Angular Asymmetry Theorems for Decay Products*

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The method of Eberhard and Good, for using decay angular distributions to determine the spins of unstable systems, is developed. Their inequality, which expresses the impossibility of getting too symmetric a decay pattern from an asymmetric initial state, is tightened. The results are generalized to include spinning decay products. The Adair analysis is generalized in the same framework.

1. INTRODUCTION

ANGULAR distributions of decay products have long provided an important, but seriously limited, means of investigating the spins of unstable systems. Suppose an object of spin s decays into two parts. In the rest frame of the unstable object, the angular distribution of either product has the form

$$I(\theta,\phi)=\sum_{L=0}^{\mathcal{L}}\sum_{M} a(L,M)Y_{LM}(\theta,\phi), \tag{1.1}$$

where

$$\mathcal{L}\leqslant 2s. \tag{1.2}$$

However, there is no direct path from the measured $I(\theta,\phi)$ to s, unless the distribution of initial spin orientations is known and not isotropic. Otherwise, possible "accidental" vanishing of many $a(L,M)$ leaves open the inequality (1.2).

Recently, Eberhard and Good[1] have proposed an ingenious way of bounding s from above, in a special case. They consider the angular distribution of π^+ from

$$K^0\rightarrow\pi^++\pi^-, \tag{1.3}$$

where the K^0 emerge in a fixed direction from

$$\pi^-+p\rightarrow\Lambda^0+K^0, \tag{1.4}$$

as a means of determining the K^0 spin. The essence of their observations is that the orientation distribution of the K^0 sample from (1.4) contains too few pure quantum states to result in isotropic decay if $s\geqslant 4$ (or $s\geqslant 2$ if the Λ spin is observed). Quantitatively, they obtain an inequality which, in our notation, reads

$$\frac{1}{Q}\leqslant\frac{2}{2s+1}+2\sum_{L,M}{}'\frac{|a(L,M)|^2}{G_0(s,L)^2}, \tag{1.5}$$

where $\sum'$ denotes the exclusion of $L=0$, $G_0(s,L)$ is given by (3.11), and Q is the number of spin states, of particles other than the K^0, which enter (1.4). Thus $Q=4$ if the protons are unpolarized and the Λ^0 spin direction is not selected, and $Q=2$ for unpolarized protons but selected Λ^0 spin direction. Inequality (1.5) can be used to eliminate too high s if the measured $\mathcal{L}$ is nonzero. If $\mathcal{L}$ is possibly zero, but the statistics are poor, the sum in (1.5) provides a quantitative statistical test which may be used to reject some or all of the values of s that are equal to or greater than Q.

Our principal purpose is to extend the method of Eberhard and Good to cover spinning decay products, and to tighten the inequality (1.5) in their special case. We hope, in addition, to elucidate their idea by our more geometric approach.

In Sec. 2, we describe the distribution of spin orientations of the unstable objects in terms of the statistical matrix ρ. The limitation to Q spin states is expressed as an inequality between ρ and Q. In Sec. 3, we analyze ρ into irreducible tensor components. These are expressed in terms of the measured angular distribution, for spinless decay products, to give two asymmetry theorems. Section 4 contains the generalization to spinning decay products. In Sec. 5, we specialize to consider the cylindrically symmetric case, obtaining a slight generalization of the Adair analysis.[2] The possibilities for extending the present work are remarked upon in Sec. 6.

Our notation for angular momentum quantities is that of Rose.[3] In statistical matrix matters, we follow Fano.[4]

2. THE SPIN DISTRIBUTION

For economy of language, we continue to refer to the particles in (1.4), but we regard the spins as arbitrary. The final state of (1.4) is an incoherent mixture of pure states α corresponding to the different initial spin states of π^-+p. The wave functions have asymptotic form

$$\psi(\alpha)=\textstyle\sum_{\mu,m} A(\alpha,\mu,m)\Lambda_{\sigma\mu}K_{sm}. \tag{2.1}$$

$\Lambda_{\sigma\mu}$ and K_{sm} are spin wave functions for spins σ and s, and for magnetic quantum numbers μ and m. A depends upon the orbital variables. Since different μ states do not interfere when measurements are made on the K^0 or its decay products, the "beam" of K^0 mesons in a fixed direction may be described as an incoherent mixture of pure states q whose spin wave functions are

$$\chi(q)=\textstyle\sum_m B(q,m)K_{sm}, \tag{2.2}$$

where q replaces α and μ. The different q states are not

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] P. Eberhard and M. L. Good, Phys. Rev. **120**, 1442 (1960).

