# Longitudinal Magnetoresistance in *n*-Type Germanium: Experimental*

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Measurements of the longitudinal magnetoresistance of single crystals of pure and doped *n*-type germamanium oriented in the ⟨100⟩, ⟨110⟩, and ⟨111⟩ directions have been made in magnetic fields up to 300 kgauss over the temperature interval 20°–300°K. The magnetoresistance ratio $\rho(H)/\rho(0)$ was found to vary linearly with magnetic field strength in the quantum limit. Magnetoresistance ratios less than one were observed and explained on the basis of the many-valley structure of the conduction band. The saturation of magnetoresistance predicted by classical transport theory was observed at the higher end of the temperature range and used to demonstrate a temperature variation of the anisotropy parameter $K$.

## I. INTRODUCTION

A GREAT deal of experimental data on the magnetoresistance of germanium exists in the literature.[1] For the most part, previous experimenters have been concerned with the low-field behavior where the effects of quantization of the electron orbits is negligible. These experiments have been quite successful in verifying the many-valley model of the conduction band in germanium and in revealing much useful information on the nature of the dominant scattering mechanisms. In recent years a considerable amount of attention has been given to the quantum mechanical calculation of magnetoresistance, at least for the case of spherically symmetric energy bands.[2–4] The extension of the theory of longitudinal magnetoresistance to the many-valley model of the conduction band in germanium has recently been made by Miller and Omar,[5] hereafter referred to as (MO). The experiments reported here were undertaken to provide data on the longitudinal magnetoresistance of oriented single crystals of *n*-germanium over a range of temperatures and magnetic field strengths in which quantum effects will play an important role. Except for the unavoidable anisotropies, *n*-germanium is an ideal material to test this theory, for obvious reasons. Previous work at high magnetic fields[6–8] has been limited to temperatures above 77°K and a less thorough investigation than is given here.

## II. THEORETICAL CONSIDERATIONS

Although a fairly complete theoretical treatment of these effects is given in the accompanying paper (MO), a survey of the important theoretical factors will be given here for the purpose of discussion.

Abeles and Meiboom[9] and Shibuya[10] have calculated the longitudinal magnetoresistance in *n*-germanium for prominent crystallographic orientations. Their calculation is based on the effective mass approximation and the classical Boltzmann equation, neglecting the effects of quantization of the electron orbits. The result of their calculations most pertinent to the present investigation is that the magnetoresistance ratio $\rho(H)/\rho(0)$ will saturate at large field strengths, the saturation value depending on crystallographic orientation and the anisotropy parameter

$$K=K_mK=m_l\tau_t/m_t\tau_l, \tag{1}$$

where $m_t$ and $m_l$ are the transverse and longitudinal masses and $\tau_t$ and $\tau_l$ are the corresponding components of the relaxation time.[11] The results are summarized for convenience in Table I, where the saturation values of $\rho(H)/(0)$ are given for different values of $K$. In the results to be presented, any deviation from saturation at high magnetic fields must be interpreted as being due to quantum effects.

For a spherically symmetric conduction band and lattice scattering, the effects of quantization of the electron orbits are given through the influence of the magnetic field on the density of states and the relaxation time. This leads to a field dependence of the magnetoresistance ratio not present in the classical treatment. In germanium, however, there is an additional quantum effect due to the many-valley structure of the conduction band. For a given orientation of the magnetic field the different ellipsoids will have a different zero-point energy due to the variation of effective mass with direction. Consequently, because of the Boltzmann factor, electrons will transfer to the ellipsoid having the lowest splitting of its Landau levels. This ellipsoid will, in turn, have a low translational effective mass and high mobility. The net result of this "quantum transfer" effect *alone* would be a decrease in the

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[1] An extensive bibliography of this work will be found in M. Glicksman, *Progress in Semiconductors* (John Wiley & Sons, New York, 1958), Vol. 3, p. 3.
[2] P. N. Argyres and E. N. Adams, Phys. Rev. **104**, 900 (1956).
[3] P. N. Argyres, J. Phys. Chem. Solids **4**, 19 (1958).
[4] E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).
[5] S. C. Miller and M. A. Omar, following paper, Phys. Rev. **123**, 74 (1961).
[6] P. Kapitza, Proc. Roy. Soc. (London) **A123**, 291 (1929).
[7] H. P. Furth and R. W. Waniek, Phys. Rev. **104**, 343 (1956).
[8] E. A. Zavadski and E. G. Fakedov, Doklady Akad. Nauk S.S.S.R. **10**, 495 (1960).
[9] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954).
[10] M. Shibuya, Phys. Rev. **95**, 1385 (1954).
[11] The generalization of the work of Abeles and Meiboom to include anisotropic relaxation time has been made by C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

