ρ^e, (A.4) results in

$$\begin{aligned}\langle u|\rho^o|v\rangle &= \langle Iu|I\rho^o I|Iv\rangle^* \\ &= -\langle v|\rho^o|u\rangle.\end{aligned} \tag{A.5}$$

Thus, ρ^o is skew symmetric in a diagonal representation of ρ^e.

Suppose first that all the eigenvalues, μ_k, of ρ^e are positive. Then ρ can be expressed as

$$\rho = (\rho^e)^{\frac{1}{2}}[1+H](\rho^e)^{\frac{1}{2}}, \tag{A.6}$$

where H is also skew symmetric in a diagonal representation of ρ^e. The eigenvalues of H, called λ_k, appear in equal, opposite pairs, so that for integer s at least one of the λ_k must vanish. The non-negative condition results in

$$\lambda_k^2 \leqslant 1. \tag{A.7}$$

$$H_{ij} = \sum_k U_{ik}\lambda_k U_{jk}^*, \tag{A.8}$$

where U is a unitary matrix.

In these terms, it is desired to choose imaginary numbers $H_{ij} = -H_{ji}$, so as to maximize

$$\mathrm{Tr}\{(\rho^o)^2\} = \sum_{j,k} \mu_j\mu_k |H_{jk}|^2, \tag{A.9}$$

subject to the constraints

$$\sum_k |H_{jk}|^2 \leqslant 1, \tag{A.10}$$

$$\sum_{j,k} |H_{jk}|^2 \leqslant 2s. \tag{A.11}$$

This is achieved by taking

$$|H_{12}| = |H_{34}| = \cdots = |H_{2s-1,2s}| = 1, \tag{A.12}$$

where the μ_k are written in the descending order (3.14), and taking all other independent H_{ij} equal to zero. With this choice, $\mathrm{Tr}\{(\rho^o)^2\}$ is given by the equality in (3.13).

If some μ_α vanish, then the non-negative condition requires that, in a diagonal representation of ρ^e,

$$\rho_{k\alpha}{}^e = \rho_{\alpha k}{}^o = 0 \tag{A.13}$$

for such α and all k. The rows and columns containing such α can be removed from ρ without affecting the remainder of the proof.

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Interactions of 1.0-, 2.0-, and 3.0-Bev Protons with Ag and Br in Nuclear Emulsion*

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Stars produced in insensitive nuclear emulsions by 1.0–3.0 Bev protons have been classified into different groups depending on whether light fragments and/or fission fragments are emitted. Alpha particle spectra and angular distributions are presented for each of the various groups. The probability for light-fragment emission increases rapidly with increasing beam energy up to 2.0 Bev. The angular distribution of the light fragments is peaked forward but also shows a preference for emission at 90° to the beam. Fission events increase from ~3% of the interactions with Ag and Br at 1.0 Bev to ~11% at 3.0 Bev. Ranges and angular distributions are also given for the recoil and fission fragments.

INTRODUCTION

A SURVEY is presented of the various types of nuclear interactions observed in silver and bromine when emulsions of low sensitivity are irradiated by 1.0–3.0 Bev protons. Numerous studies have been made in the past with nuclear emulsions exposed to cosmic rays[1–3] and to accelerator beams below the Bev region.[4–6] Recently, there have been several investigations of similar nature in which emulsions have been exposed to beams with energies up to 9 Bev.[7–10]

The emphasis in the present investigation is on events of high excitation in which multi-charged particles are produced. The distributions in energy and angle of α particles, the distributions in range and angle of recoil and fission fragments, and angular distributions of light fragments ($2<Z\leq 6$) are presented. Data are also given on the types of events observed, on α and light-fragment multiplicities, and on how both vary with bombarding energy. These data are compared with existing evaporation calculations when applicable. In

