constructed by excluding the zero momentum state from the original space.

If we construct the Hamiltonian H' (or any operator) in this V space such that

$$(\Psi_{N;\{\eta_{k'}\}}, H\Psi_{N;\{\eta_k\}})=(\Phi_{\{\eta_{k'}\}}, H'\Phi_{\{\eta_k\}}), \quad \text{(A3)}$$

the operator $a_0^\dagger$ or a_0 in H can be seen to be replaced by $[N-\sum_k \eta_k+\epsilon]^{\frac{1}{2}}$, where ϵ is a finite number. For example,

$$f(a_k^\dagger,a_k)a_0^\dagger a_0 \to f(a_k^\dagger,a_k)(N-\textstyle\sum_k \eta_k) = f(a_k^\dagger,a_k)[N-\sum_k a_k^\dagger a_k],$$

$$f(a_k^\dagger,a_k)a_0^2 \to f(a_k^\dagger,a_k) \times[(N-\textstyle\sum_k \eta_k)(N-\sum_k \eta_k-1)]^{\frac{1}{2}},$$

$$(a_0^\dagger)^2 f(a_k^\dagger,a_k) \to [(N-\textstyle\sum_k \eta_k+1)(N-\sum_k \eta_k+2)]^{\frac{1}{2}} \times f(a_k^\dagger,a_k).$$

In any case, ϵ can be neglected compared with $N-\sum_k \eta_k$ if $N-\sum_k \eta_k$ is of the order of N, so that we obtain the Hamiltonian (4), (4a).

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Test of Global Symmetry in Pion-Baryon Interactions by K^-+p Reactions*

JOGESH C. PATI†

Department of Physics, University of Maryland, College Park, Maryland, and Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California

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Under the hypothesis that the K-meson interactions do not mask the symmetries of the pion-baryon interactions appreciably, the branching ratios of the K^-+p reactions are studied to test the validity of global symmetry. The T^{-1}-matrix formalism of Matthews and Salam is adopted to calculate the branching ratios. The new Dalitz-Tuan solutions for $\bar{K}N$ scattering lengths, which incorporate the (K^+,K^0) mass difference and the new branching ratios of the various K^-+p reactions, presented at Kiev, are adopted in the analysis. The errors in the experimental branching ratios are so chosen as to satisfy the Amati-Vitale inequality. It is found that the a^- and b^+ (also a^+, though poorly) Dalitz-Tuan solutions can explain the branching ratios for K^- captured at rest. The extension of the analysis to 30-Mev incident K^- mesons under the zero-range approximation leads to very poor agreement with experiments.

I. INTRODUCTION

It is of great interest to ascertain whether the very strong pion-baryon interactions possess any symmetry higher than charge independence. It is now clear that experiments exclude[1] the possibility of very high symmetry in both pion and K-meson interactions. So the symmetries of the pion interactions, even if they exist in the bare Lagrangian, could be distorted badly by the K-meson interactions. If so, such symmetries are not useful (except, possibly, at very high energies), since we cannot calculate accurately the consequences of strong couplings. In order to test the usefulness of the proposed symmetries of the pion interactions, we would therefore consider the possibility that the K-meson interactions may not be strong enough to break the symmetries of the pion interactions appreciably, even though, to some, this may be of academic interest only. We would apply this hypothesis specifically to the K^-+p reactions.

It was first pointed out by Amati and Vitale[2] that the low-energy K^-+p reactions provide a good tool for testing the hypotheses of the restricted and global symmetries,[3] if the K-meson interactions do not break them badly. These authors established an inequality[4] involving the branching ratios of the various K^-+p reactions on the basis of the restricted symmetry alone. This inequality will be referred to as the AV inequality in this paper. Starting with the work of Amati and Vitale, a number[5–9] of works have appeared in the literature on the same problem, with stress on the individual branching ratios for the different reaction modes, in particular on the Σ^-/Σ^+ ratio. These various attempts may be classified into two groups on the basis

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[1] A. Pais, Phys. Rev. **110**, 574 (1958).

[2] D. Amati and B. Vitale, Nuovo cimento **9**, 895 (1958).

[3] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); J. Schwinger, Phys. Rev. **104**, 1164 (1956).

[4] See Eq. (14) of footnote 2, which reads

$$(W_{\Sigma^+\pi^-}+W_{\Sigma^-\pi^+}-4W_{\Sigma^0\pi^0})^2+4(W_{\Sigma^0\pi^0}W_{\Lambda^0\pi^0}-W_{\Sigma^+\pi^-}W_{\Sigma^-\pi^+})\geqslant 0,$$

where W_α is the branching ratio for the reaction α. This relation is referred to as the AV inequality in this paper.