TABLE I. Theoretical values of the saturation value of the magnetoresistance ratio $\rho(H)/\rho(0)$ for different crystallographic directions and pertinent values of the anisotropy parameter $K$.

| Formula | Crystallographic Direction ⟨100⟩ | ⟨110⟩ | ⟨111⟩ |
|---|---|---|---|
| | $(2K+1)(K+2)/9K$ | $(2K+1)^2/3K(K+2)$ | $(2K+1)(K+8)/3(7K+2)$ |
| $K$ | $[\rho(H)/\rho(0)]_{sat}$ | $[\rho(H)/\rho(0)]_{sat}$ | $[\rho(H)/\rho(0)]_{sat}$ |
| 12 | 3.24 | 1.24 | 1.94 |
| 13 | 3.46 | 1.25 | 2.03 |
| 14 | 3.68 | 1.25 | 2.12 |
| 15 | 3.90 | 1.26 | 2.22 |
| 16 | 4.13 | 1.26 | 2.31 |
| 17 | 4.34 | 1.26 | 2.41 |
| 18 | 4.57 | 1.27 | 2.50 |
| 19 | 4.79 | 1.27 | 2.60 |
| 20 | 5.02 | 1.27 | 2.70 |

resistivity. This effect was first predicted by Goldberg, Davis, and Adams,[12] who looked for the effect with respect to the light and heavy holes in germanium but were unable to determine the effect clearly because of competing quantum effects. This effect is primarily responsible for the negative magnetoresistances to be reported in the present data.

One can easily obtain an estimate of the order of magnitude of the "quantum transfer" effect and the range of magnetic field strengths and temperatures at which it should occur. The ground state energy for a given ellipsoid is given by

$$E=\tfrac{1}{2}h\omega^*, \tag{2}$$

where

$$\omega^*=eH/m^*c, \tag{3}$$

with

$$1/m^*=\left\{\frac{1}{m_t^2}\cos^2\theta+\frac{1}{m_tm_l}\sin^2\theta\right\}^{\frac{1}{2}}, \tag{4}$$

$\theta$ is the angle between the magnetic field and the longitudinal axis of the ellipsoid. By noting that $m_t=m/12.3$ and $m_l=m/0.63$, it may be seen from Eq. (3) that the ground state energy is greater at smaller values of $\theta$. Thus as the magnetic field is increased the electron population will shift to ellipsoids having a greater value of $\theta$, due to the different Boltzmann factors for these ellipsoids.

On the other hand, one can easily show that the mobility of electrons on a given ellipsoid varies with direction of the electric field as

$$\mu=\mu_t\sin^2\theta+\mu_l\cos^2\theta, \tag{5}$$

with $\mu_t=e\tau_t/m_t$ and $\mu_l=e\tau_l/m_l$. Thus the mobility will be greater at higher values of $\theta$.

For $H$ in the ⟨110⟩ direction two ellipsoids have $\theta=35.5°$ and the other two have $\theta=90°$. On assuming $K=20$, the ratio of mobilities will be $\mu_{90°}/\mu_{35.5°}=2.70$. If all electrons transfer to the 90° ellipsoids the mobility would increase by the factor 1.46, causing a 32% drop in resistivity. The relative Boltzmann factor for these two ellipsoids is $\exp[-4.9\times10^{-4}(H/T)]$. As an example, at $T=100°$K and $H=200$ kgauss the electron population of the higher state will be down by a factor $e^{-1}$. The effect should thus be easily seen with the range of fields and temperatures used in these experiments. A similar calculation for the ⟨111⟩ direction shows that a relative drop of resistivity of at most 23% would take place as a result of the "quantum transfer" effect alone. It should be noted that other factors will be affecting the resistivity along with the "quantum transfer" effect.

