Resonance Model of $\Lambda-K^0$ Production*

Akira Kanazawa†
Department of Physics, Purdue University, Lafayette, Indiana

(Received March 30, 1961)

A resonance model is proposed to explain the excitation function, the angular distribution, and the large polarization of Λ in the reaction $p+\pi^- \rightarrow \Lambda+K^0$. It is assumed that there exists a low angular momentum resonance in the channel $p+\pi^- \rightarrow$ (resonant state) $\rightarrow \Lambda+K^0$.

There are five real parameters in this model. Two of these are the coupling constants of the usual interactions. The other three are the position, half-width, and height of the assumed resonance. With reasonable choices of parameters a fairly good fit is obtained, for both a scalar and a pseudoscalar K meson, to the experimental data in the interval 910 to 1300 Mev of the pion kinetic energy in the laboratory system.

I. INTRODUCTION AND SUMMARY

There are three remarkable features observed in the reaction $p+\pi^- \rightarrow \Lambda+K^0$ in the energy interval 910–1300 Mev of the incident pion kinetic energy.[1] They are:

(a) The total cross section rises from threshold to a peak of about 0.8 mb near 960 Mev and then drops again to about 0.3 mb in the interval 1100–1300 Mev.

(b) The angular distribution of Λ is similar in the interval 910–1300 Mev and markedly peaked backwards.

(c) A large asymmetry is observed in the decay of Λ's produced in the reaction in the interval 910–990 Mev. Conclusive evidence concerning the asymmetry factor α in Λ decay is limited to the relation $\alpha\bar{P} \geqslant 0.73 \pm 0.14$, where $\bar{P}$ is the average of the polarization over the energy and angle.

The sign of α is still uncertain, but a more recent and statistically more reliable experiment indicates that it is negative.[2]

There have been several investigations[3] based on the possible existence of a new K' meson or a sharp $K-\pi$ resonance, explaining the angular distribution. The existence of a K' meson does not, however, account directly for either the large polarization or the peak in the total cross section, at least within the approximation considered by the authors of reference 3.

In this paper we try to give an explanation of all three experimental phenomena on the basis of the assumption that there is a resonant contribution in the s' integrals of the dispersion relations (8) in Sec. 2 for the invariant production amplitudes (see Fig. 1).

There are five real parameters in this model. Two of them are the coupling constants of the usual d'Espagnat-Prentki interactions. The other three are the position, half-width, and height of the assumed resonance.

The main results of the present paper are as follows:

(1) A satisfactory fit with experimental data is obtained over the energy range in question [Figs. 3 to 5(b)]. The values of parameters chosen are shown in Table II for both a pseudoscalar and a scalar K meson.

(2) If this resonant contribution is due to the suggested resonance in the $\Lambda-K$ scattering, it will be at about 100 Mev of K-meson kinetic energy in the laboratory system, with a half-width of about 50 Mev.

(3) The total angular momentum and parity (relative to the $\pi-N$) of the resonant state can be ($j=\frac{1}{2}$, odd) or ($j=\frac{3}{2}$, odd) if the K meson is pseudoscalar and ($j=\frac{1}{2}$, even) or ($j=\frac{3}{2}$, even) if the K meson is scalar.

(4) The resonant contribution cannot be attributed to the higher energy resonances observed in $\pi-N$ scattering.[4]

(5) A negative value for $\bar{P}$ is preferable, though the other case is not entirely rejected.

(6) Even relative parity between Σ and Λ is preferable.

(7) A K' meson (or $K-\pi$ resonance) would enhance the peak in the case of a pseudoscalar K meson, though its existence is completely unnecessary in order to explain the data in the case of even Σ parity.

The kinematics is given in Sec. II. In Sec. III we develop the general formalism of the resonance model. The last section is devoted to numerical analysis.

II. KINEMATICS

Consider the process

$$p+\pi^- \rightarrow \Lambda+K^0. \tag{1}$$

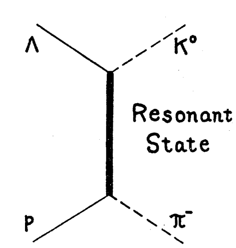


Fig. 1. The assumed resonant state in the s channel.