[2] R. K. Adair, Phys. Rev. **100**, 1540 (1955).

[3] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[4] U. Fano, Revs. Modern Phys. **29**, 74 (1957).

necessarily orthogonal, and they need not enter the beam with equal weights.

The number of q states is not greater than Q, the number of admitted α states times the number of admitted μ states.

The K^0 spin distribution, for the beam in a fixed direction, is described by the statistical matrix ρ, whose elements are

$$\rho_{mn}=\sum_q P(q)B(q,m)^*B(q,n). \tag{2.3}$$

The quantity $P(q)$ is the weight of state q in the mixture, normalized to one, so that

$$\mathrm{Tr}\{\rho\}=1. \tag{2.4}$$

The purity of the beam is measured by $\mathrm{Tr}\{\rho^2\}$. We now prove that when $Q\leqslant 2s+1$,

$$\mathrm{Tr}\{\rho^2\}\geqslant 1/Q. \tag{2.5}$$

The states $\chi(q)$ lie in a Q-dimensional subspace of the $(2s+1)$-dimensional spin space. A unitary transformation will then change the statistical matrix into ρ', a matrix with nonzero elements in only the first Q rows and columns.

$$\mathrm{Tr}\{\rho^2\}=\mathrm{Tr}\{(\rho')^2\}=\sum_{\beta=1}^{Q}(\rho_{\beta\beta}')^2. \tag{2.6}$$

Since the $\rho_{\beta\beta}'$ are Q positive numbers whose sum is one, (2.6) implies (2.5).

Equation (2.5) simply says that the K^0 spin distribution is not less pure than a mixture of Q orthogonal states with equal weights.

3. THE ASYMMETRY THEOREMS

We now wish to relate (2.5) to the measurable intensity coefficients $a(L,M)$. For clarity, we first assume that the decay products are spinless. In that case, $(2s+1)$ is odd.

The statistical matrix may be written in terms of irreducible tensor operators as

$$\rho=\sum_{L=0}^{2s}\sum_M b(L,M)T_{LM}, \tag{3.1}$$

where T_{LM} is defined by its matrix elements,[5]

$$\langle sm|T_{LM}|sn\rangle=\left(\frac{2L+1}{2s+1}\right)^{\frac{1}{2}}C(sLs;nM)\delta_{n+M,m}. \tag{3.2}$$

The vector coefficients C are the usual orthogonal transformation coefficients, with the Condon-Shortley phase convention.[3]

$$T_{LM}{}^\dagger=(-1)^M T_{L,-M}, \tag{3.3}$$

$$\mathrm{Tr}\{T_{LM}T_{L'M'}{}^\dagger\}=\delta_{LL'}\delta_{MM'}, \tag{3.4}$$

$$b(L,M)=\mathrm{Tr}\{\rho T_{LM}{}^\dagger\}, \tag{3.5}$$

$$b(0,0)=(2s+1)^{-\frac{1}{2}}. \tag{3.6}$$

Since ρ is Hermitean,

$$b(L,-M)=(-1)^M b(L,M)^*, \tag{3.7}$$

$$\mathrm{Tr}\{\rho^2\}=\sum_{L,M}|b(L,M)|^2. \tag{3.8}$$

$$\begin{aligned} a(L,M)&=\int Y_{LM}(\theta,\phi)^*I(\theta,\phi)d\Omega \\ &=\mathrm{Tr}\{\rho Y_{LM}{}^*\} \\ &=\left(\frac{2s+1}{2L+1}\right)^{\frac{1}{2}}\langle s\|Y_L\|s\rangle b(L,M). \end{aligned} \tag{3.9}$$

By use of the reduced matrix elements $\langle s\|Y_L\|s\rangle$ given by Rose,[6] this becomes

$$a(L,M)=G_0(s,L)b(L,M), \tag{3.10}$$

where

$$G_0(s,L)=\left(\frac{2s+1}{4\pi}\right)^{\frac{1}{2}}C(sLs;00). \tag{3.11}$$

The quantity $G_0(s,L)$ vanishes for integral values of s with odd values of L.