One other important effect should be mentioned here. At low temperatures ($<30°K$), where thermal freeze-out of carriers is occurring, one would expect magnetically induced freeze-out due to the influence of the magnetic field on the ionization energy of the impurity states. A rough estimate based on the Yafet, Keyes, and Adams[13] hydrogen-atom model in a magnetic field predicts an approximate doubling of the ionization energy in the fields used in these experiments. However, separate measurements of the Hall coefficient by Diesel and Love (to be published) have shown no change of carrier concentration induced by the field above 20°K. The success of Miller and Omar in accounting for the present data without assuming changes in carrier concentration is further evidence of this fact.

## III. EXPERIMENTAL PROCEDURE

### A. Method of Measurement

The magnetic field was produced by the discharge of a 2160-μf capacitor bank charged to 3000 v through a wire-wound solenoid. The solenoid consisted of 1800 turns of No. 18 copper wire imbedded in epoxy resin. It had an internal diameter of $\frac{11}{16}$ in., an outside diameter of $2\frac{1}{16}$ in., and a length of $2\frac{1}{2}$ in. The field reached the peak value of 180 kgauss in 7 msec. Constant current to the sample during the field pulse was provided by a power supply through a large series resistance. The voltage across the potential leads from the sample was displayed on the vertical sweep of a Tek-

[12] Goldberg, Davis and Adams, Phys. Rev. **105**, 865 (1957).

[13] Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids **1**, 137 (1956).

tronix type 535 oscilloscope. The horizontal sweep was driven by a signal proportional to the magnetic field obtained from a low resistance in series with the magnet. In some cases data were taken to 300 kgauss using a low inductance helically machined copper beryllium magnet of 28 turns with dimensions $\frac{5}{8}\times1\frac{3}{4}\times1\frac{13}{16}$ in. The time constant to peak magnetic field in this magnet was 120 $\mu$sec. In this case the magnetic field was obtained from a pickup coil whose output was passed through an integrating circuit to drive the horizontal sweep. These techniques have been adequately described in the literature.[14,15] The short time constant of this field pulse leads to appreciable corrections due to electromagnetic pickup in the measuring circuit of the sample. This correction was generally negligible for the data taken with the wire-wound solenoid. The correction for data taken with the other magnet is obtained by pulsing the field with zero current through the sample and subtracting the resulting signal from that obtained with sample current. The oscilloscope displays were photographed with a Polaroid Land camera. The resulting accuracy of the measurements is limited by the linearity of the oscilloscope and is better than 5%.

Kapitza,[16] in his classic measurements on the magnetoresistance of bismuth, found that residual voltages of thermomagnetic origin were present during a field pulse when the sample current was suddenly turned off. A check for such residual voltages was made in these experiments by providing a pulsed current (10-$\mu$sec pulses, 10% duty cycle) to the sample from a Dumont type 404R pulse generator. No such effects were found at temperatures above 100°K. At the lower temperatures it was not possible to use the pulse generator as a current supply due to the occurrence of a large magnetoresistance in the contact resistance of the leads. This was sometimes a full order of magnitude or more greater than the magnetoresistance of the sample itself.

In order to control the temperature of the sample it was mounted on a Bakelite sample holder and placed on the inside of a double-walled, vacuum-jacketed, silvered, glass transfer tube at the center of the magnet. The magnet was immersed in a liquid nitrogen bath. The temperature was controlled by the flow of cold helium gas from a storage vessel through the transfer tube and measured by a thermocouple of copper with an alloy of gold plus 2.1% cobalt. Calibration curves on this thermocouple were obtained through the courtesy of the Cryogenic Engineering Laboratories of the National Bureau of Standards in Boulder.