* This work is supported by the National Science Foundation.

† On leave from Hokkaido University, Sapporo, Japan.

[1] J. Steinberger, *1958 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958). F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, and J. Steinberger, Nuovo cimento **10**, 468 (1958).

[2] R. Birge and W. Fowler, Phys. Rev. Letters **5**, 254 (1960).

[3] A. Pais, Phys. Rev. **112**, 624 (1958). M. Gell-Mann, *Proceedings of the 1960 Annual International Conference on High-Energy Physics of Rochester* (Interscience Publishers, Inc., New York, 1960). J. Tiomno, A. L. L. Videira, and N. Zagury, Phys. Rev. Letters **6**, 120 (1961).

[4] An investigation has been done by T. Sakuma and S. Furui, taking into account the $\pi-N$ resonances in addition to the Born terms (private communication).

The four-momentum and the mass of the p (Λ) are denoted by p (p') and m (m'), while those of the π^- (K^0) are denoted by k (k') and μ (μ'), respectively. The S-matrix element of the process is given by

$$S=-i(2\pi)^4\delta(p'+k'-p-k)\left(\frac{mm'}{p_0p_0'k_0k_0'}\right)^{\frac{1}{2}}T, \quad (2)$$

with

$$T=\bar{u}_\Lambda(p')\Gamma[A+\tfrac{1}{2}i\gamma(k+k')B]u_p(p), \quad (3)$$

where u_p and u_Λ are the Dirac sponsors of p and Λ, respectively. A and B are invariant functions of three variables,

$$s=-(p+k)^2,\quad t=-(k-k')^2,\quad \text{and}\quad u=-(p-k')^2,$$

which satisfy the restriction $s+t+u=m^2+m'^2+\mu^2+\mu'^2$. Γ is 1 if the K meson is pseudoscalar and γ_5 if the K meson is scalar, where we have adopted the convention of the same parity for p and Λ.

In the center-of-mass system the amplitude T can be written in the following form:

$$T=\left\langle\Lambda\left|f_1+i\frac{\boldsymbol{\sigma}\cdot\mathbf{k}'\times\mathbf{k}}{\bar{k}\bar{k}'}f_2\right|p\right\rangle \quad \text{for } \Gamma=1, \quad (4)$$

or

$$T=\left\langle\Lambda\left|\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\bar{k}}f_1+\frac{\boldsymbol{\sigma}\cdot\mathbf{k}'}{\bar{k}'}f_2\right|p\right\rangle \quad \text{for } \Gamma=\gamma_5. \quad (4')$$

Here $|p\rangle$ and $|\Lambda\rangle$ stand for the Pauli spinors of p and Λ, respectively. $\mathbf{k}$ ($\mathbf{k}'$) is the three-momentum of π^- (K^0) and $\bar{k}$ ($\bar{k}'$) is its magnitude. f_1 and f_2 are linear combinations of the invariant amplitudes A and B with real coefficients. They are explicitly given by

$$\begin{aligned} f_1&=NN'(A-y_eB)+f_2\cos\theta,\\ f_2&=-NN'xx'(A+z_eB), \quad \text{for } \Gamma=1, \end{aligned} \quad (5)$$

or

$$\begin{aligned} f_1&=NN'x(A+y_0B),\\ f_2&=NN'x'(-A+z_0B), \quad \text{for } \Gamma=\gamma_5, \end{aligned} \quad (5')$$

with

$$\begin{aligned} &N=[(E+m)/2m]^{\frac{1}{2}},\quad N'=[(E'+m')/2m']^{\frac{1}{2}},\\ &x=\bar{k}/(E+m),\quad x'=\bar{k}'/(E'+m'),\\ &y_e=W-\tfrac{1}{2}(m+m'),\quad y_0=W-\tfrac{1}{2}(m'-m),\\ &z_e=W+\tfrac{1}{2}(m+m'),\quad z_0=W+\tfrac{1}{2}(m'-m). \end{aligned} \quad (6)$$

Here $W=\sqrt{s}$ is the total center-of-mass energy of the system and $E(E')$ is the energy of p (Λ). θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$.