[5] A. Salam, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

[6] B. D. Espagnat and J. Prentki, Nuovo cimento, **15**, 130 (1960).

[7] K. Kawarabayashi, Progr. Theoret. Phys. (Kyoto) **20**, 117 (1958).

[8] M. L. Gupta, Nuovo cimento **16**, 737 (1960).

[9] M. Ross and G. L. Shaw, Bull. Am. Phys. Soc. **5**, 504 (1960). See also, Phys. Rev. **115**, 1773 (1959).

of their starting assumptions: one,[5–8] in which the symmetries of the pion interactions are assumed not to be broken by the K interactions, so that one can obtain the pion-hyperon scattering phase shifts directly from the pion-nucleon scattering phase shifts (in case of the global symmetry); and the other,[9] in which the effect of the presence of the $(\bar{K}N)$ channel on the pion-hyperon-production amplitudes is taken into account approximately. Without further apology we would, as mentioned before, quickly include ourselves in the former group for the purpose of the present paper. The relationship of this work with that of Ross and Shaw[9] will be taken up in detail in a subsequent paper.

In view of the fact that several attempts have already been made under the category of the first group, one could ask: What is the purpose of the present work? To answer this question we begin by summarizing the previous attempts.[5–8]

It was pointed out by Salam[5] at Kiev that the energy dependence of $\Sigma^-:\Sigma^+:\Sigma^0$ branching ratios is not consistent with global symmetry. This analysis was based, however, on the old branching ratios $(\Sigma^-:\Sigma^+:\Sigma^0:\Lambda^0 \approx 2:1:1:\frac{1}{4})$ which violate the AV inequality. Hence the disagreement with global symmetry is not unexpected. Furthermore, this analysis adopted the old Dalitz-Tuan[10] scattering lengths, which neglected (K^+,K^0) mass difference. The work of d'Espagnat and Prentki[6] and Kawarabayashi[7] are also based on the old branching ratios, and hence their conclusions need not hold. Recently Gupta[8] has shown that it is possible to obtain a set of real values for the phase shifts[11] $\alpha_{\frac{3}{2}}$ and $\alpha_{\frac{1}{2}}$ of the pion-hyperon system, if one adopts the new[12] branching ratios. From this, he concludes that restricted symmetry may be a useful concept. In his analysis, however, he chooses the mean values of the branching ratios, which contradict the AV inequality and hence the restricted symmetry. It is not immediately obvious how sensitive is his analysis to the choice of errors[13] in the experimental branching ratios. Hence, the conclusion on the usefulness of the restricted symmetry seems to be rather ambiguous. Furthermore, like the previously mentioned authors, Gupta also uses the old Dalitz-Tuan scattering lengths.

In view of the above, it was felt necessary to test the validity of the global symmetry by adopting the new branching ratios[12] and the new Dalitz-Tuan[14] scattering lengths which include the (K^+,K^0) mass difference. We start with such choice[15] of errors in the branching ratios, which satisfy the AV inequality.

We assume the following in the analysis:

(i) A low-energy K^- (laboratory kinetic energy of $K^- \lesssim 30$ Mev) is captured predominantly in the S state.

(ii) K^- has odd[16] parity relative to the hyperon-nucleon system.

(iii) The (Σ,Λ) parity is even.

(iv) The (Σ,Λ) mass difference may be neglected in the dynamics of calculations.

(v) The presently known baryon spectrum is complete.

(vi) The pion-baryon coupling constants satisfy global symmetry, which is not masked appreciably by the K-meson interactions.

(vii) The new Dalitz-Tuan[14] scattering lengths hold, and the zero-range approximation is valid up to a K^- laboratory kinetic energy of at least 30 Mev.

In Sec. IIA we discuss the scheme of pion-baryon interactions to define the restricted and global symmetries. In Sec. IIB we develop the T^{-1}-matrix formalism of Matthews and Salam for calculating the branching ratios of the various K^-+p reactions. In Sec. III, global symmetry is applied to determine the pion-hyperon scattering lengths from pion-nucleon scattering lengths and the results are used to determine the T-matrix elements for the various reaction channels. In Sec. IV the results of the calculation of branching ratios for the various K^-+p reactions are given for capture of K^- at rest and at 30 Mev laboratory kinetic energy. It is found that there exist solutions for a^- and b^+ (also rather poorly for a^+) Dalitz-Tuan scattering lengths which can explain the $\Sigma^-:\Sigma^+:\Sigma^0$ branching ratios reasonably well for K^- capture at rest. However, the agreement for 30-Mev K^- mesons is very poor. This casts doubts on the validity of the global symmetry. Thus the conclusion regarding the global symmetry hypothesis remains essentially the same as before,[5] even with the new[12] data and the new Dalitz-Tuan scattering lengths.

