for the N^{14}—F^{19} collision could not be estimated by optical model calculations, such as were used to analyze N^{14}—C and N^{14}—Be^9 elastic scattering.[33] Still available were the heavy-ion barrier-penetration calculations of Thomas,[32] which used a square-well potential with radius 1.5 $A^{\frac{1}{3}}$ f. Interpolation from Thomas' results was accomplished as follows:

(1) For each of the collisions C^{12}—Al^{27}, O^{16}—Al^{27}, N^{14}—Al^{27} in Thomas' Table I, values of $\bar{I}$ were interpolated for the same values of ϵ/B as for the N^{14}—F^{19} collision.

(2) By extrapolation on the basis of the parameter η values of $\bar{I}$ were determined for the two energies of the N^{14}—F^{19} collision.

(3) $\bar{I}$ and $\langle I(I+1)\rangle_{\rm av}$ were related by means of Thomas' Fig. 5, which shows partial wave absorption cross sections for Al^{27}—C^{12} collisions at four energies. From this figure $\langle I(I+1)\rangle_{\rm av}/(\bar{I})^2$ was calculated as a function of η; interpolation for the N^{14}—F^{19} case gave the results shown in Table I.

These stages of interpolation are summarized in the following equation:

$$[I(I+1)]_{\text{N-F},\epsilon/B,\eta}=[\bar{I}]^2{}_{\text{X-Al},\epsilon/B,\eta}[I(I+1)/(\bar{I})^2]_{\text{C-Al},\eta}. \tag{A.3}$$

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Beta-Gamma Directional Correlation in Eu^{154}†

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The energy dependence of the beta-gamma angular correlation between the outer beta-ray group of Eu^{154}, $W_0=1.86$ Mev, and the 123-kev gamma ray of the daughter Gd^{154} has been measured with a shaped magnetic field beta-gamma coincidence spectrometer. A negative correlation coefficient, accurate to about 5%, is obtained which ranges from -0.10 to -0.17 in the energy region 0.80 to 1.60 Mev. It is shown that the modified B_{ij} approximation must be relaxed to explain the data. The nuclear parameters (Kotani's notation) which result are: $x=-0.24\pm0.05$, $u=+0.05\pm0.03$, $Y=+0.76\pm0.08$. These values are compared with those which have been reported for Eu^{152}: $x=u=0$, $Y=0.69\pm0.06$.

INTRODUCTION

OF particular interest in beta-gamma angular correlation studies are certain $3^-(\beta)2^+$ decays from odd-odd to even-even nuclei which are characterized by abnormally long comparative half-lives. Kotani[1] has suggested that this situation may arise from cancellation among the nuclear matrix elements or from additional selection rules, such as those associated with K forbiddenness. Deviations from the ξ approximation are expected in these cases, which should be manifest in experiments as an energy-dependent shape factor and a large beta-gamma correlation coefficient ϵ. The highest energy beta groups of Eu^{152} and Eu^{154} appear to be examples of this effect, since both have large comparative half-lives ($\log ft\simeq 12$) and both are nonunique first-forbidden decays with nonstatistical beta-ray shapes.[2] Beta-gamma angular correlation studies in Eu^{152} support this view. It has been shown,[3] for example, that the energy dependence of the large negative correlation coefficient is adequately described by the modified B_{ij} approximation ($x=u=0$, $Y\neq 0$), and that the matrix element ratio,[4] $Y=0.69\pm0.06$, is in good agreement with the value obtained in the same approximation from the spectrum shape determination of Langer and Smith.[2] In this case it seems well established that u and x are suppressed relative to B_{ij} by some selection rule effect. The slowness of the transition is probably associated with the fact that $_{64}Gd^{152}$, although spherical, is close to the transition region (neutron numbers 88–90) between spherical and deformed even-even nuclei.[5]

For a more detailed understanding of the selection rule effect it seems desirable to have available the beta-gamma correlation data in the companion case of Eu^{154}. In this decay, the highest energy beta group is a once-forbidden transition from a 3^- initial state in Eu^{154} to a 2^+ final state in Gd^{154}, and is followed by a 123-kev $E2$ gamma-transition to the Gd^{154} ground state.[6] The large comparative half-life, $\log ft=12.4$, is probably associated with the change in the deformation parameter