TABLE II. Information on samples studied and range of measuring currents with comments on Ohmic behavior.

| Sample | Resistivity (ohm-cm) | Orientation and shape | Range of measuring current and comments |
|---|---|---|---|
| Ge 100-2 | 35 | ⟨100⟩, bridge | 1. Data taken with $i$=0.50 ma.<br>2. Found slightly non-Ohmic contact at 76°K after measurement. |
| Ge 110-2$B$ | 35 | ⟨110⟩, bridge | 1. Contact Ohmic up to 15 ma at 76°K.<br>2. $\rho(H)/\rho(0)$ independent of current as measured between 0.047 ma and 10.6 ma at 79°K and 180 kgauss.<br>3. Data taken with $i$=0.047 to 4 ma. |
| Ge 111-3 | 35 | ⟨111⟩, bridge | 1. Contact Ohmic up to 25 ma at 76°K.<br>2. $\rho(H)/\rho(0)$ independent of current as measured between 0.058 ma and 6.3 ma at 79°K and 180 kgauss.<br>3. Data taken with $i$=0.62 ma. |
| Ge 100-10 (Sb-doped) | 2.9 | ⟨100⟩, bridge | 1. $\rho(H)/\rho(0)$ independent of current as measured between 0.49 ma and 44.4 ma at 79°K and 180 kgauss. Data taken with $i$=0.62 ma.<br>2. $\rho(H)/\rho(0)$ independent of current as measured between 5 ma and 50 ma at 35°K and 300 kgauss produced by a second magnet. Data taken with $i$=50 ma. |
| Ge 100-10$R$ (Sb-doped) | 2.9 | ⟨100⟩, rod | 1. Ge 100-10$R$ is the same sample as Ge 100-10 with potential leads lapped off to make it a rod shape. Potential leads were connected to two very small spots.<br>2. Contact Ohmic up to 30 ma at 76°K.<br>3. $\rho(H)/\rho(0)$ independent of current as measured (a) between 0.38 ma and 10.6 ma at 83°K; (b) between 1.97 ma and 20 ma at 25°K.<br>4. Data taken with $i$=1.97 ma. |
| Ge 110-1$B$ (Sb-doped) | 2.7 | ⟨110⟩, rod | 1. Contact Ohmic up to 59 ma at 76°K.<br>2. Data taken with $i$=15.8 ma for 64°K and $i$=4 ma for other temperatures. |
| Ge 111-10$B$ (Sb-doped) | 6.0 | ⟨111⟩, bridge | 1. Contact Ohmic up to 86 ma at 29°K and to 9 ma at 21°K.<br>2. Data taken with $i$=1 ma. |

## B. Sample Preparation

Samples of pure and doped germanium crystals oriented with longitudinal axes in the ⟨100⟩, ⟨110⟩, and ⟨111⟩ directions were prepared. Some of the samples were cut in the form of a rectangular rod and others in a bridge shape. The length of the samples was 10 mm with cross section approximately 1×1 mm. On the bridge samples the arms were several millimeters out and less than a half-millimeter thick. All samples were checked for proper orientation with x-ray back-reflection Laue pictures. The surfaces were lapped and etched

[14] H. P. Furth and R. W. Waniek, Rev. Sci. Instr. **27**, 195 (1956).
[15] S. Foner and H. Kolm, Rev. Sci. Instr. **27**, 547 (1956).
[16] P. Kapitza, Proc. Roy. Soc. (London) **A119**, 358 (1958).

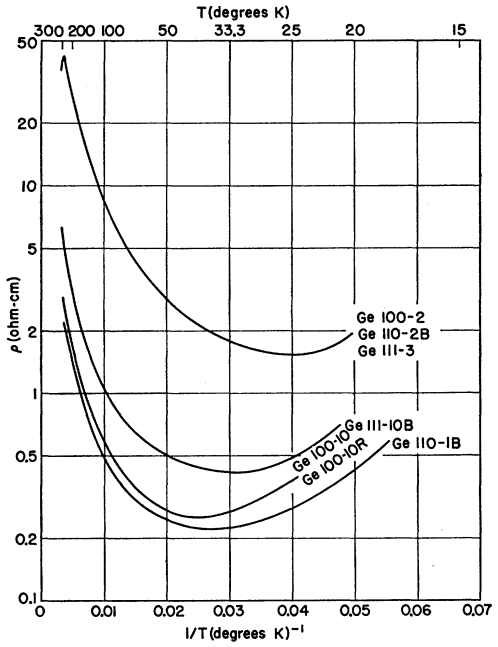


FIG. 1. Semi-log plot of the resistivity versus temperature for all samples studied.

with CP4 solution before soldering leads to the sample. The solder used was an alloy of tin with 50% lead and 0.5% antimony. The contacts were checked for Ohmic behavior at liquid nitrogen temperatures out to currents well beyond the range of measuring currents used in the experiments. Table II gives the room temperature resistivity, doping element, orientation, and shape of the sample crystals studied. The last column gives the range of measuring currents used together with comments on the Ohmic behavior of the samples.