II. FORMALISM

A. Pion-Baryon Interactions

It is well-known[17] that under the hypothesis of charge independence, Yukawa-type interactions, equality of the $(\Sigma\Lambda\pi)$ and $(\Sigma\Sigma\pi)$ coupling constants, and assumptions (iii) and (iv), the pion-baryon interactions take the form

$$H_\pi = g_1\bar{N}_1\boldsymbol{\tau} i\gamma_5 N_1\cdot\boldsymbol{\pi} + g_2[\bar{N}_2\boldsymbol{\tau} i\gamma_5 N_2 + \bar{N}_3\boldsymbol{\tau} i\gamma_5 N_3]\cdot\boldsymbol{\pi} + g_4\bar{N}_4\boldsymbol{\tau} i\gamma_5 N_4\cdot\boldsymbol{\pi}, \quad (1)$$

[10] R. H. Dalitz and S. F. Tuan, Ann. Phys. **8**, 100 (1959).

[11] $\alpha_{\frac{3}{2}}$ and $\alpha_{\frac{1}{2}}$ denote the pion-hyperon phase shifts in $I=\frac{3}{2}$ and $\frac{1}{2}$ states, respectively, and are defined in Sec. III.

[12] L. Alvarez, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

[13] It is to be noted that the mean values of the experimental branching ratios[12] $(\Sigma^-:\Sigma^+:\Sigma^0:\Lambda^0 \approx 45:21:27:7)$ lead to a value of $p \equiv (k_\Sigma/k_\Lambda)[\sigma(\Lambda)/\sigma_{T=1}(\Sigma)] \approx 0.41$, which is a parameter in Gupta's analysis. Gupta chooses $p \approx 0.5$. However, if we choose the errors in the experimental branching ratios, so that the AV inequality is satisfied, then $p \approx 1.36$.

[14] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).

[15] It is easy to check that this choice is very limited.

[16] The analysis is not sensitive, however, to the choice of this parity, essentially because both $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}'(\pi-N)$ phase shifts are small at the energies involved.

[17] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); A. Pais, Phys. Rev. **110**, 574 (1958).

where

$$N_1=\binom{p}{n},\quad N_2=\binom{\Sigma^+}{Y},\quad N_3=\binom{Z}{\Sigma^-},\quad N_4=\binom{\Xi^0}{\Xi^-},$$

$$Y\equiv(\Lambda-\Sigma^0)/\sqrt{2};\quad Z\equiv(\Lambda+\Sigma^0)/\sqrt{2}. \tag{2}$$

The symmetry involved in Eq. (1), i.e., the isospinor description of all baryons, is referred to as "restricted symmetry." The hypothesis of "global symmetry" [assumption (vi)] is to enlarge the above symmetry by assuming

$$g_1=g_2=g_4. \tag{3}$$

It is clear that H_π given by Eq. (1) conserves the usual isotopic spin $\mathbf{T}$ and the doublet[18] spin $\mathbf{I}$, which is $\frac{1}{2}$ for members of the four doublets N_1, N_2, N_3, and N_4 and 1 for the pions. Thus the πN_1, πN_2, πN_3, and πN_4 systems can be in $I=\frac{1}{2}$ and $\frac{3}{2}$ states and the scattering of each of these systems in a given angular momentum state can be described by two scattering amplitudes,[19] one for the $I=\frac{1}{2}$ and the other for the $I=\frac{3}{2}$ state. This is the outcome of "restricted symmetry."

If we further assume global symmetry [Eq. (3)], the πN_2 and πN_3 scattering amplitudes in $I=\frac{1}{2}$ and $\frac{3}{2}$ states can be equated to the corresponding πN_1 scattering amplitudes. The precise energy at which these should be compared is not clear, since the mass difference between nucleons and hyperons is large. However for low-energy K^- absorption the relative pion-hyperon energy is nearly that of resonance for the πN_1 system. At these energies the $J=\frac{1}{2}$ πN_1 phase shifts are small and slowly varying, so we will choose some mean value of these phase shifts.