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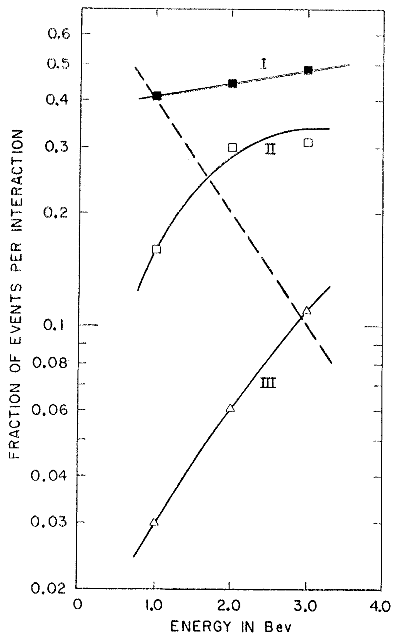

FIG. 1. Yields of various types of events from Ag and Br as a function of incident proton energy. I, stars with only alpha particles and a recoil; II, stars with light fragments, but excluding fission events; III, fission events. The dashed line represents the estimated fraction of events not observed because no alpha particles were emitted.

general, most calculations of high-energy nuclear interactions[11–13] do not consider fission or light-fragment emission. Recently, however, there have appeared[14,15] two Monte Carlo evaporation calculations which consider the emission of light fragments (up to Be^7) and which show fair agreement with certain experiments. Other experimental data,[16,17] however, on energy spectra of Li^8 fragments do not show good agreement with evaporation calculation.

EXPERIMENTAL

Ilford D.1 200 μ nuclear emulsions were exposed at the Brookhaven Cosmotron in the manner described previously.[7] The proton beam entered the emulsions at an angle of 10° to the surface. The plates were processed so that only α particles and heavier fragments were recorded and the interactions with Ag and Br nuclei were selected by the same criteria as those used before. Tracks were accepted with dip angles up to 50° in the undeveloped emulsion and suitable corrections were made for the rejection of tracks with larger dip angles and for those leaving the emulsion. As before, these corrections did not assume angular isotropy of the tracks. Some of the results were also calculated from those tracks whose dip angles were no greater than 30°. No appreciable differences were found outside of statistical uncertainty. Calibration curves for both α particles and Li^8 "hammer" tracks were used in the identification of the emitted particles. Alpha particles were visible up to energies of 50 Mev whereas the detection of light fragments was restricted only by the limits of the emulsion thickness.

In routine area scanning 803 stars were observed at 1.0 Bev, 633 stars at 2.0 Bev, and 514 stars at 3.0 Bev. These were classified into three types of events: group I, in which only α particles and a recoil were observed; group II, in which light fragments ($2<Z\leq 6$) were observed in addition to the alphas and recoil; group III, in which the fission events are represented. The latter are characterized by two short, very heavily ionizing tracks instead of a single recoil. Group III was then divided into two subgroups depending on the ratio of the fission fragment ranges. The events in which the ratios were $l_L/l_H\leq 2$ were called near-symmetric fissions, or subgroup III_{sym}. Those events in which the ratio of ranges, $2<l_L/l_H\leq 5$, were called asymmetric fissions, or subgroup III_{asy}. In order to improve the statistics on the fission events the plates were further scanned for $\sim$200 additional events of each of the two subgroups, at each of the three proton energies. Detailed data on the fission events will be presented in a forthcoming publication.[18]

RESULTS AND DISCUSSION

A summary of the various particles emitted, the types of events, and their relative abundances are presented in Table I and Figs. 1 and 2. Although the absolute numbers of protons that passed through the emulsions were not measured in these experiments, approximate cross sections can be estimated since the average total

