Beta-Decay Coupling Constant and the *ft* Value of O^{14}

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A calculation is made of the effect of a charge-dependent internucleon potential on the $O^{14}(0^+, T=1) \to N^{14}(0^+, T=1)$ β-decay matrix element. It is found that a not unreasonable strength and form for such a potential can lead to a reduction in the matrix element $\approx 1\%$ which is of the right order of magnitude to resolve the present discrepancy between the β- and μ-decay polar vector coupling constants.

THE remarkable near equality of the polar vector coupling constant G_V in beta decay and the coupling constant G_μ in μ decay led Feynman and Gell-Mann,[1] in the context of a universal Fermi interaction, to propose that the strangeness-conserving polar vector part of the current responsible for the weak interactions should be conserved. It is then expected that when electromagnetic radiative corrections are taken into account[2–4] an exact equality in G_V and G_μ is obtained. In spite of uncertainties in the evaluation of these corrections it has proved difficult, if not impossible, to obtain such an equality from the experimental data[5] and an outstanding discrepancy of at least 1–2% in G^2 seems to remain.

Now G_V is most accurately determined from the $0^+(T=1, T_z=1) \to 0^+(T=1, T_z=0)$ beta transition of $O^{14} \to N^{14}$, and it is conceivable that the above discrepancy arises from a deviation of the $O^{14} \to N^{14}$ nuclear matrix element from the usually accepted value of $\sqrt{2}$. The deviation can be a consequence of (a) charge-dependent effects leading to isotopic spin impurities, (b) relativistic effects, and (c) contributions from second-forbidden matrix elements. Estimates of (a) taking into account the Coulomb potential[6] and of (b)[7,8] and (c)[8] have suggested, however, that these effects are an order of magnitude too small to reconcile G_V with G_μ.

The purpose of this paper is to report a calculation of the additional effects stemming from a charge dependence of the internucleon potential. Such a charge dependence is to be expected both from the mass difference of the neutral and charged pions[9,10] and the difference in the electromagnetic radiative corrections to the $nn\pi^0$, $np\pi^-$, and $pp\pi^0$ vertices.[10]

Consider a charge-independent nuclear Hamiltonian H_0 with eigenfunctions ψ_ν and eigenvalues E_ν, each eigenfunction corresponding to some definite isotopic spin T. States of the same and different isotopic spin are then mixed in by a charge-dependent perturbation H' which we take to be

$$H' = H_c + H_n,$$

with

$$H_c = \tfrac{1}{4}\sum_{i<j}\left\{\frac{e^2}{r_{ij}}(1+\tau_z^{(i)})(1+\tau_z^{(j)}) - \frac{e^2}{r_{ij}}\right\},$$

$$H_n = V_0 \sum_{i<j}\{[p(\tau_z^{(i)}+\tau_z^{(j)})+q\tau_z^{(i)}\tau_z^{(j)}]+[r(\tau_z^{(i)}+\tau_z^{(j)}) + s\tau_z^{(i)}\tau_z^{(j)}]\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}\}\exp(-r_{ij}^2/\mu^2).$$

H_c is the charge-dependent part of the usual Coulomb potential and H_n the most general (apart from radial dependence) static, charge-dependent, central potential that can be constructed. p, q, r, and s measure the extent of the charge dependence of the internucleon potential but H_n is *not* to be regarded as the actual charge-dependent part of the potential. Rather it is an *effective* potential suitable for use in shell model calculations. V_0 is the strength of the central part of the effective two-body charge-independent potential[11] and the remaining symbols have their usual meanings.

In essentially the notation of MacDonald[6] the square of the Fermi matrix element for the $O^{14} \to N^{14}$ transition is given explicitly by

$$M_F^2 = 2(1-\delta),$$

where

$$\delta = \textstyle\sum_\nu [(a_\nu^{(1)} - b_\nu^{(1)})^2 + (a_\nu^{(0)})^2 + (a_\nu^{(2)})^2 + (b_\nu^{(2)})^2 - 2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}].$$

Here $a_\nu^{(T)}$ and $b_\nu^{(T)}$ are the amplitudes with which states ψ_ν with isotopic spin T are mixed into the relevant zero-order states of N^{14} and O^{14}, respectively. They are calculated by first-order perturbation theory, treating H' as a perturbation, and have the form $\langle \nu T | H' | 01\rangle / \Delta E_\nu$, where ΔE_ν is an appropriate energy denominator and the zero-order states in O^{14} and N^{14} are labeled $\nu=0$, $T=1$.