B. T^{-1}-Matrix Formalism of Matthews and Salam

Let us consider the reactions involving two particles in, and two particles out, with n possible channels, and let us assume that only S-wave interaction is important in the energy region of interest. Following Matthews and Salam,[20] we define the T matrix in terms of the S matrix by

$$S_{fi}=\delta_{fi}+2i(k_f)^{\frac{1}{2}}T_{fi}(k_i)^{\frac{1}{2}}, \tag{4}$$

where k_i and k_f denote the relative momenta measured in the c.m. frame in the initial and final channels, respectively. The unitarity of the S matrix implies that it can be written in terms of a Hermitean matrix as follows:

$$S=(1+ik^{\frac{1}{2}}Kk^{\frac{1}{2}})/(1-ik^{\frac{1}{2}}Kk^{\frac{1}{2}}), \tag{5}$$

where

$$(k^{\frac{1}{2}})_{if}\equiv\delta_{if}(k_f)^{\frac{1}{2}}. \tag{6}$$

By Eqs. (4) and (5)

$$(T^{-1})_{if}=(K^{-1})_{if}-ik_i\delta_{if}. \tag{7}$$

By time-reversal invariance, S and hence K are symmetric. But K is Hermitean by unitarity of the S matrix. Hence K is real and symmetric. So also is K^{-1}. This reduces the number of parameters needed to describe the T^{-1} matrix [see Eq. (7)] and makes the kinematic structure of T^{-1} particularly simple. This is the motivation for adopting the T^{-1}-matrix formalism.

For a system with only one channel, K^{-1} is related to the S-wave phase shift α by

$$K^{-1}=k\cot\alpha=Z, \tag{8}$$

where k is the relative momentum in the two-particle center-of-mass system, and Z is the inverse of the scattering length.

C. T^{-1} Matrices for K^-+p Reactions

The following three[21] channels are to be considered for K^-+p processes in the energy region of interest:

$$\begin{aligned}&K^-+p\to\bar{K}+N,\\&K^-+p\to\Sigma+\pi,\\&K^-+p\to\Lambda+\pi.\end{aligned} \tag{9}$$

The initial state can have isotopic spin 0 or 1. So the T^{-1} matrix for the above processes can be decomposed to $(T^{-1})^0$ and $(T^{-1})^1$, where the superscripts denote the isotopic spin of the system. By Eq. (7) and the fact that K^{-1} is real and symmetric, we can write

$$(T^{-1})^0=\begin{array}{c c}& \begin{array}{cc}\bar{K}N & \Sigma\pi\end{array}\\ \begin{array}{c}\bar{K}N\\ \Sigma\pi\end{array} & \begin{bmatrix}a_0-ik_1 & h_0\\ h_0 & b_0-ik_2\end{bmatrix}\end{array}, \tag{10}$$

$$(T^{-1})^1=\begin{array}{c c}& \begin{array}{ccc}\bar{K}N & \Sigma\pi & \Lambda\pi\end{array}\\ \begin{array}{c}\bar{K}N\\ \Sigma\pi\\ \Lambda\pi\end{array} & \begin{bmatrix}a-ik_1 & h & g\\ h & b-ik_2 & f\\ g & f & c-ik_2\end{bmatrix}\end{array}, \tag{11}$$

k_1, k_2, and k_3 denote the relative momenta in the center-of-mass system in the $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ channels, respectively. The nine parameters a_0, h_0, b_0, a, h, g, b, f, and c are real and determine the (T^{-1}) matrix completely. Denoting the $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ channels by the numbers 1, 2, and 3, respectively, we have from Eq. (10)

$$\begin{aligned}T_{11}{}^0&=\frac{1}{[a_0-h_0{}^2/(b_0-ik_2)]-ik_1}\\&=1/(Z_0-ik_1),\end{aligned} \tag{12}$$

where Z_0 is the inverse of the complex Dalitz-Tuan scattering length for the $T=0$ state of the $\bar{K}N$ system.

[18] A. Pais, Phys. Rev. **112**, 624 (1958).

[19] These amplitudes are necessarily the same for πN_2 and πN_3 systems because of Eq. (1).

[20] P. T. Matthews and A. Salam, Nuovo cimento **13**, 381 (1959).

[21] We neglect $(\Lambda\pi\pi)$ and $(\Sigma\pi\pi)$ channels, since their cross sections are very small compared to $(\Lambda\pi)$ and $(\Sigma\pi)$ channels in the energy region under consideration.