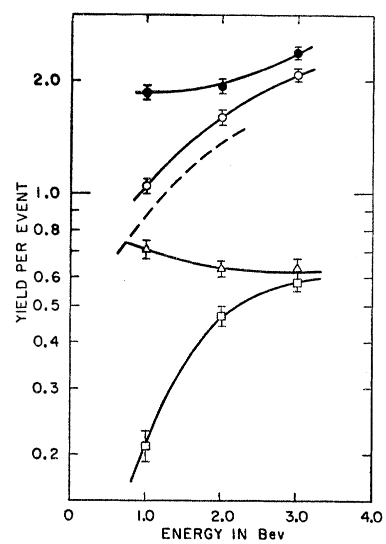

FIG. 2. Yields per interaction of alpha particles and of light fragments in various types of events as a function of incident proton energy. ○, alpha particles per non-fission event; ●, alpha particles per fission event; □, light fragments per non-fission event; △, light fragments per fission event. The dashed line shows the increase in calculated average excitation energy.

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TABLE I. Summary of data from stars produced by 1.0–3.0 Bev protons.

Beam energy (Bev)	Type of event	Stars observed	Corr. No. of events	Fraction of total events	No. of alphas	Alphas per event	No. of light fragments	Light fragments per event	Light fragments per alpha
1.0	0	...	537	0.40	...	...	...	...	...
	I	544	544	0.41	1011	1.86±0.06	...	...	...
	II	218	218	0.16	354	1.62±0.09	275	1.26±0.08	0.78±0.09
	0+I+II	762	1299	0.97	1365	1.05±0.05	275	0.21±0.02	0.20±0.02
	III	41	41	0.03	76	1.86±0.08	29	0.71±0.04	0.38±0.04
	Total	803	1340	1.00	1441	1.08±0.06	304	0.23±0.01	0.21±0.02
2.0	0	...	157	0.20	...	...	...	...	...
	I	351	351	0.44	742	2.12±0.09	...	...	...
	II	234	234	0.30	444	1.90±0.10	349	1.49±0.09	0.79±0.08
	0+I+II	585	742	0.94	1186	1.60±0.07	349	0.47±0.03	0.29±0.02
	III	48	48	0.06	93	1.94±0.08	30	0.63±0.03	0.31±0.03
	Total	633	790	1.00	1279	1.62±0.09	379	0.48±0.02	0.30±0.02
3.0	0	...	57	0.10	...	...	...	...	...
	I	272	272	0.48	647	2.38±0.11	...	...	...
	II	177	177	0.31	407	2.30±0.13	295	1.67±0.10	0.73±0.07
	0+I+II	449	506	0.89	1054	2.08±0.08	295	0.58±0.03	0.28±0.02
	III	65	65	0.11	155	2.39±0.09	41	0.63±0.04	0.26±0.03
	Total	514	571	1.00	1209	2.11±0.12	336	0.59±0.03	0.28±0.02

inelastic cross section for Ag and Br is ~1000 mb. Estimates based on Monte Carlo evaporation calculations[12,14,19] were made of the events missed when no α particles or light fragments were emitted. The fractions not included at 1.0, 2.0, and 3.0 Bev are 0.4, 0.2 and 0.1, respectively. These unobserved events in which only singly charged particles are produced were assigned to group 0. The alpha particle yields, of course, include He^3 and He^6. Since there is some difficulty in distinguishing Li tracks from the more numerous α tracks, the light fragment yields are somewhat uncertain. The errors given for all the data are only statistical standard deviations.

It may be seen from Fig. 1 and Table I that the probability for fission increases rapidly from 1.0 to 3.0 Bev. The probability for the light fragment emission in non-fissioning nuclei (Fig. 2 and Table I) also increases rapidly with bombarding energy to 2.0 Bev and somewhat more slowly to 3.0 Bev. The probability for α particle emission in this group also increases from 1.0 to 3.0 Bev, and parallels the rise in average excitation energy.[11] This may be seen by the dotted curve in Fig. 2. Since the fission events tend to be associated with higher excitation, they show more α particles and light fragments per event than do the non-fission events. In the fission events the α particle yields increase more slowly with bombarding energy and the light fragment yields may actually show a slight decrease. Figure 3 shows the alpha and light fragment prong distributions for all events combined and, separately, for near-symmetric fission events (III_{sym}). It is again seen that in the combined data the multiplicities of both α prongs and light fragment prongs increase with bombarding energy, but for the III_{sym} events little change in prong multiplicities is evident.