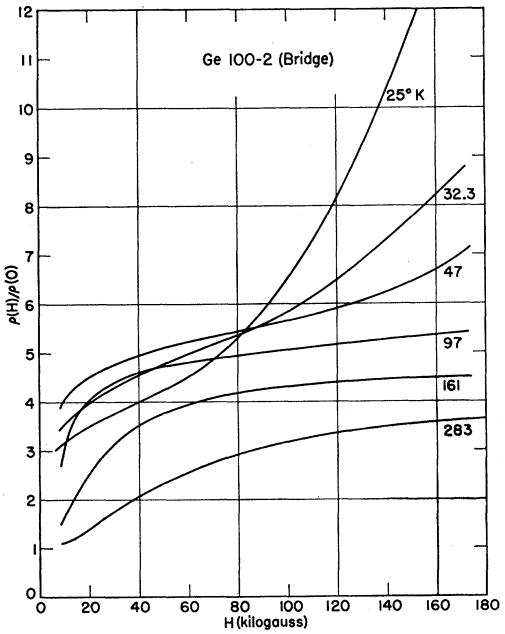


FIG. 2. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 100-2. The data has been averaged over both directions of current and field.

## IV. EXPERIMENTAL RESULTS

The resistivities of all samples studied are given for the range of temperatures from 20° to 300°K in Fig. 1. In specifying the samples the first three digits denote the crystallographic orientation. The three samples Ge 100-2, Ge 110-2*B*, and Ge 111-3 were all cut from the same ingot of pure germanium and had identical resistivities at all temperatures in the absence of a magnetic field. The doped samples were obtained from different ingots and varied in their resistivities.

Figure 2 shows a plot of the magnetoresistance ratio $\rho(H)/\rho(0)$ of Ge 100-2 vs magnetic field with temperature as a parameter for temperatures ranging from 285° down to 25°K. The saturation effect predicted by the classical theory is clearly evident in this figure. However, complete saturation is obtained at only one temperature, 161°K. The value of $K$ obtained from this curve is 18. The rise above saturation at lower temperatures is due to quantum effects. It is clear that $K$ increases with decreasing temperature for this sample. It has previously been noted that no quantum transfer effect should be present for this sample, since the magnetic field makes equal angles with all of the ellipsoids of the conduction band. This is confirmed by the fact that no region of decreasing resistance with increasing magnetic field exists in these curves.

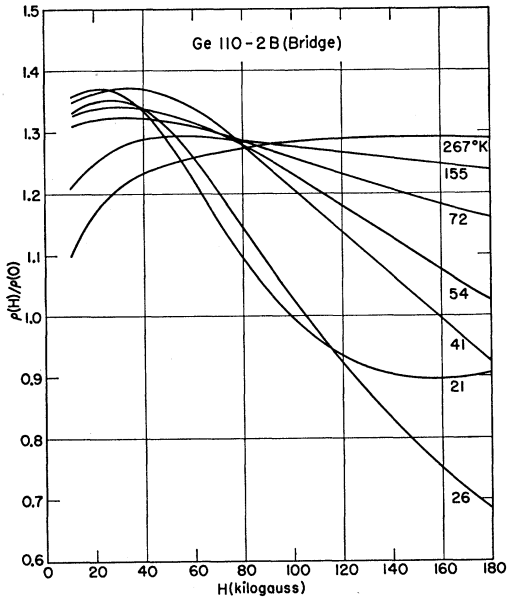


FIG. 3. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 110-2*B*.

Similar magnetoresistance plots for Ge 110-2*B* are shown in Fig. 3. At the highest temperature (267°K)

saturation is obtained, but a glance at Table I shows that the ⟨110⟩ direction is not suitable for an accurate determination of $K$. It will be noticed that all of the curves at lower temperatures show regions of decreasing magnetoresistance with increasing magnetic field. The relative drops of resistivity are of the right order of magnitude to agree with the quantum transfer effect discussed above.