Writing $Z_0 \equiv X_0 - iY_0$, we have

$$X_0 = a_0 - (h_0^2 b_0/b_0^2 + k_2^2),$$
$$Y_0 = h_0^2 k_2/(b_0^2 + k_2^2). \tag{13}$$

From Eq. (10) we also have

$$T_{12}^0 = \frac{-h_0/(b_0 - ik_2)}{[a_0 - h_0^2/(b_0 - ik_2)] - ik_1} = -\frac{(Y_0/k_2)^{\frac{1}{2}} e^{i\phi_0}}{Z_0 - ik_1}, \tag{14}$$

where

$$\tan\phi_0 = k_2/b_0. \tag{15}$$

By Eq. (11) we have

$$T_{11}^1 = \frac{A_{11}}{D} = \frac{1}{Z_1 - ik_1},$$
$$T_{12}^1 = \frac{A_{12}}{D} = \frac{A_{12}/A_{11}}{Z_1 - ik_1}, \tag{16}$$
$$T_{13}^1 = \frac{A_{13}}{D} = \frac{A_{13}/A_{11}}{Z_1 - ik_1},$$

where D denotes the determinant of the $(T^{-1})^1$ matrix, and $Z_1 \equiv X_1 - iY_1$ is the inverse of the complex Dalitz-Tuan scattering length for $T=1$ state of the $\bar{K}N$ system. The quantities A_{ij}s are given by

$$A_{11} = (b - ik_2)(c - ik_3) - f^2,$$
$$A_{12} = -h(c - ik_3 - xf), \tag{17}$$
$$A_{13} = h\{f - (b - ik_2)\},$$

where

$$x \equiv g/h. \tag{18}$$

If we assume the validity of restricted symmetry [Eq. (1)] and that the K-meson interactions do not break it appreciably, the four real parameters b_0, b, f, and c for pion-hyperon scattering can be expressed in terms of two parameters $(K^{-1})_{\frac{1}{2}}$ and $(K^{-1})_{\frac{3}{2}}$ corresponding to I-spin$=\frac{1}{2}$ and $\frac{3}{2}$, respectively, as follows[2]:

$$b_0 = \langle \Sigma\pi | K^{-1} | \Sigma\pi \rangle_0 = (K^{-1})_{\frac{1}{2}},$$
$$b = \langle \Sigma\pi | K^{-1} | \Sigma\pi \rangle_1 = \tfrac{1}{3}(K^{-1})_{\frac{3}{2}} + \tfrac{2}{3}(K^{-1})_{\frac{1}{2}},$$
$$f = \langle \Sigma\pi | K^{-1} | \Lambda\pi \rangle_1 = \tfrac{1}{3}\sqrt{2}[(K^{-1})_{\frac{3}{2}} - (K^{-1})_{\frac{1}{2}}], \tag{19}$$
$$c = \langle \Lambda\pi | K^{-1} | \Lambda\pi \rangle_1 = \tfrac{1}{3}[2(K^{-1})_{\frac{3}{2}} + (K^{-1})_{\frac{1}{2}}],$$

The subscripts 1 and 0 correspond to the usual isotopic spin 1 and 0, respectively, while $\frac{1}{2}$ and $\frac{3}{2}$ correspond to I-spin$=\frac{1}{2}$ and $\frac{3}{2}$, respectively.

If we assume global symmetry [Eq. (3)], we can evaluate $(K^{-1})_{\frac{1}{2}}$ and $(K^{-1})_{\frac{3}{2}}$ from the known experimental data on pion-nucleon scattering. This gives us four of the nine parameters needed to specify the (T^{-1}) matrix elements. Four more parameters are given by the two complex Dalitz-Tuan scattering lengths. To determine the remaining one parameter, we use one experimental number involving the branching ratios for Σ and Λ production. We choose this number to be $\sigma_{\text{absorption}}(T=1)/\sigma(\Lambda)$. Thus we determine all the parameters of the (T^{-1}) matrix.