The differential cross section and the polarization $P(\theta)$ are given in terms of f_1 and f_2 by

$$\begin{aligned} \frac{d\sigma}{d\Omega}&=\frac{1}{(4\pi)^2}\frac{mm'}{W^2}\frac{\bar{k}'}{\bar{k}}(|f_1|^2+|f_2|^2\sin^2\theta),\\ P(\theta)&=\frac{2\sin\theta\,\mathrm{Im}(f_1f_2^*)}{|f_1|^2+|f_2|^2\sin^2\theta}, \quad \text{for } \Gamma=1, \end{aligned} \quad (7)$$

or

$$\begin{aligned} \frac{d\sigma}{d\Omega}&=\frac{1}{(4\pi)^2}\frac{mm'}{W^2}\frac{\bar{k}'}{\bar{k}}[|f_1|^2+|f_2|^2+2\,\mathrm{Re}(f_1f_2^*)\cos\theta],\\ P(\theta)&=2\sin\theta\,\mathrm{Im}(f_1f_2^*)/\\ &\quad[|f_1|^2+|f_2|^2+2\,\mathrm{Re}(f_1f_2^*)\cos\theta], \quad \text{for } \Gamma=\gamma_5. \end{aligned} \quad (7')$$

The polarization is positive if the spin of Λ is parallel to $\mathbf{k}'\times\mathbf{k}$, and negative if antiparallel.

It is worthwhile to note here that in order to obtain a large polarization f_1 and f_2 must be of the same order of magnitude and nearly out of phase in either case of K-meson parity.

III. RESONANCE MODEL

Let us start with investigating what behavior might be expected for the invariant amplitudes in order to reproduce the experimental situation mentioned in Sec. I.

We assume, after Mandelstam,[5] that the invariant amplitude A satisfies the dispersion relation

$$\begin{aligned} A(s,t,u)&=\frac{R_s}{s-m^2}+\frac{R_u}{u-m_\Sigma^2}\\ &+\frac{1}{\pi}\int_{(m+\mu)^2}\frac{ds'}{s'-s}\left[\frac{1}{\pi}\int_{(\mu+\mu')^2}dt'\frac{\rho_{12}(s',t')}{t'-t}\right.\\ &\left.+\frac{1}{\pi}\int_{(m'+\mu)^2}du'\frac{\rho_{13}(s',u')}{u'-u}+\rho_1(s')\right]+\cdots, \end{aligned} \quad (8)$$

and the amplitude B satisfies a similar relation. The dots represent the other double- and single-integral terms that occur in the Mandelstam representation. If there exists another K' meson,[3] we must add a corresponding pole term $R_t/(t-m_{K'}^2)$ to (8), where $m_{K'}$ is the mass of the K' meson.

Invariance under time reversal requires the reality of all the residues R_s, R_u, and R_t and of the weight functions ρ's in all the integrals. Now it is clear from (5) and (5′) that the Born terms alone, even taking into account the K' meson, fail to give any polarization irrespective of the K-meson parity, since f_1 and f_2 are, to this order, just in phase. The large polarization indicates that the contribution from the s' integral in (8) must be large, at least over the energy range in question, since only the s' integral can contribute an imaginary part to the amplitudes in the channel in question.

Now we assume that there is a resonant state with definite angular momentum and parity in the s channel, schematically shown in Fig. 1, whose contribution dominates the integral terms in (8), or explicitly that

[5] S. Mandelstam, Phys. Rev. **112**, 1344 (1958) and **115**, 1741 and 1752 (1959).

TABLE I. $a_1(s,\theta)$ and $a_2(s,\theta)$ for several choices of the total angular momentum j and parity P of the resonant state. The parity is defined relative to the $\pi-p$ intrinsic parity. l is the orbital angular momentum of the resonant state if it is involved in the $\Lambda-K$ scattering. $C(s)$ is a certain slowly varying real function of s.