In shell model notation the zero-order states have the configuration $(1s)^4(1p)^{10}$ and there are two types of

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admixed states to be considered: (i) states which involve just a recoupling of the $1p$-shell nucleons, and (ii) states in which nucleons are excited out of the $1s$ and $1p$ shells into higher levels. We are concerned here with excited states of type (i) since such states have associated with them a much smaller energy denominator than those of type (ii) and are therefore expected to give the dominant contribution to δ. There is, in fact, only one such state in N^{14} and O^{14}, namely, another $T=1$, 0^+ state.

The two $T=1$, 0^+ states can be written as linear combinations of the $^1S_0$ and $^3P_0$ $1p$-shell states, the appropriate combination being determined by the assumed nature of the charge-independent internucleon potential. A treatment of these states has been given by Visscher and Ferrell[11] who obtain

$$\psi_{m_T}=[\alpha\, {}^1S_0+\beta\, {}^3P_0]\chi_{1m_T},$$
$$\phi_{m_T}=[\beta\, {}^1S_0-\alpha\, {}^3P_0]\chi_{1m_T},$$

where χ_{1m_T} is an isotopic spin triplet function with $m_T=1$ corresponding to O^{14} and $m_T=0$ to N^{14}. The amplitudes α and β can be determined either by diagonalizing the relevant matrix after assuming some form for H_0 or semiempirically. Visscher and Ferrell[11] adopt both approaches but the values they obtain for α and β are not significantly different.

The calculation of $a^{(1)}$ and $b^{(1)}$ due to the mutual admixing of ψ_m and ϕ_m by H' is straightforward and we obtain finally

$$\Delta E(a^{(1)}-b^{(1)})=\alpha\beta\{8(r-s)L+[10(q-p)+6(r-s)]K\},$$

where L and K are the usual $1p$-shell direct and exchange integrals evaluated for a potential of strength V_0. There is no contribution from the Coulomb potential H_c since we are dealing either with two neutron holes or a neutron and a proton hole. Our results are not sensitive to the values of L and K and we take the values quoted by Visscher and Ferrell,[11] $L=-7.05$ Mev and $K=-1.12$ Mev to give

$$\Delta E(a^{(1)}-b^{(1)})=\alpha\beta[63.1(s-r)-11.2(q-p)].$$

The main evidence regarding the charge-dependent nature of nuclear forces comes from the difference in the singlet scattering lengths for proton-proton and neutron-proton scattering. This difference can be accounted for if the pp and np internucleon potentials differ by $\approx 3\%$,[12,13] and we therefore assume that the parameters $p, \cdots s$ may be of the order of magnitude 0.03. Clearly, the largest effect is then obtained if H_n has a strong spin dependence ($s, r \gg p, q$). Such a dependence is, in fact, obtained in the lowest order charge-dependent potential deduced by Riazuddin[10] although such a low-order calculation is probably not very significant. Nevertheless in order to obtain some estimate of the possible size of charge-dependent effects, we put $q-p=0$ and $s-r=0.03$. Taking $\Delta E=6.3$ Mev, the experimentally observed separation between the two 0^+, $T=1$ levels in N^{14} and $\alpha\beta=0.49$, the semiempirical value used by Visscher and Ferrell,[11] then gives $\delta\approx 2\%$ which is an effect of the required order of magnitude.

Admixtures of states of type (ii) due to H' could also lead to further small contributions to δ but because of the associated large energy denominators are unlikely to be very appreciable. Calculations to estimate this additional effect are under way.

Clearly the foregoing figures should be taken as order of magnitude estimates only. However the calculations reported here do indicate that the O^{14} decay is sensitive to a charge dependence of the internucleon potential and that a not unreasonable value for this dependence can lead to $\delta\approx 1-2\%$ which may well be sufficient to reconcile G_V with G_μ.

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