[19] J. Hudis (private communication).

Since the distributions in energy and angle of α particles and the distribution in angle of the light fragments showed no significant variations with bombarding energy, the data from 1.0, 2.0, and 3.0 Bev have been combined. Figure 4 shows the α spectra in the laboratory system for events of groups I, II, and III. It has been shown in a previous paper[7] that the α energy distribution in non-fission events may be made consistent with evaporation calculations, if one makes center-of-mass transformations assuming the direction of motion to be generally along the recoil path and using a velocity of 0.015–0.02 c. Center-of-mass motion accounts for the apparent excess of α particles well below the Coulomb barrier for emission from Ag and Br. Although

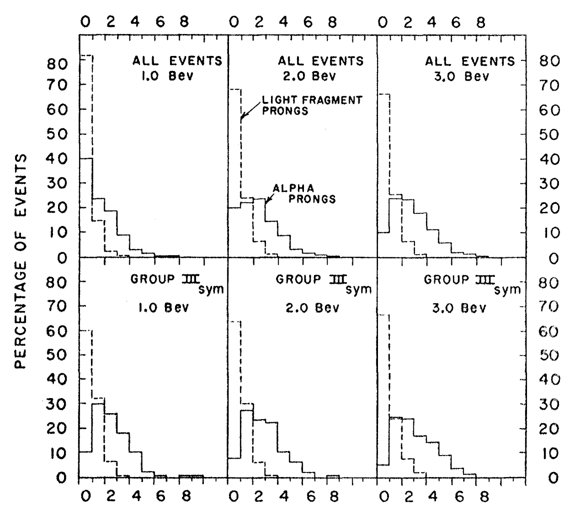

FIG. 3. Prong distributions for alpha particles (solid lines) and light fragments (dashed lines) from all events and, separately, from near-symmetric fission events.

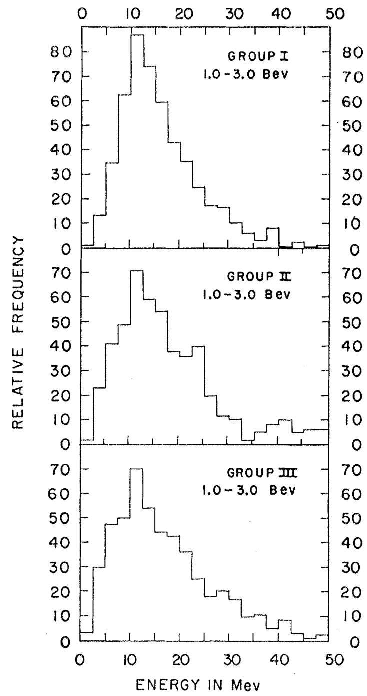

FIG. 4. Alpha spectra for group I, II, and III events combined for the three bombarding energies.

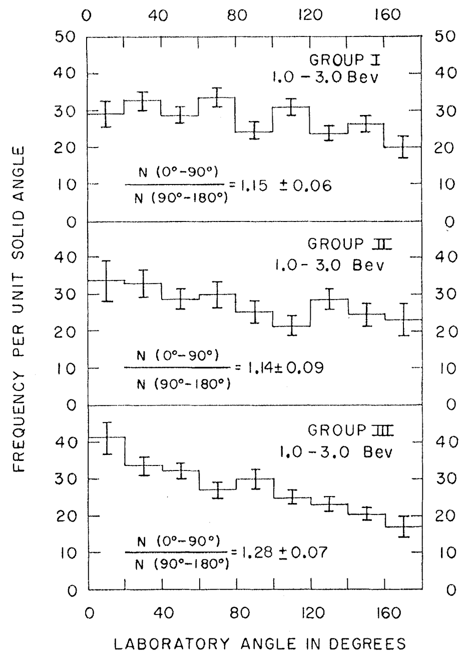

FIG. 5. Angular distributions of alpha particles for group I, II, and III events combined for the three bombarding energies.