		Pseudoscalar K meson			Scalar K meson		
j	P	l	a_1	a_2	l	a_1	a_2
$\frac{1}{2}$	$+$	S	$C(s)$	0	P	0	$C(s)$
	$-$	P	$C\cos\theta$	C	S	C	0
$\frac{3}{2}$	$+$	D	$C(3\cos^2\theta-1)$	$-3C\cos\theta$	P	$2C\cos\theta$	C
	$-$	P	$2C\cos\theta$	C	D	$C(3\cos^2\theta-1)$	$3C\cos\theta$
$\frac{5}{2}$	$+$	D	$C(3\cos^2\theta-1)$	$3C\cos\theta$	F	$2C\cos\theta$	$-C(5\cos^2\theta-1)$
	$-$	F	$C(20\cos^2\theta+9)\cos\theta$	$C(20\cos^2\theta-11)$	D	$-C(5\cos^2\theta-1)$	$2C\cos\theta$

the invariant amplitudes can be approximated by

$$A=A_p+\frac{1}{\pi}\int_{(m+\mu)^2} ds'\frac{\rho^A(s',\theta)}{s'-s},$$
$$B=B_p+\frac{1}{\pi}\int_{(m+\mu)^2} ds'\frac{\rho^B(s',\theta)}{s'-s}, \tag{9}$$

where A_p and B_p represent the pole (Born) terms. ρ^A and ρ^B are real functions of s' and θ, which have a peak at the resonance energy $s'=s_0$. If we, further, assume the Breit-Wigner form to ρ's, (9) can be reduced to

$$A(s,\theta)=A_p+\alpha(s,\theta)\frac{\Gamma_0^2}{\pi}\int_{(m+\mu)^2} ds' \times\frac{1}{(s'-s)[(s'-s_0)^2+\Gamma_0^2]},$$
$$B(s,\theta)=B_p+\beta(s,\theta)\frac{\Gamma_0^2}{\pi}\int_{(m+\mu)^2} ds' \times\frac{1}{(s'-s)[(s'-s_0)^2+\Gamma_0^2]}. \tag{10}$$

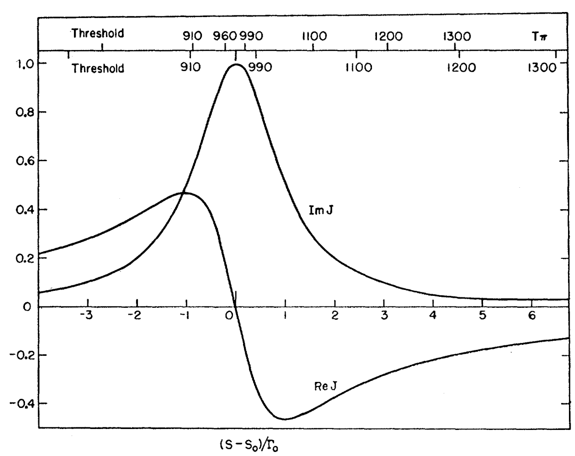


FIG. 2. The real and imaginary parts of J as a function of $(s-s_0)/\Gamma_0$. The upper scale of T_π corresponds to the case $s_0=152.8$ and $\Gamma_0=7$, while the lower scale is for $s_0=151.5$ and $\Gamma_0=5$.

Here $\alpha(s,\theta)$ and $\beta(s,\theta)$ are slowly varying functions of s. Γ_0 corresponds to the half-width of the assumed resonance.

Putting (10) into (5) or (5′), we obtain finally, for $\Gamma=1$,

$$f_1=f_{p1}+f_{p2}\cos\theta+a_1(s,\theta)J(s,s_0,\Gamma_0),$$
$$f_2=f_{p2}+a_2(s,\theta)J(s,s_0,\Gamma_0), \tag{11a}$$

with

$$f_{p1}=NN'(A_p-y_eB_p),$$
$$f_{p2}=-NN'xx'(A_p+z_eB_p),$$
$$a_1(s,\theta)=NN'(\alpha-y_e\beta)+a_2(s,\theta)\cos\theta,$$
$$a_2(s,\theta)=-NN'xx'(\alpha+z_e\beta), \tag{12a}$$
$$J(s,s_0,\Gamma_0)=\frac{\Gamma_0^2}{\pi}\int_{(m+\mu)^2}\frac{ds'}{(s'-s)[(s'-s_0)^2+\Gamma_0^2]},$$

or, for $\Gamma=\gamma_5$,

$$f_1=f_{p1}+a_1(s,\theta)J(s,s_0,\Gamma_0),$$
$$f_2=f_{p2}+a_2(s,\theta)J(s,s_0,\Gamma_0), \tag{11b}$$