III. APPLICATION OF GLOBAL SYMMETRY

The quantities $(K^{-1})_{\frac{1}{2}}$ and $(K^{-1})_{\frac{3}{2}}$ referring to the pion-hyperon scattering can be written in terms of the relevant phase shifts as

$$(K^{-1})_{\frac{1}{2}} = \eta_\pi \cot\alpha_{\frac{1}{2}},$$
$$(K^{-1})_{\frac{3}{2}} = \eta_\pi \cot\alpha_{\frac{3}{2}},$$

where

$$\eta_\pi = k_\pi/m_\pi. \tag{20}$$

k_π denotes the relative momentum in the c.m. frame of the pion-hyperon system. We adopt the units, in which $\hbar = c = 1$ and represent all lengths in *units of pion Compton wavelength*. By global symmetry, we relate the pion-hyperon phase shifts $\alpha_{\frac{1}{2}}$ and $\alpha_{\frac{3}{2}}$ to the corresponding pion-nucleon phase shifts. As a convention, we compare these phase shifts for the same pion-baryon relative momentum in the c.m. frames. Since these phase shifts are small, $\eta_\pi \cot\alpha_{\frac{1}{2}}$ and $\eta_\pi \cot\alpha_{\frac{3}{2}}$ may be approximated by $\eta_\pi/\alpha_{\frac{1}{2}}$ and $\eta_\pi/\alpha_{\frac{3}{2}}$, respectively, which are nearly constants for low energies. The outgoing pion kinetic energy in $K^- + p \to \Sigma(\Lambda) + \pi$ is nearly 90 (150) Mev in the c.m. frame for $\Sigma(\Lambda)$ production. Using the values[22] of $\eta_\pi/\alpha_{\frac{1}{2}}$ and $\eta_\pi/\alpha_{\frac{3}{2}}$ for S-wave πN scattering around these energies, we adopt[23]

$$(K^{-1})_{\frac{1}{2}} \approx +6,$$
$$(K^{-1})_{\frac{3}{2}} \approx -10. \tag{21}$$

By Eqs. (19) and (21), we have

$$b_0 = +6,$$
$$b = \tfrac{2}{3},$$
$$f = -7.5, \tag{22}$$
$$c = -14/3.$$

Capture of K^- at Rest

The relative momenta k_1, k_2, and k_3 in the c.m. frame in $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ channels are given by

$$k_1^2 = \frac{m_p^2(E_k^2 - m_k^2)}{m_k^2 + m_p^2 + 2m_p E_k^{\text{lab}}},$$
$$k_{2,3}^2 = \frac{[E^{*2} - (m_{\Sigma,\Lambda} + m_\pi)^2][E^{*2} - (m_{\Sigma,\Lambda} - m_\pi)^2]}{4E^{*2}}, \tag{23}$$

[22] G. Puppi, *1958 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958). B. Pontecorvo, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

[23] The analysis is not very sensitive to slight alteration of the values of $(K^{-1})_{\frac{1}{2}}$ and $(K^{-1})_{\frac{3}{2}}$.

where E^* denotes the total incident energy in the c.m. frame and E_K the total energy of the K meson in the laboratory system. For capture of K^- at rest, we obtain

$$\begin{aligned} k_1&=0,\\ k_2&\simeq 1.3m_\pi,\\ k_3&\simeq 1.8m_\pi. \end{aligned} \tag{24}$$

By Eqs. (17), (22), and (24) we have

$$\begin{aligned} A_{11}&\approx -62.3+5.9i,\\ A_{12}&\approx -h(-14/3+7.5x-1.8i),\\ A_{13}&\approx +h(-7.5-\tfrac{2}{3}x+1.3xi). \end{aligned} \tag{25}$$

The only remaining unknowns in the T matrix are the parameters h and x or, equivalently, h and g. We proceed to determine x first by using the following relation:

$$\frac{\sigma_{\text{absorption}}(T=1)}{\sigma(\Lambda)}=\frac{k_2|A_{12}|^2+k_3|A_{13}|^2}{\frac{1}{2}k_3|A_{13}|^2}=\frac{2(\Sigma^++\Sigma^-)-4\Sigma^0+2\Lambda^0}{\Lambda^0},$$

or

$$k_2|A_{12}|^2/k_3|A_{13}|^2 =[2(\Sigma^++\Sigma^-)-4\Sigma^0]/2\Lambda^0\equiv\alpha \quad \text{(say)}. \tag{26}$$

By Eqs. (25) and (26) we can evaluate x provided we use the experimental value of α. The branching ratios reported at Kiev[12] are:

$$\begin{aligned} &K^-+p\to\Sigma^-+\pi^+, && 45\pm1;\\ &K^-+p\to\Sigma^++\pi^-, && 21\pm1;\\ &K^-+p\to\Sigma^0+\pi^0, && 27\pm2.5;\\ &K^-+p\to\Lambda^0+\pi^0, && 7\pm1.5. \end{aligned} \tag{27}$$

It is pertinent to notice that if we choose the mean values B_0 of the above branching ratios, then there is not much point in proceeding further, since B_0 violates the AV inequality. We therefore choose the following branching ratios (roughly consistent with the errors in Eq. (27)) for our analysis.

$$\Sigma^-:\Sigma^+:\Sigma^0:\Lambda^0\approx 43.5:20:29:7.5. \tag{28}$$

The above choice is rather limited in order to satisfy the AV inequality. By Eq. (28) we have[24]

$$\alpha\simeq 0.75. \tag{29}$$

Equations (25), (26), and (29) yield a quadratic equation for x, solving which we obtain

$$x\simeq\begin{pmatrix}+1.81\\-0.344\end{pmatrix}. \tag{30}$$

[24] Note that the mean values of the branching ratios in Eq. (27) lead to $\alpha\approx1.7$.