Data for the third orientation studied in pure germanium are given in Fig. 4 for Ge 111-3. The saturation values for the magnetoresistance ratio of this sample show that $K$ increases with decreasing temperature below 280°K. The data below 200°K is complicated by the presence of quantum effects so that it becomes difficult to determine the correct temperature variation of $K$. It should be mentioned that any temperature

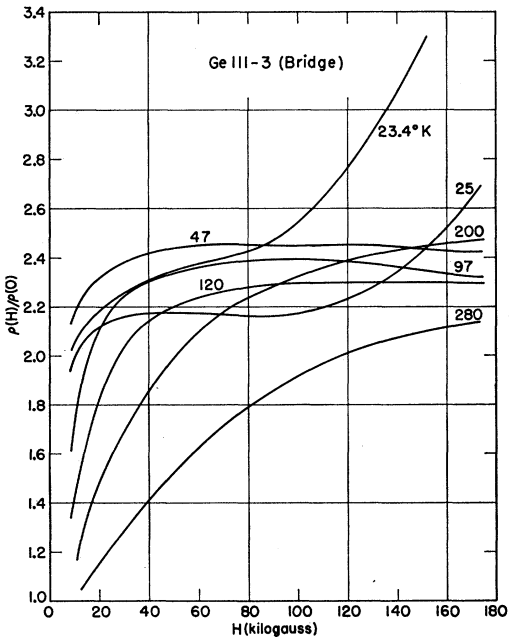


FIG. 4. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 111-3.

variation of $K$ can only be due to the relaxation time factor in the expression for $K$ since cyclotron resonance experiments have shown clearly that the effective mass does not change with temperature. The relaxation time factor has only been introduced in a phenomenological manner. Both the theories of Abeles and Meiboom and Miller and Omar do not contain this factor and hence cannot account for the relatively small variations in $K$ which do occur.

A number of samples of doped germanium have been studied in both bridge and rod shapes in order to determine the effects of ionized impurity scattering on the magnetoresistance. One sample, Ge 100-10, was studied in considerable detail. Data taken with 180-kgauss pulses are shown in Fig. 5. This sample was

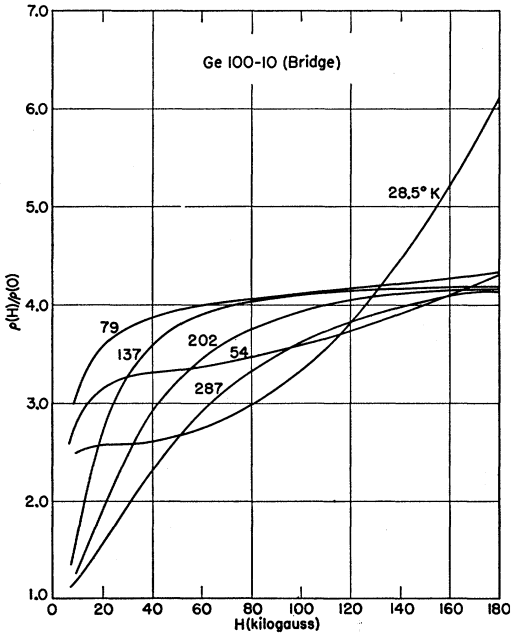


FIG. 5. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 100-10.

then placed in the magnet producing 300 kgauss but with shorter time constant. The results are shown in Fig. 6. Two important conclusions follow from these results. The first is that the variation of magnetoresistance in the quantum limit is linear with field strength, and the second is that the measured magnetoresistance

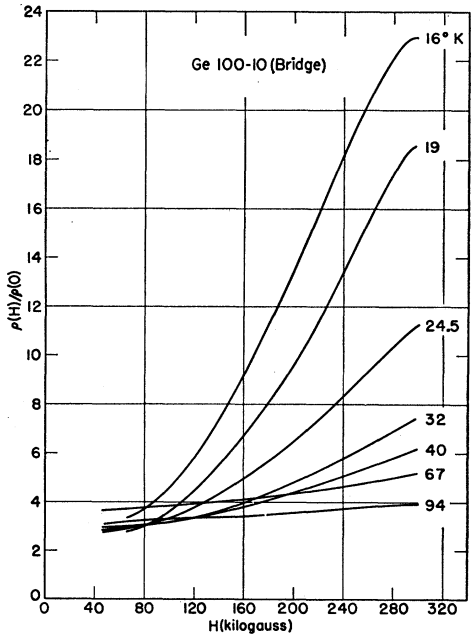


FIG. 6. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 100-10. Data taken with short time constant high-field magnet.

ratios do not depend on the time constant of the field as a comparison with the data of Fig. 5 will show.