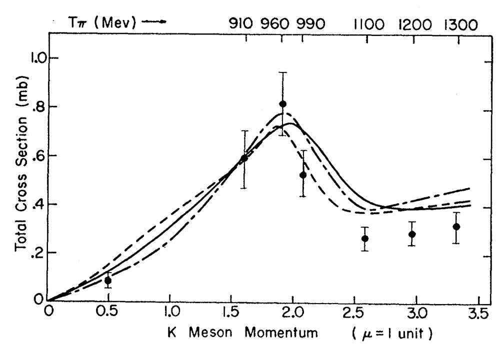


FIG. 3. The total cross section of the reaction $p+\pi^-\rightarrow\Lambda+K^0$ in the energy range from the threshold to 1300 Mev of the pion kinetic energy T_π (lab system). The unbroken curve is the calculated cross section in the case of a pseudoscalar K meson with a ($j=\frac{1}{2}$, odd) resonant state, while the broken curve is for a ($j=\frac{3}{2}$, odd) state. The dot-dash curve stands for the cross section of a scalar K meson with a ($j=\frac{1}{2}$, even) resonant state. The cross section with a ($j=\frac{3}{2}$, even) state is quite similar to the corresponding one for the pseudoscalar K meson. The values of parameters chosen are given in Table II. The experimental data are given in references 1.

TABLE II. The choices of parameters in the case of even Σ parity. The $\pi-N$ coupling constant is fixed as $f^2/4\pi=15$. The unit $\mu=1$ is used. The ratio $C/f_{p1}(s=s_0,\theta=\pi/2)$ gives an idea of the magnitude of the resonant contribution. The K-meson kinetic energy (lab system) T_{K0} and ΔT_{K0}, corresponding to s_0 and $\pm\Gamma_0$, are also shown.

K meson	j	$G^2/4\pi$	$f'G'/fG$	$(C/fG)\times10^3$	$C/f_{p0}(s_0,\pi/2)$	s_0	Γ_0	T_{K0} (Mev)	ΔT_{K0} (Mev)
ps	$\frac{1}{2}$	5.0	-0.538	10.0	0.465	152.8	7	106.5	±30
	$\frac{3}{2}$	5.0	-0.538	6.8	0.316	151.5	5	91.5	±25
s	$\frac{1}{2}$	5.0	$+0.570$	10.4	1.07	152.8	7	106.5	±30
	$\frac{3}{2}$	5.0	$+0.570$	7.0	0.722	151.5	5	91.5	±25