We shall henceforth represent the results by a *two-row* matrix; the top row corresponding to $x=1.81$ and the bottom row to $x=-0.344$. By Eqs. (25) and (30) we have

$$\begin{aligned} A_{11}&\simeq -62.3+5.9i,\\ A_{12}&\simeq -h\left[\begin{pmatrix}8.99\\-7.25\end{pmatrix}-1.81i\right],\\ A_{13}&\simeq h\left[-\begin{pmatrix}8.74\\7.31\end{pmatrix}+i\begin{pmatrix}2.35\\-0.45\end{pmatrix}\right]. \end{aligned} \tag{31}$$

We determine the only remaining unknown parameter h by using the unitarity condition on the T^1 matrix elements, which gives

$$\operatorname{Im} T_{11}{}^1=k_1|T_{11}{}^1|^2+k_2|T_{12}{}^1|^2+k_3|T_{13}{}^1|^2. \tag{32}$$

By Eqs. (16), (31), and (32), we obtain

$$h^2\simeq\begin{pmatrix}15.3\\23.2\end{pmatrix}y_1. \tag{33}$$

The inverses of new Dalitz-Tuan[14] scattering lengths in units of pion mass are the following:

$$\begin{aligned} Z_1\equiv X_1-iY_1&=(0.831-i0.208) && (a^+)\\ &=(-1.37-i0.246) && (a^-)\\ &=(+1.84-i0.577) && (b^+)\\ &=(-1.30-i1.97) && (b^-) \end{aligned} \tag{34}$$

and,

$$\begin{aligned} Z_0\equiv X_0-iY_0&=(+0.415-i1.66) && (a^+)\\ &=(-0.160-i0.853) && (a^-)\\ &=(+0.442-i0.442) && (b^+)\\ &=(-0.706-i0.235) && (b^-) \end{aligned} \tag{35}$$

By Eqs. (33) and (34),

$$h=(\pm)\begin{array}{c} \begin{array}{cccc} a^+ & a^- & b^+ & b^- \end{array}\\ \begin{pmatrix} 1.78 & 1.94 & 2.97 & 5.48\\ 2.19 & 2.39 & 3.65 & 6.75 \end{pmatrix} \end{array}. \tag{36}$$

The top row, as mentioned before, corresponds to $x=+1.81$ and the bottom row to $x=-0.344$. The four columns correspond to the four possible Dalitz-Tuan solutions for the $\bar{K}N$ scattering lengths.

We have now determined all the parameters of the T^0 and T^1 matrices and can proceed to calculate the $\Sigma^-:\Sigma^+:\Sigma^0$ branching ratios from the following expressions for the matrix elements of the various K^-+p reactions.

$$\begin{aligned} &\Sigma^-+\pi^+: && (1/\sqrt{6})T_{12}{}^0+\tfrac{1}{2}T_{12}{}^1;\\ &\Sigma^++\pi^-: && (1/\sqrt{6})T_{12}{}^0-\tfrac{1}{2}T_{12}{}^1;\\ &\Sigma^0+\pi^0: && (1/\sqrt{6})T_{12}{}^0;\\ &\Lambda^0+\pi^0: && (1/\sqrt{2})T_{13}{}^1. \end{aligned} \tag{37}$$

IV.