We define the even-L and odd-L parts of ρ, called ρ^e and ρ^o, according to (3.1). The intensity coefficients completely determine ρ^e through (3.10):

$$\rho_{mn}{}^e=\sum_L\frac{a(L,m-n)}{G_0(s,L)}\langle sm|T_{L,m-n}|sn\rangle. \tag{3.12}$$

While there is no way to measure ρ^o, its contribution to (3.8) can be given an upper bound. The bound can be understood from the condition that ρ is a nonnegative matrix, whose even and odd parts under time inversion[7] are ρ^e and ρ^o. Then ρ^o must be smaller than ρ^e in the sense that $\langle u|\rho^o|u\rangle\leqslant\langle u|\rho^e|u\rangle$ for every spin function u. Otherwise, even if $\langle u|\rho^o|u\rangle$ is positive, it can be reversed by replacing u with its time-inverted spin function, to make $\langle u|\rho|u\rangle$ negative.

Quantitatively, the nonnegative condition yields

$$\mathrm{Tr}\{(\rho^o)^2\}\leqslant 2(\mu_1\mu_2+\mu_3\mu_4+\cdots+\mu_{2s-1}\mu_{2s}), \tag{3.13}$$

where the μ_k are the eigenvalues of the measured ρ^e, written in descending order, i.e.,

$$1\geqslant\mu_1\geqslant\mu_2\geqslant\cdots\geqslant\mu_{2s}\geqslant\mu_{2s+1}\geqslant 0. \tag{3.14}$$

This is proved in the Appendix. While formal use is made of time inversion, no dynamical assumptions are introduced.

Inequality (3.13) leads directly to the first asymmetry theorem,

$$\begin{aligned} 1/Q\leqslant(\mu_1+\mu_2)^2+(\mu_3+\mu_4)^2+\cdots \\ +(\mu_{2s-1}+\mu_{2s})^2+\mu_{2s+1}{}^2. \end{aligned} \tag{3.15}$$

[5] The T_{LM} are discussed further by M. Peshkin, Phys. Rev. **121**, 636 (1961). The normalization is different. Equations (2.16) should read $Q_2=\frac{3}{2}J_z{}^2-\frac{1}{2}J^2$.

[6] Reference 3, p. 89.

[7] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957), p. 51.

It is advantageous for some purposes to isolate the isotropic contribution to (3.15) by introducing

$$\nu_k = \mu_k - (2s+1)^{-1}. \tag{3.16}$$

$$\frac{1}{Q} - \frac{4s+1}{(2s+1)^2} \leqslant -\frac{2\nu_{2s+1}}{2s+1} + (\nu_1+\nu_2)^2 + \cdots + (\nu_{2s-1}+\nu_{2s})^2 + \nu_{2s+1}^2. \tag{3.17}$$

The right-hand side of (3.17) vanishes for isotropic $I(\theta,\phi)$, and is otherwise positive.

Theorem (3.15) can be weakened by using the Cauchy inequality to obtain

$$1/Q \leqslant 2\,\mathrm{Tr}\{(\rho^e)^2\} - \mu_{2s+1}^2, \tag{3.18}$$

and again by discarding the last term to leave the inequality (1.5) of Eberhard and Good.

The only characteristic of the K^0 spin distribution used so far is (2.5). A stronger result than (3.15) can be obtained when ρ possesses a mirror symmetry. We choose the quantization (z) axis perpendicular to the production plane in (1.4). Then, if any polarization of the initial state of (1.4) is in the z direction, that state is invariant under space inversion followed by 180° rotation about the z axis. If the production reaction conserves parity, and if additionally any selection of the Λ^0 spin is made with reference to the z axis, then ρ must share the invariance property of the initial state. Since ρ deals only with angular momentum, it is automatically invariant under space inversion. Under the rotation,

$$T_{LM} \to (-1)^M T_{LM}. \tag{3.19}$$

Then the invariance requires that only even M contribute to ρ in (3.1).