with

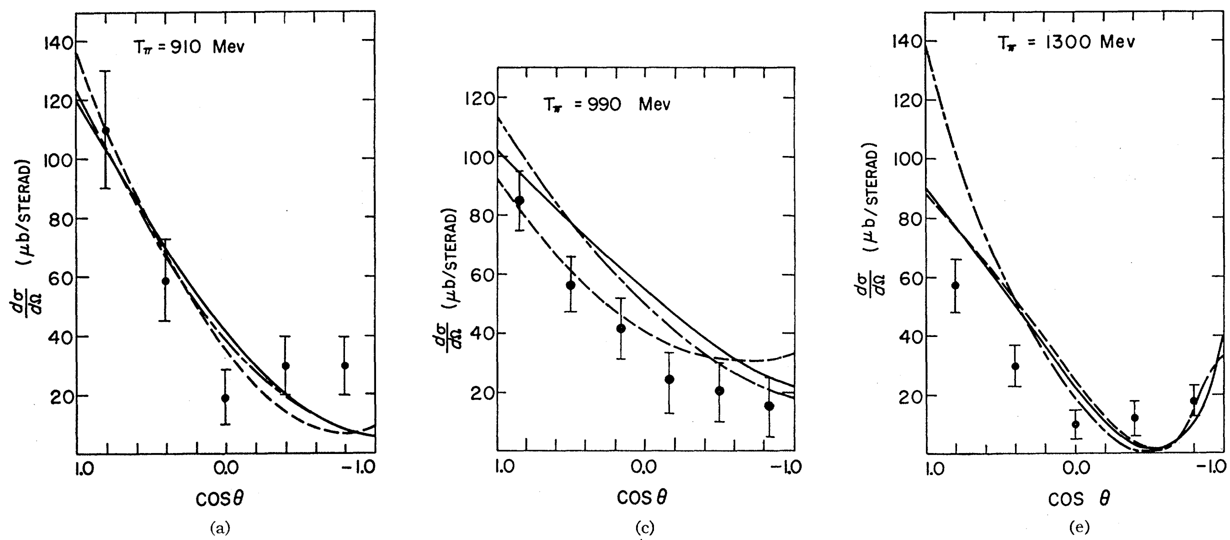
$$f_{p1}=NN'x(A_p+y_0B_p),\quad f_{p2}=NN'x'(A_p+z_0B_p),$$
$$a_1(s,\theta)=NN'x(\alpha+y_0\beta),\quad a_2(s,\theta)=NN'x'(\alpha+z_0\beta),\quad (12b)$$

and the same J as in (12a).

The angular momentum and parity of the resonant state fixes the angular dependence and relative magnitude of a_1 and a_2 in (11a) and (11b). They are given in Table I for several angular momentum states.

IV. NUMERICAL ANALYSIS

We discuss first the Born terms f_{p1} and f_{p2} in (11a) and (11b). We are here concerned with four vertices (πNN), $(\pi\Sigma\Lambda)$, $(KN\Lambda)$, and $(KN\Sigma)$. The renormalized



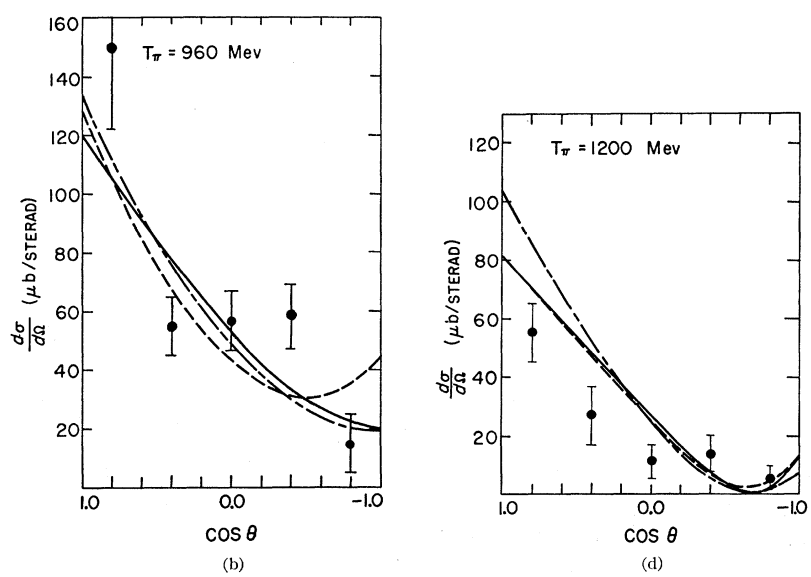


FIG. 4. (a)–(e) The differential cross sections for the pion kinetic energies $T_\pi=910$, 960, 990, 1200, and 1300 Mev.[1] The notation for the various curves are the same as in Fig. 3.

coupling constants corresponding to these vertices are denoted by f, f', G, and G', respectively. f and G, and f' and G', always appear as products in the Born terms. For the four combinations of K and Σ parities, they are given by

(ps, even):

$$\begin{aligned} f_{p1}&=NN'[-(W-m)F(s)+(W+m_\Sigma-m-m')\times F'(u)],\\ f_{p2}&=-NN'xx'[(W+m)F(s)-(W-m_\Sigma+m+m')F'(u)]; \end{aligned} \tag{13a}$$