Ge 100-10 was a bridge-shaped specimen. It was converted to a rod-shaped specimen by lapping off the bridge arms, and then measurements of the magnetoresistance were taken again. Quite strong differences in the data occurred in the region below 100°K. A plot of the temperature dependence of all the data taken on this sample at zero magnetic field and at the highest field strengths used is given in Fig. 7. The differences between rod- and bridge-shaped samples are clearly seen in this figure. Also to be noted is the fact that the temperature dependence of the resistivity is very little different in the strong field from that in zero field. This has been found to be the case for all samples studied.

Another disturbing factor relating to measurements on rod-shaped samples is the fact that the measured magnetoresistance depends on the direction of both the sample current and the magnetic field, this effect being greatest at the lowest temperatures. An extreme example of this effect is illustrated in Fig. 8, where the dependence of $\rho(H)/\rho(0)$ on the direction of the current and field is plotted versus the magnetic field for data taken at $T=21$°K on Ge 110-1$B$. These differences disappear on going to higher temperatures ($\sim$100°K). This effect is very much smaller in bridge-shaped samples, although variations of up to 30% have been observed in the most extreme case. The data on Ge 110-1$B$ shown in Fig. 9 have been averaged over both directions of current and field. The same is true for the data on Ge 110-2$B$ shown in Fig. 3. All other data were taken with only one direction of the field but were found to be independent of the direction of current at several check points. The occurrence of such spurious effects makes the data somewhat unreliable quantitatively. However, all of the phenomena reported here have been seen on numerous samples and are undoubtedly qualitatively correct. Finally, the data on the bridge-shaped sample Ge 111-10$B$ are plotted in Fig. 10. This data shows clear signs of a quantum transfer effect taking place in the two curves at 21° and 33°K.

TABLE III. Comparison of resistivities of pure and doped samples of germanium with and without a large applied magnetic field at room temperature and 25°K.

| Sample | $\rho_{\text{pure}}/\rho_{\text{doped}}$ $H=0$ $T=296$°K | $H=0$ $T=25$°K | $H=170$ kgauss $T=25$°K |
|---|---|---|---|
| Ge 100-10 | 12.0 | 4.0 | 8.2 |
| Ge 110-1 | 13.0 | 5.5 | 8.7 |
| Ge 111-10$B$ | 5.8 | 3.4 | 4.4 |

The question arises as to the influence of ionized impurity scattering on the magnetoresistance of the doped samples. Theoretical considerations (MO) indicate that although ionized impurity scattering may be dominant at zero magnetic field, a strong field will "switch off" impurity scattering and will leave acoustic scattering dominant for the concentration of impurities used in these experiments. Some evidence that this is taking place is provided in Table III. In this table ratios of the resistivity of the doped samples to the pure samples for the three directions are compared at different temperatures and field strengths. The ratios at room temperature and zero magnetic field should correspond to the ratio of carrier concentrations. The fact that these ratios are much lower at 25°K shows that ionized impurity scattering is practically dominant. On the other hand, all of these ratios are closer to their room-temperature value in the strong magnetic field at 25°K. This would be true if the carrier mobilities are

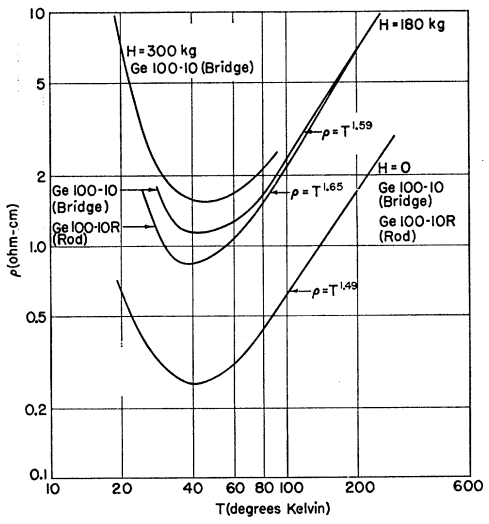


FIG. 7. Semi-log plot of resistivity versus temperature with and without an applied magnetic field for Ge 100-10.