Because of this mirror symmetry of ρ^e, its eigenspin functions contain only even or only odd m, and the equally mirror-symmetric ρ^o does not mix the two kinds. We call the eigenvalues of the odd-m functions $\kappa_1 \cdots \kappa_s$, in descending order, and those of the even-m functions $\kappa_{s+1} \cdots \kappa_{2s+1}$, in separately descending order. The proof given in the Appendix applies separately to the two sets, and results in the second asymmetry theorem,

$$1/Q \leqslant (\kappa_1+\kappa_2)^2 + \cdots + (\kappa_{2s-1}+\kappa_{2s})^2 + \kappa_{2s+1}^2 \tag{3.20}$$

for even s, or

$$1/Q \leqslant (\kappa_1+\kappa_2)^2 + \cdots + (\kappa_{s-2}+\kappa_{s-1})^2 + \kappa_s^2 + (\kappa_{s+1}+\kappa_{s+2})^2 + \cdots + (\kappa_{2s}+\kappa_{2s+1})^2 \tag{3.21}$$

for odd s. The second theorem can be written in a form analogous to (3.17), through the analog of (3.16).

As a practical matter, it is evident that the asymmetry theorems are stronger than the inequality (1.5) of Eberhard and Good, but more troublesome. The second theorem, when applicable, is stronger than the first, and less troublesome, since it involves diagonalizing smaller matrices. If $I(\theta,\phi)$ is isotropic with good statistical accuracy, (1.5) gives $s<Q$, and (3.17) shows that the asymmetry theorems do no better. In case of isotropic $I(\theta,\phi)$ with poor statistical accuracy, the asymmetry theorems have a quantitative advantage. Suppose, for example, that $b(2,0)$ appears to be negative in the $Q=2$ experiment, and that all other nonisotropic terms are negligible. Then the maximum probable value of $|b(2,0)|^2$ which still rejects $s=2$ is 0.05 by use of the Eberhard-Good inequality, 0.08 by use of the first asymmetry theorem, and 0.11 by use of the second asymmetry theorem.

4. SPINNING DECAY PRODUCTS

The asymmetry theorems are easily generalized to include spinning decay products, provided that the resultant spin τ and the orbital angular momentum l of the decay products are both definite. These conditions are necessarily met in the important case where one decay product has spin one-half and the other is spinless, if the decay process conserves parity. An interesting example is the $Q=2$ experiment,

$$K^- + p \to \pi + Y^*, \tag{4.1}$$

followed by

$$Y^* \to \Lambda^0 + \pi, \tag{4.2}$$

to determine the Y^* spin.[8] We again consider all the spins to be arbitrary, but refer to this special case for brevity. Q always refers to the production process, and does not include the $2\tau+1$ states of the decay products.

The even-L reduced matrix elements are given in the coupling scheme $l+\tau=s$ by[9]

$$\langle l+\tau=s \| Y_L \| l+\tau=s\rangle = (-1)^{\tau+l-s}[(2l+1)(2s+1)]^{\frac{1}{2}} \times W(lsls;\tau L)\langle l\|Y_L\|l\rangle, \tag{4.3}$$

where W is the Racah coefficient, and $G_0(s,L)$ is replaced by

$$G_\tau(s,L) = (-1)^{\tau+l-s}(2s+1)W(lsls;\tau L) \times \left(\frac{2l+1}{4\pi}\right)^{\frac{1}{2}} C(lLl;00). \tag{4.4}$$

In particular,

$$G_{\frac{1}{2}}(s,L) = \frac{2s+1}{(l+s+\frac{1}{2})(l+s+\frac{3}{2})} \times \left[\frac{(2l+1)(l+s+L+\frac{3}{2})(l+s-L+\frac{1}{2})}{4\pi}\right]^{\frac{1}{2}} \times C(lLl;00). \tag{4.5}$$

For integral values of s, (4.4) replaces (3.11), but the asymmetry theorems and the Eberhard-Good inequality are otherwise unchanged. For half-integral

[8] This experiment has also been considered by R. H. Capps, Phys. Rev. **122**, 929 (1961), and private communication, and by R. Gatto and H. P. Stapp, Phys. Rev. **121**, 1553 (1961).
[9] Reference 3, p. 119.