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[1] T. Kotani, Phys. Rev. **114**, 795 (1959).
[2] L. M. Langer and D. R. Smith, Phys. Rev. **119**, 1308 (1960).
[3] H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. **120**, 1762 (1960).
[4] The notation of reference 1 will be used throughout this paper. A discussion of the modified B_{ij} approximation as it applies to these measurements is given in reference 3.
[5] R. K. Sheline, Rev. Modern Phys. **32**, 1 (1960).
[6] *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C.).

accompanying the beta decay, since $_{64}Gd^{154}$, with 90 neutrons, also lies in the transition region. The shape correction factor has been reported by Langer and Smith[2] in the form $C=q^2+\lambda_1 p^2+20\pm5$ and is therefore of the nonstatistical, nonunique type. The experimental results reported here on the energy dependence of the beta-gamma correlation coefficient for the outer beta group, end-point energy $W_0=4.63$ (m_0c^2 units), and the 123-kev gamma ray also indicate that the decay is governed predominantly by the B_{ij} matrix element and the parameter Y. It will be shown that the matrix element ratio x is also operative to some degree.

EXPERIMENTAL PROCEDURE

In this investigation a shaped magnetic field beta-gamma coincidence spectrometer, previously described,[3] was employed. The instrument was adjusted for 5% transmission, 5% energy resolution, and a coincidence resolving time of about 40 mμsec. Sources were prepared by liquid deposition on 0.25-mil Mylar film and were approximately 0.10 mg/cm^2 thick. The source material was obtained by bombarding EuO_3, enriched to 95% in Eu^{153}, with neutrons in the Brookhaven reactor. A lapse of eight months before taking data reduced the Eu^{156} fraction to a negligible amount. The small amount of Eu^{152} present was not a problem, since only coincidences with the 123-kev gamma ray were recorded.

The anisotropy, $A=[N(180°)-N(90°)]/N(90°)$, was measured at nine beta-ray energies from 800 to 1600 kev with an accuracy of about 5%. The data taken at the lower energies were corrected for the presence of the 0.97-Mev beta group from a Fermi analysis of the coincidence data. As a by-product, the intensity of this beta group to the intensity of the 1.86-Mev group was found to be 0.61. The proposed decay scheme[6] of Eu^{154} also suggests a beta group with an end point at 1.6 Mev. Since Langer and Smith[2] find no evidence for this first inner group, no corrections are necessary. Other corrections, control experiments, and data reduction procedures are the same as those described in an earlier paper.[3]

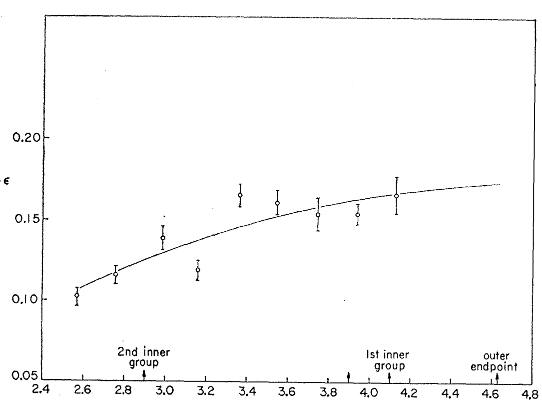

FIG. 1. Energy dependence (m_0c^2 units) of the measured correlation coefficient for the outer beta group and the 123-kev gamma ray of Eu^{154}. The solid curve is a theoretical one with $x=-0.24$, $u=+0.05$, and $Y=0.76$. There is no experimental evidence for the proposed inner group at 4.1. The arrow at 3.9 indicates the end-point energy of the Eu^{152} contaminant.

RESULTS

The beta-gamma angular correlation data are summarized in Table I. A plot of the correlation coefficient versus energy is shown in Fig. 1. The coefficient is negative, exhibits a rather small energy dependence, and is considerably smaller than in the Eu^{152} correlation involving the 1.48-Mev beta group and the 0.344-Mev cascade gamma ray. In this latter case[3] ϵ was found to range from -0.25 to -0.37 in an energy span of 2.3 to 3.5 m_0c^2. When the present data are analyzed according to the modified B_{ij} approximation, one finds that the values of Y which result from each data point vary from 2.5 to 2.9 over the energy interval 2.6 to 4.2 (m_0c^2), instead of being energy independent as required. Moreover, these values do not agree with $Y=1.29$ which follows from Langer's shape factor measurement in this same approximation. It would, therefore, appear that the modified B_{ij} approximation is not adequate in this case, and that contributions from x and u are required to explain the data.