many of the extremely low energy α particles observed in the fission events may also be accounted for in the same manner, there is still an appreciable number of "sub-barrier" α particles in the center-of-mass system. A more detailed discussion of α emission from fissioning nuclei will be presented in a forthcoming paper. Figure 5 shows the angular distribution of α particles in the laboratory system for each of the three groups with data for 1.0, 2.0, and 3.0 Bev combined. It has also been shown in reference 7 that this distribution for non-fissioning nuclei is consistent with the random emission of α particles from a moving system. Forward to backward ratios and mean angles derived from the angular distributions are given in Tables II and III. The increase in forward peaking from groups I and II to group III indicates the corresponding increase in momentum transfer from the incident proton.

Figure 6 shows the angular distribution of the light fragments for non-fission and fission events. The corresponding forward to backward ratios and mean angles are given in Tables II and III, respectively. The expected forward peaking in the laboratory system is again observed, but in addition there also appears to be a preference for emission near 90°. As seen from Fig. 6, the angular distribution is essentially unchanged when the limiting dip angle for accepting light fragments is reduced from 50° to 30°. The total corrected number of light fragments from data at the two limiting dip angles agreed within 8 percent. Thus there does not seem to be appreciable experimental bias in which steeper tracks are more likely to be incorrectly identified. Such preference for sidewise emission was also observed in cosmic ray produced stars by Perkins[3] for light fragments with energies below 9 Mev per nucleon.

This kind of angular distribution suggests that an appreciable fraction of the light fragments is produced in a fast process occurring at about the time of the nuclear cascade rather than in the slow evaporation process. Their emission appears to be associated with high excitation and large momentum transfer. Since these fragments are likely to have a very short mean free path in nuclear matter they must be produced near the surface where an inelastic collision may have occurred be-

TABLE II. Forward to backward ratios[a] for alpha particles, light fragments, recoils, and fission products.

Bev	Alphas			Light fragments			Recoils or fission fragments		
Group	1.0	2.0	3.0	1.0	2.0	3.0	1.0	2.0	3.0
I	1.14±0.10	1.11±0.11	1.20±0.13	...	...	...	5.6 ±0.7	2.8 ±0.4	3.5 ±0.5
II	1.28±0.19	1.23±0.16	0.94±0.12	1.5±0.2	1.8±0.3	1.8±0.3	3.1 ±0.6	1.8 ±0.3	2.3 ±0.4
III_{sym}	1.38±0.21	1.22±0.16	1.42±0.17	1.7±0.5	1.5±0.4	1.2±0.3	1.54±0.17	1.42±0.14	1.62±0.17
III_{asy}	1.51±0.25	0.89±0.13	1.30±0.17	1.4±0.3	1.6±0.4	1.5±0.4	1.53±0.18	1.42±0.14	1.36±0.14

[a] Not per unit solid angle.

TABLE III. Average angle per unit solid angle to the beam.

Group \ Bev	Alphas 1.0	Alphas 2.0	Alphas 3.0	Light fragments 1.0	Light fragments 2.0	Light fragments 3.0	Recoils or fission fragments 1.0	Recoils or fission fragments 2.0	Recoils or fission fragments 3.0
I	84°±4°	88°±5°	81°±5°	...	...	...	39°±3°	57°±4°	51°±4°
II	88 ±8	81 ±7	83 ±6	73°±7°	73°±5°	77°±7°	53 ±5	58 ±5	65 ±5
III_{sym}	73 ±7	82 ±7	72 ±6	75 ±13	74 ±10	75 ±13	69 ±5	71 ±4	71 ±4
III_{asy}	73 ±9	88 ±7	80 ±5	87 ±8	83 ±10	80 ±13	79 ±3	72 ±4	81 ±4

tween the incident proton and one of the nucleons. The resulting meson may be re-absorbed in the immediate vicinity and part of its energy transferred to a newly formed aggregate of nucleons—the light fragment. Such a fragmentation process[20] would be less probable at the forward surface of the nucleus because the incident proton after traversing most of the nuclear diameter is considerably degraded in energy. Thus there would be a deficiency of fragments ejected at small angles and a sidewise peaking is produced. Of course, that fraction of the light fragments which comes out by nuclear evaporation is expected to show only forward peaking.