values of s, (1.5) is again unchanged except that G_0 is replaced by $G_{\frac{1}{2}}$, but the first asymmetry theorem becomes

$$1/Q \leqslant (\mu_1+\mu_2)^2+\cdots+(\mu_{2s}+\mu_{2s+1})^2. \qquad (4.6)$$

For half-integral values of s, the spin eigenfunctions of ρ^e separate according to even and odd values of $(m+\frac{1}{2})$. In case of mirror symmetry, the second theorem gives

$$1/Q \leqslant (\kappa_1+\kappa_2)^2+\cdots+(\kappa_{2s}+\kappa_{2s+1})^2 \qquad (4.7)$$

for even $(s+\frac{1}{2})$, and

$$1/Q \leqslant (\kappa_1+\kappa_2)^2+\cdots+(\kappa_{s-\frac{3}{2}}+\kappa_{s-\frac{1}{2}})^2+\kappa_{s+\frac{1}{2}}^2 + (\kappa_{s+\frac{3}{2}}+\kappa_{s+\frac{5}{2}})^2+\cdots+(\kappa_{2s-1}+\kappa_{2s})^2+\kappa_{2s+1}^2 \qquad (4.8)$$

for odd $(s+\frac{1}{2})$. Inequality (4.7) is stronger than (4.6) because all the κ_k are not in descending order.

The results in this section may be used to test l as well as s in some cases, as may the condition

$$\mathfrak{L} \leqslant 2l, \qquad (4.9)$$

which replaces (1.7). For example, suppose reactions (4.1) and (4.2) are observed with polarized protons, so that $Q=1$, and that the decay distribution is isotropic except for a suspected $L=2$ contribution. Then,

$$\textstyle\sum_M |a(1,M)|^2 < 1/20\pi \qquad (4.10)$$

eliminates $s=\frac{3}{2}$. The unpolarized $Q=2$ experiment can only eliminate spin $\frac{5}{2}$.

5. THE CASE OF CYLINDRICAL SYMMETRY

Adair[2] pointed out some time ago that there is an advantage to taking the unstable particles in the forward or backward direction. In the reaction (1.4), for example, the magnetic quantum number of the Λ^0 is determined by those of the proton and the K^0, if the beam direction is taken as the quantization axis. Quite generally, Q is reduced to Q_i, that of the initial state of the production process, as long as any polarization is in the direction of the beam.

The cylindrical symmetry of the problem guarantees that only $M=0$ terms contribute to the statistical matrix. Then ρ^e and ρ^o are simultaneously diagonal in the representation (2.3), i.e.,

$$\rho_{-m,-m}{}^e = \rho_{mm}{}^e, \qquad (5.1)$$

$$\rho_{-m,-m}{}^o = -\rho_{mm}{}^o. \qquad (5.2)$$

The non-negative condition reduces to

$$|\rho_{mm}{}^o| \leqslant \rho_{mm}{}^e, \qquad (5.3)$$

and the first asymmetry theorem becomes

$$1/Q_i \leqslant 2\,\mathrm{Tr}\{(\rho^e)^2\} - (\rho_{00}{}^e)^2 \qquad (5.4)$$

for integral values of s. For half-integral values of s, the last term is absent.

The qualitative advantage of (5.4) over (3.15) or (4.6) lies in the reduction of Q. For integral values of s, there is an additional quantitative advantage if ρ_{00} is not the smallest matrix element of ρ^e.

In case of mirror symmetry, ρ^o vanishes. Then the second asymmetry theorem becomes

$$1/Q_i \leqslant \mathrm{Tr}\{(\rho^e)^2\} = \frac{1}{2s+1} + \textstyle\sum_L{}' \dfrac{|a(L,0)|^2}{G_\tau(s,L)^2}, \qquad (5.5)$$

where again $\sum'$ excludes $L=0$. Restriction (5.5) is evidently much stronger than any of the previous results. If $Q_i=2$, the equality holds in (5.5).

6. POSSIBLE EXTENSIONS

It is natural to inquire whether the asymmetry theorems presented here can be tightened further. Since ρ^o is constructed in (A.12) of the Appendix to realize the equality, they plainly cannot be improved upon without additional physical assumptions about ρ^o.

Somewhat different theorems can be constructed in the same spirit by using $1/Q^3 \leqslant \mathrm{Tr}\{\rho^4\}$. These, however, involve ρ^o in a complicated way, and the chance of obtaining an improvement over the present results seems remote. Expression (5.5), which applies when ρ^o vanishes, is not improved by using ρ^4.

Finally, if the decay goes through more than one l value, or into more than two particles, we anticipate that the asymmetry theorems will be weakened, but still useful. Analogous results for the charge distribution in isospin-invariant processes[10] encourage this expectation.

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APPENDIX

For given non-negative ρ^e, it is desired to choose ρ^e so as to maximize $\mathrm{Tr}\{(\rho^o)^2\}$, subject to the constraint that ρ be non-negative.