A clue as to how the modified B_{ij} approximation may be relaxed is obtained by noting that Langer and Smith empirically fit the experimental beta-spectrum data by a shape factor of the form $C(W)=\frac{1}{12}(\lambda_1 p^2+q^2)+D/12$, where p and q are the electron and neutrino momenta, λ_1 is a Coulomb correction factor, and D is an adjustable constant. More generally, $C(W)$ has the form $C(W)=k(1+aW+b/W+cW^2)$, where the factors k, a, b, and c are functions of the nuclear matrix element ratios u, x, and Y, in decays for which the spin sequence is $3^-(\beta)2^+(\gamma)0^+$. Inspection of Kotani's theoretical expressions[7] shows that if the *form* of the shape factor reported by Langer is assumed to be valid then

$$\tfrac{1}{9}(4x^2+5u^2)\simeq0,\quad \tfrac{4}{3}uY/W_0\simeq0,\quad bk\simeq0. \qquad (1)$$

Accordingly, we introduce a modified "modified B_{ij} approximation" in which certain quadratic terms are suppressed in $C(W)$ and $\epsilon(W)$, but first-degree terms are

TABLE I. Summary of the Eu^{154} correlation data.

E (kev)	W (m_0c^2)	$-A$	$-\epsilon$
800	2.567	0.147±0.006	0.103±0.005
897	2.756	0.165±0.009	0.116±0.006
1014	2.985	0.194±0.011	0.138±0.007
1103	3.159	0.167±0.009	0.118±0.006
1207	3.362	0.230±0.008	0.166±0.006
1302	3.548	0.223±0.010	0.161±0.008
1401	3.744	0.214±0.013	0.154±0.010
1503	3.940	0.215±0.008	0.154±0.006
1598	4.128	0.230±0.016	0.166±0.011

[7] See reference 1, p. 805.

retained. The theoretical expressions with these assumptions are[4]:

$$\mathcal{E}=12[R_3k+(ek)W]/(\lambda_1p^2+q^2+D),\qquad \mathcal{E}=\epsilon W/p^2, \tag{2}$$

$$R_3k=-(\lambda_2/21)(2x+3)\zeta_1,\qquad \zeta_1=Y+(u-x)(W_0/3), \tag{3}$$

$$ek=(1/42)(2x^2/3+5u-2x-\lambda_1),\qquad \lambda_1=\lambda_2\simeq 0.8, \tag{4}$$

$$\zeta_1^2+(2W_0^2-1)x^2/9=D/12,\quad W_0=4.63(m_0c^2). \tag{5}$$

In these expressions, $\mathcal{E}$ is the "reduced" correlation coefficient, W_0 is the beta-ray end-point energy, and λ_1 and λ_2 are Coulomb correction factors which are essentially the same and constant over the energy region of the measurements. The quadratic term in (4) is retained because of the smallness of ek and in (5) because of the large numerical coefficient. A least-squares analysis of (2) using the data of Table I and retaining $D/12$ as an adjustable parameter yields

$$R_3k=-0.060(D/12)-0.0136,\qquad ek=+0.0039(D/12)-0.0078. \tag{6}$$

The values of u, x, and Y may be obtained immediately from (3)–(6) if the measured value of Langer and Smith, $D=20\pm5$, is used. Because of the rather large error assigned to D, it seems advisable to first demonstrate that the values of x and u, within the accuracy of our data and the approximations used, do not depend sensitively on D. The dependence of x on $D/12$, independent of the choice of u and consistent with the experimental values of ϵ, may be obtained by combining (3), (5), and (6). The dashed curve of Fig. 2 shows this dependence. It is seen that x has the value -0.20 ± 0.07, for a variation of $D/12$ from 0.2 to 2.2. Hence, for a shape factor ranging from almost unique to almost statistical, x does not change markedly. The solid curves in Fig. 2 show the dependence of x on $D/12$ for several choices of u, as given by (4) and (6). The intersections of these curves with the dashed curve are the possible solutions of (3)–(6) for a given $D/12$. Again, it is seen that no rapid variation of u occurs, and that u is a small quantity. Accordingly, it seems justified to adopt the values of u and x, and therefore Y, which correspond to $D/12=20/12\pm5/12$, namely:

$$x=-0.24\pm0.05,\quad u=+0.05\pm0.03,\quad Y=+0.76\pm0.08. \tag{7}$$

As a check on the validity of our assumptions, the theoretical shape factor has been calculated using the above values of x, u, and Y without suppressing *any* terms in Kotani's formulas; i.e., all quadratic terms, including those in b, have been retained. A comparison of this curve with Langer's experimental result is shown in Fig. 3. The agreement between the two curves is well within the experimental error denoted by the dashed curves. In

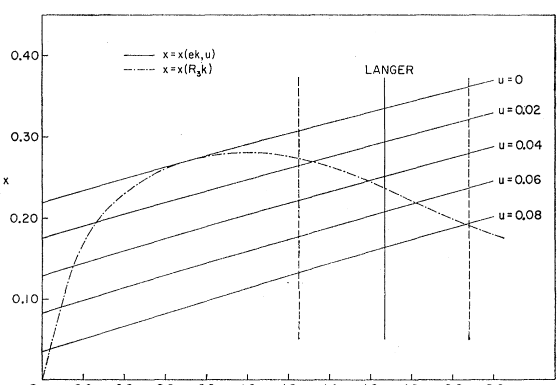

FIG. 2. Possible values of x and u, consistent with the experimental values of ϵ, as a function of $D/12$. The dashed curve is obtained by combining (3), (5), and (6), given in the text. The solid lines yield possible values of x vs $D/12$ for several assignments of u and are obtained from (4) and (6). The dashed straight lines represent Langer's quoted error in $D/12$.

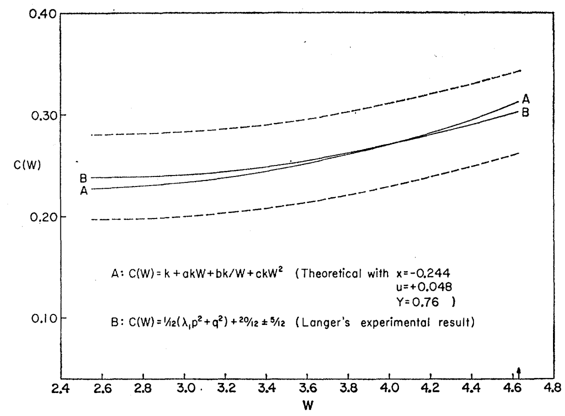

FIG. 3. Comparison of the theoretical shape factor and the measured shape factor of Langer and Smith. Kotani's complete expression has been computed for the values in (7). The dashed lines represent the error assigned to the experimental shape measurement.

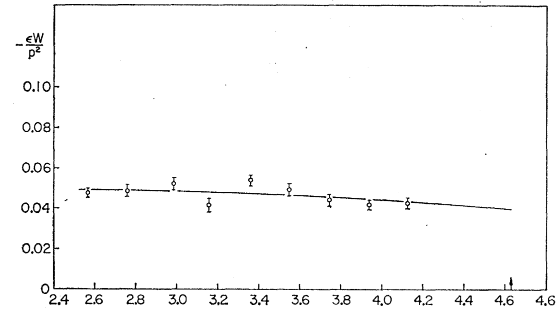

FIG. 4. Plot of the experimental "reduced" correlation coefficient versus energy. The curve is Kotani's theoretical expression, all quadratic terms included, with $x=-0.24$, $u=+0.05$, $Y=0{\cdot}76$.

TABLE II. Comparison of the nuclear parameters involved in the outer beta transitions of Eu^{154} and Eu^{152}.