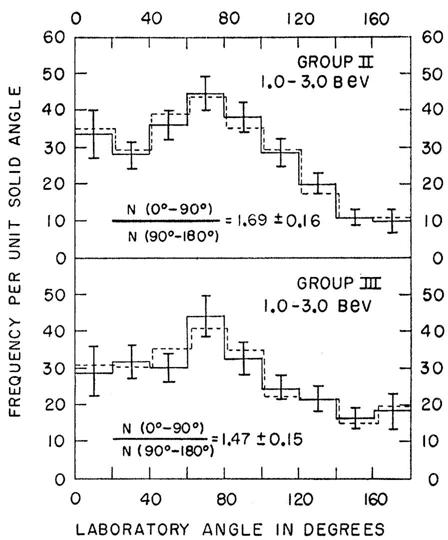

FIG. 6. Angular distributions of light fragments (1.0–3.0 Bev) for non-fission and fission events. Solid line, dip angle ≤50°; dashed line, dip angle ≤30°.

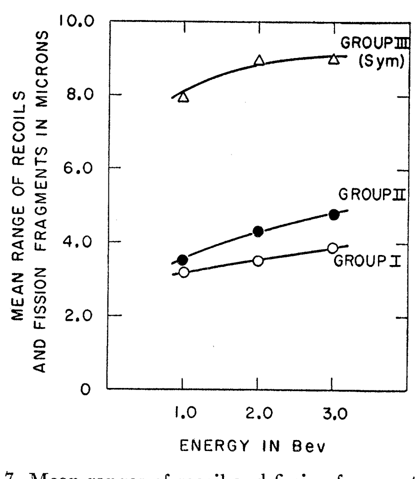

FIG. 7. Mean ranges of recoil and fission fragments as a function of bombarding energy.

The average lengths of the recoil tracks from group I and group II events and for fission fragments from group III_{sym} events are plotted in Fig. 7 as a function of bombarding energy. The group II curve lies above the group I curve because of the greater momentum transfer and because of the larger mass of the ejected particles. Figure 8 shows the distributions of recoil ranges from group I and group II events and of the fission fragment ranges from group III_{sym} events at the three bombarding energies. As expected the distributions are broader for group II than for group I. At 2.0 Bev bombarding energy an average recoil of mass 70 has a range of $\sim 3.5\ \mu$ in nuclear photographic emulsion corresponding to a velocity of $\sim 0.02c$ and an energy of ~ 14 Mev. Fission fragment ranges (group III_{sym}) average about $9\ \mu$ and vary from $\sim 3\ \mu$ to $\sim 20\ \mu$.

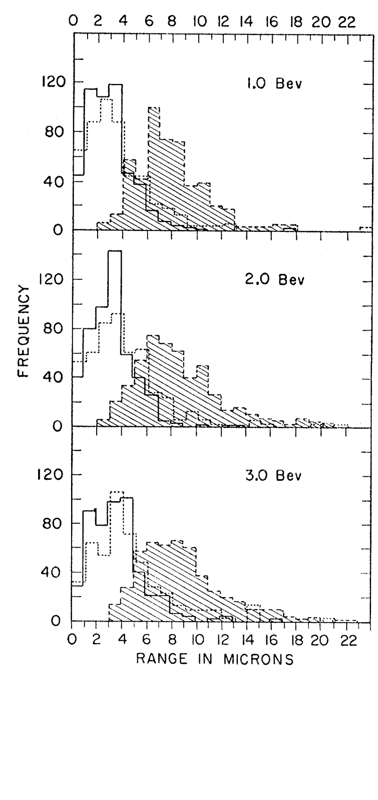

FIG. 8. Distributions of recoil and fission fragment ranges at each of the three bombarding energies. Solid line, group I recoils; dotted line, group II recoils; dashed line (shaded area), group III fission fragments.