A. Results for Capture at Rest

The results of the calculation of the branching ratios for $a^{\pm}$ and $b^{\pm}$ solutions of $\bar{K}N$-scattering lengths are given below.

$$\Sigma^-:\Sigma^+:\Sigma^0 \simeq \begin{pmatrix} 1.2 & 1.5 \\ 2.0 & 0.7 \end{pmatrix} : \begin{pmatrix} 1.5 & 1.2 \\ 0.7 & 2.0 \end{pmatrix} : 1 \quad (a^+)$$

$$\simeq \begin{pmatrix} 0.7 & 1.4 \\ 1.3 & 0.9 \end{pmatrix} : \begin{pmatrix} 1.4 & 0.7 \\ 0.9 & 1.3 \end{pmatrix} : 1 \quad (a^-)$$

$$\simeq \begin{pmatrix} 0.7 & 1.5 \\ 1.6 & 0.5 \end{pmatrix} : \begin{pmatrix} 1.5 & 0.7 \\ 0.5 & 1.6 \end{pmatrix} : 1 \quad (b^+)$$

$$\simeq \begin{pmatrix} 0.7 & 2.3 \\ 2.8 & 0.28 \end{pmatrix} : \begin{pmatrix} 2.3 & 0.7 \\ 0.28 & 2.8 \end{pmatrix} : 1 \quad (b^-) \qquad (38)$$

The first and the second row in each parenthesis corresponds to $x=+1.81$ and -0.344, respectively [see Eq. (30)], while the first and second column in each bracket corresponds to positive and negative signs of h, respectively, [see Eq. (36)]. Comparing with the observed branching ratios $\Sigma^-:\Sigma^+:\Sigma^0 \approx 1.5:0.7:1$ [see Eq. (28)], we notice that there exist solutions, in particular for the a^- and b^+ Dalitz-Tuan scattering lengths, which can explain reasonably well the observed branching ratios at threshold. The agreement is not too bad for the a^+ Dalitz solution (consider the set $2:0.7:1$) while it is rather poor for b^-. From this, we may conclude that the data on the branching ratios at threshold are consistent with global symmetry.

We next extend the same procedure to capture of K^- in flight with low enough energy so as to permit the assumptions (i) and (vii). As an example, we consider capture of K^- at 30 Mev of laboratory kinetic energy.

B. Capture of K^- at 30 Mev

We assume the zero-range approximation [assumption (vii)], by which we treat the parameters a_0, h_0, b_0, a, h, g, b, f, and c as constants. Thus we take their values at 30 Mev to be those at threshold. The change in the matrix elements occur only through a change in the values of the relative momenta k_1, k_2, and k_3. Their values for $E_K=(m_K+30)$ Mev are [see Eq. (23)]:

$$k_1 \approx 0.809 m_\pi, \quad k_2 \approx 1.44 m_\pi, \quad k_3 \approx 1.93 m_\pi. \qquad (39)$$

By following the same procedure for the calculation of the branching ratios, as at threshold, we obtain

$$\Sigma^-:\Sigma^+:\Sigma^0 \simeq \begin{pmatrix} 0.6 & 2 \\ 2.4 & 0.3 \end{pmatrix} : \begin{pmatrix} 2 & 0.6 \\ 0.3 & 2.4 \end{pmatrix} : 1, \quad (a^+)$$

$$\simeq \begin{pmatrix} 0.4 & 2 \\ 1.7 & 0.7 \end{pmatrix} : \begin{pmatrix} 2 & 0.4 \\ 0.7 & 1.7 \end{pmatrix} : 1, \quad (a^-)$$

$$\simeq \begin{pmatrix} 0.7 & 1.9 \\ 2.3 & 0.3 \end{pmatrix} : \begin{pmatrix} 1.9 & 0.7 \\ 0.3 & 2.3 \end{pmatrix} : 1, \quad (b^+)$$

$$\simeq \begin{pmatrix} 0.3 & 3.7 \\ 4.0 & 0.0 \end{pmatrix} : \begin{pmatrix} 3.7 & 0.3 \\ 0.0 & 4.0 \end{pmatrix} : 1, \quad (b^-). \qquad (40)$$

All of the above results are clearly in contradiction with the experimental branching ratios[25] ($\Sigma^-:\Sigma^+ \approx 1:1$) at 30 Mev. Thus one may conclude, granting the validity of the assumptions (i), (ii), (iii), (iv), (v), and (vii), that,

(i) either global symmetry does not exist even in the bare pion-baryon interactions, or

(ii) it exists in the bare interaction, but is heavily masked by the K-meson interactions, so that it is not a useful symmetry at low energies. At very high energies,[26] experiments may still confirm its validity.

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[25] There is not any reliable data for the Σ^0-branching ratio around 30 Mev kinetic energy of K^-. For the Σ^-/Σ^+-ratio at 30 Mev, see M. F. Kaplan, *1958 Annual International Conference on High Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958).

[26] There is some hope that the symmetries of the bare renormalisable interactions may be exhibited in the limit of certain high energies [M. Gell-Mann and F. Zachariasen, Phys. Rev. (to be published)].