(ps, odd):

$$\begin{aligned} f_{p1}&=NN'[-(W-m)F(s)-(W-m_\Sigma)F'(u)],\\ f_{p2}&=-NN'xx'[(W+m)F(s)+(W+m_\Sigma)\times F'(u)]; \end{aligned} \tag{13b}$$

(s, even):

$$\begin{aligned} f_{p1}&=-NN'x[(W+m)F(s)+(W+m_\Sigma-m'+m)F'(u)],\\ f_{p2}&=-NN'x'[(W-m)F(s)+(W-m_\Sigma+m'-m)F'(u)]; \end{aligned} \tag{13c}$$

(s, odd):

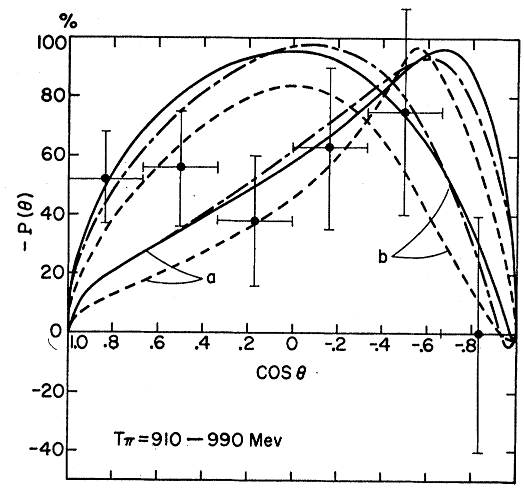
$$\begin{aligned} f_{p1}&=-NN'x[(W+m)F(s)-(W-m_\Sigma-m'+m)F'(u)],\\ f_{p2}&=-NN'x'[(W-m)F(s)-(W+m_\Sigma+m'-m)F'(u)]; \end{aligned} \tag{13d}$$

with $F(s)=Gf/(s-m^2)$, $F'(u)=G'f'/(u-m_\Sigma^2)$.

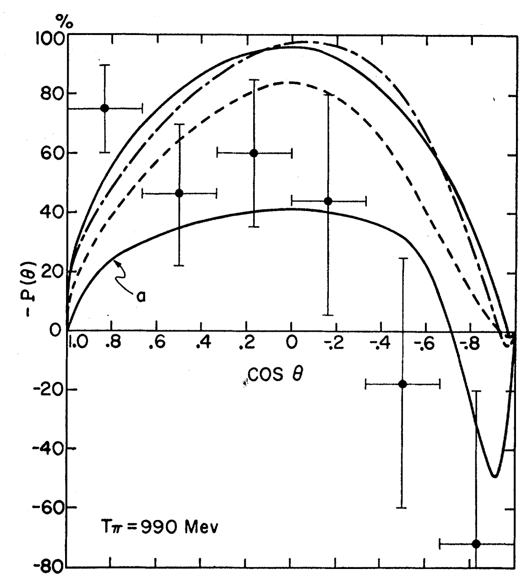
Now we examine the case of pseudoscalar K meson in detail. First it should be noted that f_{p2} is negligibly small compared with f_{p1} over the energy range considered, simply because of the kinematical factor xx'. We can see, further, that the only complex quantity in (11a) is the integral J, the real and imaginary parts of which are shown in Fig. 2. Then, if we neglect f_{p2}, the differential cross section and the polarization become, except for over-all kinematical factors,

$$\frac{d\sigma}{d\Omega}\propto[f_{p1}^2+2a_1f_{p1}\,\mathrm{Re}J+(a_1^2+a_2^2\sin^2\theta)|J|^2], \tag{14}$$
$$P(\theta)\propto-2a_1f_{p1}\sin\theta\,\mathrm{Im}J/(d\sigma/d\Omega).$$

Now it is clear from (14) and Table I that the only possible quantum number of the resonant state is ($j=\frac{1}{2}$, odd) or ($j=\frac{3}{2}$, odd), since the ($j=\frac{1}{2}$, even) state gives no appreciable polarization, and the ($j=\frac{3}{2}$, even) and the higher angular momentum states give a wrong angular dependence for the polarization and a poor angular distribution, too. This excludes the possible identification of this resonance with one of the known higher energy resonances observed in pion-nucleon scattering. A similar situation results in the scalar K-meson case. Thus, the assumed resonant contribution might be attributed to a resonance in $\Lambda-K$ scattering at low energy.