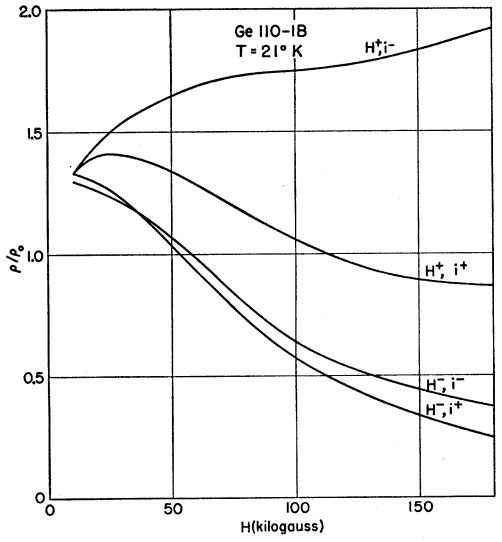


FIG. 8. Curves showing the dependence of $\rho(H)/\rho(0)$ on the direction of current and magnetic field for Ge 110-1$B$.

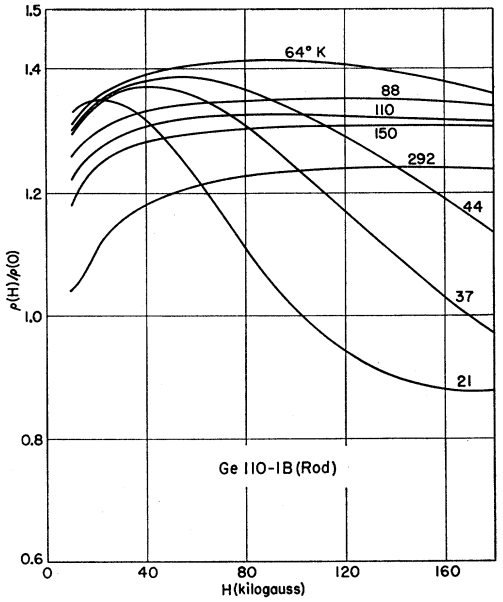


FIG. 9. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 110-1$B$. Data has been averaged over both directions of current and field.

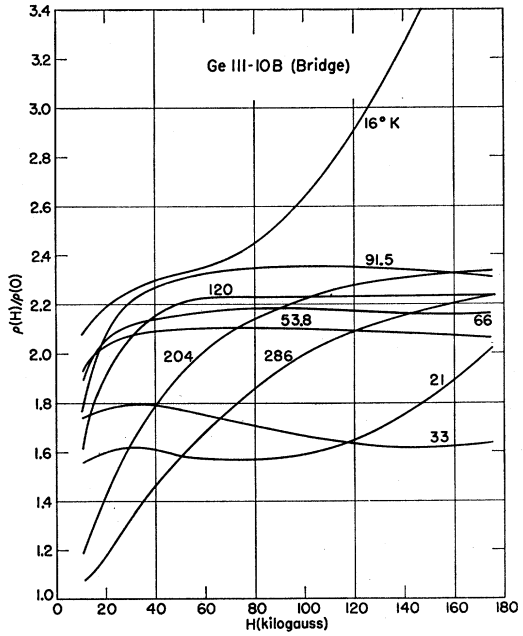


FIG. 10. Curves of $\rho(H)/\rho(0)$ vs $H$ with temperature as a parameter for Ge 111-10$B$.

the same for both pure and doped samples, i.e., if the ionized impurity scattering of the doped samples is "switched off" by the high magnetic field.

## V. DISCUSSION OF RESULTS

The measurements reported here have been of the nature of a survey of the typical magnetoresistive behavior encountered in single crystals of germanium at high magnetic fileds and over a range of temperatures from room temperature down to approximately 20°K. There are many features of the data which require further investigation. The qualitative nature of these results can be understood on the basis of the theoretical work of Miller and Omar. The reader is referred to their work for a more quantitative explanation.

## ACKNOWLEDGMENTS

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