

The plot of Δi as a function of the square of the temperature for tungsten over the region from 4.2°K to 400°K is linear. This supports the correctness of the theory of temperature dependence in the field emission region.

It has been demonstrated that the field at the emitter tip can be measured with high sensitivity using the current increment method (CIM). Besides sensitivity there are advantages of in-place determination and of being able to make measurements under controlled tip coverages and tip shapes not amenable to electrostatic field calculations.

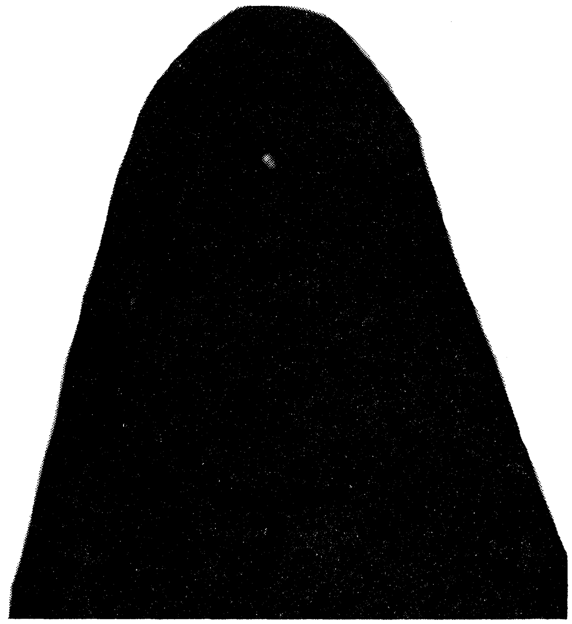

FIG. 5. Electron microscope picture of the tungsten emitter. Magnification approximately 13 500×.

ACKNOWLEDGMENT

We would like to thank Dr. L. Marton, Chief of the Electron Physics Section of the National Bureau of Standards for making available to us the facilities with which these experiments were performed.

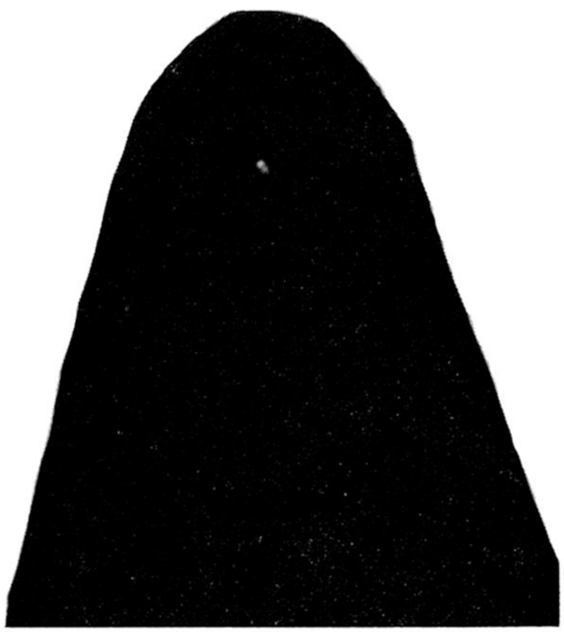

FIG. 5. Electron microscope picture of the tungsten emitter. Magnification approximately 13 500×.