(a)



(b)

FIG. 5. (a) The polarization of Λ in the energy interval 910–990 Mev of the pion kinetic energy.[1] The group a of curves corresponds to the energy 910 Mev, while the group b is that for 990 Mev. The notation for the various curves is that given in Fig. 3. The data are not $P(\theta)$ but $\alpha P(\theta)$, where α is the asymmetry factor in the decay of the Λ. (b) The polarization of Λ at 990-Mev pion kinetic energy[1]. The curve a is associated with the energy 1240 Mev, which shows the typical figure for an energy higher than the resonance energy.

There are five real parameters in this model. They are fG, $f'G'$, s_0, Γ_0, and $C(s)$, the last of which is a slowly varying function of s and is assumed to be constant for simplicity.

The markedly forward peak (backward for Λ) of the angular distribution must be mainly due to this behavior of the Born term f_{p1}, since otherwise the distribution turns out to be nearly symmetric with respect to $\cos\theta$. In this respect, even parity of the Σ is preferable to odd parity, since with the even parity we can easily obtain the strong forward peak of f_{p1} over the energy range in question if the ratio $f'G'/fG$ is adjusted properly.

It is worthwhile to note here that a K' meson would be helpful in obtaining the peak only in this pseudoscalar K-meson case, though it is not at all necessary for the case of even Σ parity.

In the case of even Σ parity, a fairly good fit with experimental data is obtained by fixing the parameters as shown in Table II. The choice of either ($j=\frac{1}{2}$, odd) or ($j=\frac{3}{2}$, odd) is satisfactory. The fit can be seen in Figs. 3 to 5 (b).

The resonance energy chosen corresponds to 90–100 Mev for K-meson kinetic energy (lab system) in the $\Lambda-K$ scattering. The half-width is about 50–60 Mev. The resonant contribution is the same order of magnitude as the Born terms, which is well indicated by the ratio C/f_{p1} ($s=s_0$, $\theta=\pi/2$). As for the coupling constants, if we fix $f^2/4\pi=f'^2/4\pi=15$, then we obtain a best fit with $G^2/4\pi\sim 5$ and $G'^2/4\pi\sim 1.5$,[6] which are reasonable.

With a positive ratio C/fG (Table II), we have a negative mean polarization $\bar{P}$, as shown in Figs. 5(a) and 5(b). A negative ratio with a positive $\bar{P}$ is only possible with a rather poor over-all fit to the data.

Finally we briefly discuss the case of a scalar K meson. In case of even Σ parity, Eq. (13c) indicates that f_{p2} is about ten times smaller than f_{p1} over the energy range in question. Further, if we note that each of two terms in f_{p1} of (13c) is almost the same numerically as the corresponding term in f_{p1} of (13a), except for their relative sign, we can see that the analysis may proceed similarly to that in the pseudoscalar case. In fact, we can get the same good fit with almost the same absolute values of the parameters for each of the corresponding two resonant states (Table II). In Figs. 3 to 5(b), only the fit in the case of ($j=\frac{1}{2}$, even) is shown for simplicity. It is very difficult to obtain a good fit with odd Σ parity in this scalar K-meson case.

ACKNOWLEDGMENTS

The author should like to thank Professor M. Sugawara for discussions and for the hospitality extended to him at Purdue University. He wishes to express his gratitude to Professor R. Spitzer for his careful reading of a draft of the present paper. He also wishes to thank Professor S. Gartenhaus and Professor J. Uretsky for conversations and Dr. T. Sakuma of Hokkaido University, Sapporo, Japan, for a communication.

[6] See, for example, R. H. Capps, Phys. Rev. **121**, 291 (1961).