	$f_c t$[a]	x	u	Y	$\lvert\int B_{ij}\rvert/R$[b]	$\lvert\int \mathbf{r}\rvert/R$	$\lvert\int i\boldsymbol{\sigma}\times\mathbf{r}\rvert/R$	$\lvert\int i\alpha\rvert$
Eu^{154}	4.18×10^{33}	-0.24	$+0.05$	0.76	1.20×10^{-3}	3.45×10^{-4}	$\sim6\times10^{-5}$	5.56×10^{-5}
Eu^{152}	3.22×10^{32}	~0	~0	0.69	4.34×10^{-3}	$<10^{-5}$	$<10^{-5}$	$\sim6.0\times10^{-5}$

[a] In units of $\hbar=m=c=1$.
[b] R, the nuclear radius in natural units, is 1.66×10^{-2}.

Fig. 4 a comparison of the measured "reduced correlation coefficient," $\epsilon W/p^2$, with Kotani's theoretical prediction is shown. The curve is the theoretical energy dependence for (7) with all terms retained. It is of interest to note that according to (6), ek is essentially zero for $D=20$, and hence the energy dependence shown in Fig. 4 is very small. The modified B_{ij} approximation would require $ek=-\lambda_1/42$. This value, while small, is substantially different from the experimental result, $ek=-0.0013\pm0.001$. On the other hand, the ξ approximation, for which ek is zero, does not apply, since the shape factor is nonstatistical. This case thus appears to be one which is intermediate between the modified B_{ij} approximation and the ξ approximation.

It should be pointed out that in principle the small value of ϵ which we measure, and the consequent failure of the modified B_{ij} approximation, could be accounted for by an attenuation effect. The half-life of the 123-kev level in Gd^{154} has been measured by Sunyar[8] to be 1.2×10^{-9} sec, so that without further checks on the results this alternative cannot be ruled out. When a modified B_{ij} approximation analysis is applied to the data with an attenuation factor a included (i.e., ϵ is replaced by $a\epsilon$), the value $a=0.61$ is required to bring Y into agreement with Langer's result. Moreover, since these values of Y vary more widely with energy than for $a=1$, we conclude that the modified B_{ij} approximation is even less applicable. This fact, coupled with the rather unreasonably large attenuation (40%) needed for agreement with Langer, suggests that an appreciable washing-out effect is unlikely. Some support to this conclusion comes from Wiedenbeck's[9] gamma-gamma correlation measurements involving the 123-kev level. No appreciable attenuation is indicated in this case.

[8] A. W. Sunyar, Phys. Rev. **98**, 653 (1955).
[9] G. D. Hickman and M. L. Wiedenbeck, Phys. Rev. **111**, 539 (1958).

DISCUSSION

The results of the previous section may be used to assign numbers to the matrix elements responsible for the transition, since the corrected ft value, $f_c t=4.2\times10^{33}$ ($\hbar=m=c=1$), determines the standard matrix element, which in this case is taken to be B_{ij} ($z=1$). For this purpose, the currently accepted relations are employed: $C_A=-1.20C_V$, $C_V=g=2.97\times10^{-12}$ ($\hbar=m=c=1$). The matrix element values listed in Table II clearly indicate the lack of overlap of the initial- and final-state wave functions. For comparison, the corresponding magnitudes for Eu^{152} are presented.[3] In both cases B_{ij} is dominant, but is much smaller (by a factor of from 25 to 100) than expected for a unique ($\Delta I=2$) transition. The relativistic matrix element appears to be of the same order of magnitude in the two cases and is also very small, about a factor of 1000 less than expected for perfect overlap. In Eu^{152} the remaining matrix elements are several orders of magnitude smaller than B_{ij}. The rough upper limits given for these are based on the largest deviations of x and u from zero which experimental error allows. In Eu^{154}, however, a significantly larger contribution from $\int\mathbf{r}$ is indicated. In this case this matrix element is reduced only by a factor of 3 from B_{ij}. It should be mentioned that these numbers are subject to some change if some attenuation of the correlation exists, or if values other than $\lambda_i=0.8$ are used for the Coulomb correction factors.

We conclude that a selection rule effect is operative on both of these decays, as suggested by Kotani,[1] and that the effect is associated with a change in the deformation parameter which accompanies the beta decay. This change is expected to be somewhat less in the case of Eu^{154}, since this nucleus lies two neutrons closer to the deformed region. The results given in Table II are consistent with this idea. More data in this region are needed for a detailed understanding of the selection rule effect. Work is in progress in this laboratory on other elements in this region.