The antilinear "time reversal" operator I is defined by

$$IK_{sm} = (-1)^{s+m} K_{s,-m}, \qquad (A.1)$$

$$I(\alpha u+\beta v) = \alpha^* Iu + \beta^* Iv, \qquad (A.2)$$

where α, β are numbers and u, v spin functions.

$$I\rho^e I = \rho^e, \qquad (A.3)$$

$$I\rho^o I = -\rho^o. \qquad (A.4)$$

From (A.3), the eigenfunctions of ρ^e can be chosen to be even under I. Then, if u and v are eigenfunctions of

[10] Reference 5, Eq. (4.12).

ρ^e, (A.4) results in

$$\langle u|\rho^o|v\rangle = \langle Iu|I\rho^o I|Iv\rangle^* \\ = -\langle v|\rho^o|u\rangle. \tag{A.5}$$

Thus, ρ^o is skew symmetric in a diagonal representation of ρ^e.

Suppose first that all the eigenvalues, μ_k, of ρ^e are positive. Then ρ can be expressed as

$$\rho = (\rho^e)^{\frac{1}{2}}[1+H](\rho^e)^{\frac{1}{2}}, \tag{A.6}$$

where H is also skew symmetric in a diagonal representation of ρ^e. The eigenvalues of H, called λ_k, appear in equal, opposite pairs, so that for integer s at least one of the λ_k must vanish. The non-negative condition results in

$$\lambda_k^2 \leqslant 1. \tag{A.7}$$

$$H_{ij} = \sum_k U_{ik}\lambda_k U_{jk}^*, \tag{A.8}$$

where U is a unitary matrix.

In these terms, it is desired to choose imaginary numbers $H_{ij} = -H_{ji}$, so as to maximize

$$\mathrm{Tr}\{(\rho^o)^2\} = \sum_{j,k} \mu_j\mu_k |H_{jk}|^2, \tag{A.9}$$

subject to the constraints

$$\sum_k |H_{jk}|^2 \leqslant 1, \tag{A.10}$$

$$\sum_{j,k} |H_{jk}|^2 \leqslant 2s. \tag{A.11}$$

This is achieved by taking

$$|H_{12}| = |H_{34}| = \cdots = |H_{2s-1,2s}| = 1, \tag{A.12}$$

where the μ_k are written in the descending order (3.14), and taking all other independent H_{ij} equal to zero. With this choice, $\mathrm{Tr}\{(\rho^o)^2\}$ is given by the equality in (3.13).

If some μ_α vanish, then the non-negative condition requires that, in a diagonal representation of ρ^e,

$$\rho_{k\alpha}{}^e = \rho_{\alpha k}{}^o = 0 \tag{A.13}$$

for such α and all k. The rows and columns containing such α can be removed from ρ without affecting the remainder of the proof.

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Interactions of 1.0-, 2.0-, and 3.0-Bev Protons with Ag and Br in Nuclear Emulsion*

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Stars produced in insensitive nuclear emulsions by 1.0–3.0 Bev protons have been classified into different groups depending on whether light fragments and/or fission fragments are emitted. Alpha particle spectra and angular distributions are presented for each of the various groups. The probability for light-fragment emission increases rapidly with increasing beam energy up to 2.0 Bev. The angular distribution of the light fragments is peaked forward but also shows a preference for emission at 90° to the beam. Fission events increase from ~3% of the interactions with Ag and Br at 1.0 Bev to ~11% at 3.0 Bev. Ranges and angular distributions are also given for the recoil and fission fragments.

INTRODUCTION

A SURVEY is presented of the various types of nuclear interactions observed in silver and bromine when emulsions of low sensitivity are irradiated by 1.0–3.0 Bev protons. Numerous studies have been made in the past with nuclear emulsions exposed to cosmic rays[1–3] and to accelerator beams below the Bev region.[4–6] Recently, there have been several investigations of similar nature in which emulsions have been exposed to beams with energies up to 9 Bev.[7–10]

The emphasis in the present investigation is on events of high excitation in which multi-charged particles are produced. The distributions in energy and angle of α particles, the distributions in range and angle of recoil and fission fragments, and angular distributions of light fragments ($2<Z\leq 6$) are presented. Data are also given on the types of events observed, on α and light-fragment multiplicities, and on how both vary with bombarding energy. These data are compared with existing evaporation calculations when applicable. In

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