[20] R. Wolfgang, E. W. Baker, A. A. Caretto, J. B. Cumming, G. Friedlander, and J. Hudis, Phys. Rev. 103, 394 (1956).

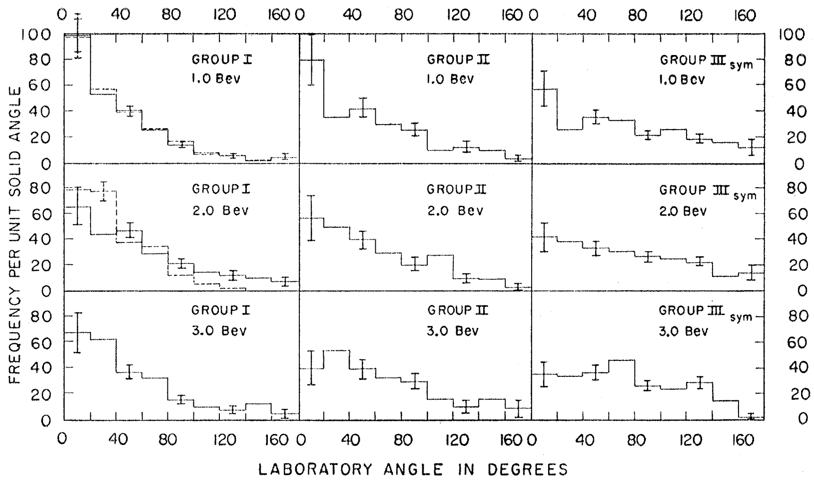

FIG. 9. Angular distributions of recoil and fission fragments at each of the three bombarding energies. The dashed lines are from calculations which do not include the effect of evaporation.

The angular distributions of recoil and fission fragments are shown for the three bombarding energies in Fig. 9. The corresponding forward to backward ratios and mean angles are given in Tables II and III, respectively. The dashed lines are from calculations by Porile[21] derived from earlier Monte Carlo cascade calculations of Metropolis *et al.*[11] and do not include the effect of nuclear evaporation. The calculated distributions, therefore, should show more forward peaking than the observed distributions since the evaporation of particles would tend to weaken the correlation between the direction of the recoil and that of the incident beam. This is clearly evident at 2.0 Bev; at 1.0 Bev the agreement between the calculated and the observed distributions is probably fortuitous. The angular distributions for the recoils from group II events are broader because of the larger recoil momenta of the emitted light fragments.

CONCLUSION

It appears that at least several different processes are involved when protons of a few Bev in energy interact with medium weight nuclei such as Ag and Br. The nuclear cascade and evaporation mechanisms certainly play a major role, but fission also occurs, and "fragmentation" as described above seems to contribute in some cases. The evidence from emulsion data indicates that the majority of alpha particles are emitted by evaporation. However, the light fragments may come from both evaporation and fragmentation. This kind of overlap among different mechanisms makes it difficult to determine the extent of each one. It should prove useful to study a given phenomenon by different techniques since each may emphasize a given process to a different extent. For example, radiochemical analyses give cross sections for producing specific products from the interaction of Bev protons with Ag, but they may give little information on the fission cross section of Ag, since many of the products can be formed either by fission or as spallation residues. Analysis of nuclear emulsion stars does give an estimate of total fission cross section of Ag and Br although the mass and charge can be determined only roughly. Further work is in progress by both of these techniques.

[21] N. T. Porile, Phys. Rev. **120**, 572 (1960).

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