demonstrated that the O_2^- molecule-ion is present in alkali halide crystals treated with oxygen. Measurements of the resonance signals from portions of crystals used in these experiments were kindly made for us by Känzig, and it was found that the signals increased as the intensity of fluorescence increased. In one case an accurate value for the ratio of signals from the two crystals KBr+KCN and KBr "pure," both grown in oxygen, was obtained. The ratio was 7, whereas the ratio of fluorescent intensities, which can be measured more accurately, was 6.6. Thus we assume, from the optical results and the paramagnetic resonance measurements, that the center responsible for the absorption and fluorescence is an O_2^- molecule-ion substituted for a halide ion in the crystal, aligned along the ⟨110⟩ directions, as found by Känzig and Cohen.

Some of the results are more difficult to explain, however. One would expect that some polarization of fluorescence would be observed at 4.2°K. However, the local temperature in the vicinity of the excited O_2^- molecule-ion may be quite high, since a rather large Stokes shift (2.5 ev) is observed. Perhaps half the energy, over 1.2 ev, may be given off as phonons before the center emits. This process may destroy any polarization.[10] Also, there is no other evidence to account for the fact that the fluorescence emission of KCl at 4.2°K has a quite different structure from the other two alkali halides NaCl and KBr. Finally, the very small absorption coefficients associated with oxygen might be due either to forbidden electronic transitions, or to the fact that the observed band is actually a subsidiary band, the main band lying in wavelength regions beyond our limits of measurement.

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[10] We are indebted to the referee for this suggestion.

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Galvanomagnetic Effects in Semiconductors at High Electric Fields

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A treatment of magnetoconductivity is developed for high electric fields and general energy-band structure using a partial solution of the Boltzmann equation in a form similar to that set up by McClure for low electric fields. The present treatment is valid when the scattering processes are such that the distribution function varies but a small amount over an entire constant-energy surface, or, in the case of the many-valley band structure, over the part of a constant-energy surface within each valley. In the latter case, different distribution functions must be used for the different valleys. The elements of the magnetoconductivity matrix that results are expressed in terms of carrier concentration, total or within each valley, and averages over the carriers of a quantity involving the momentum relaxation time and the **S** tensor defined by McClure. This tensor, which depends on the shape of the constant-energy surfaces and on the magnetic-field strength, is evaluated for the individual valleys in a nondegenerate many-valley semiconductor. The magnetoconductivity matrix is then in a form convenient for calculation of conductivity and galvomagnetic effects for either low or high electric fields. It is used to obtain expressions for anisotropy voltage and Hall coefficient in high electric fields involving the number of carriers in each valley, orientation of the valleys, and valley averages over quantities involving relaxation time and energy.

I. INTRODUCTION

In connection with many of the investigations of conductivity in high electric fields, the Hall effect has been of interest because of the possibility of change in carrier concentration due to impact ionization or other processes. It has become apparent, however, that the Hall coefficient R, measured with due care, may change in high electric fields even though carrier concentration does not. The change in the distribution function, which is the basis of the so-called hot carrier effects, can itself cause a change in R.[1] Changes in the band structure, such as change in curvature of energy vs crystal momentum, as the carriers move to higher energy states can also cause a change in R. This seems to be a sizeable effect in p-germanium[2] where the curvature of the light-

[1] For a calculation of the change in Hall coefficient with electric field, under the assumption that scattering is by acoustic modes only, see M. S. Sodha and P. C. Eastman, Phys. Rev. **110**, 1314 (1958).

[2] J. Zucker and E. M. Conwell, Bull. Am. Phys. Soc. **4**, 185 (1959).

hole band changes even at relatively small energies.[3] In a many-valley semiconductor, such as *n*-germanium, the anisotropy of the constant-energy surfaces gives rise to additional complications. In the absence of a magnetic field there is, at high electric field, a considerable transverse voltage, the anisotropy voltage.[4] The different degree of heating of the different valleys that gives rise to the anisotropy voltage could itself affect the Hall coefficient, even more so under some circumstances by causing a net shift of carriers from one set of valleys to another.

In this paper, a treatment of magnetoconductivity in high electric fields is developed from which expressions for the Hall coefficient and other galvanomagnetic properties at high electric fields can be derived. The treatment is based on a partial solution of the Boltzmann equation in a form similar to that set up by McClure for low electric fields.[5] In Sec. II the solution is given. The validity of this solution at high electric fields requires that the distribution function vary but little over an entire constant-energy surface. The conditions that the scattering processes must satisfy to make this so are set up. These conditions will not, in general, be satisfied in a many-valley semiconductor when intervalley scattering is not sufficiently frequent. In Sec. III, it is shown for the latter case that, under the less restrictive condition that the distribution function vary but little over the portion of a constant-energy surface within each valley, a magnetoconductivity matrix of form similar to that obtained in Sec. II can be set up for each valley. The magnetoconductivity matrix for the whole is, of course, the sum of the matrices for the individual valleys. Calculation of the individual **S** tensors is carried out, making it possible to express the magnetoconductivity in terms of averages over functions of relaxation time, energy, magnetic field, and orientation of the valleys. In Sec. IV, expressions are obtained for the anisotropy voltage and Hall coefficient in terms of the elements of the magnetoconductivity matrices of Secs. II and III. For the many-valley case, it is shown that anisotropy and Hall voltages can readily be separated at low magnetic fields. Expressions are then given for each of them in terms of the number of carriers in each valley and averages for each valley of the usual functions of relaxation time and energy. In an Appendix, the individual **S** tensors are calculated for the case of *n*-germanium with current in the (110) plane, a situation that has been used considerably in experiments.[4]

II. MAGNETOCONDUCTIVITY AT HIGH ELECTRIC FIELDS

The solution of the Boltzmann equation set up by McClure for low electric fields and arbitrary magnetic fields assumes the existence of a relaxation time that is a function of energy only. The form of the solution he takes to be the usual[6]:

$$f=f_0(\epsilon)-\phi\partial f_0/\partial\epsilon, \tag{1}$$

where f_0 is the Maxwell-Boltzmann distribution for the cases of interest here, and ϵ is the energy. As a consequence of the electric field being small,

$$\phi\partial f_0/\partial\epsilon\ll f_0. \tag{2}$$

Beyond this, McClure's treatment differs from the conventional one in the introduction of a variable $s(\mathbf{P})$ representing the time at which a carrier precessing about a constant-energy surface in the presence of a magnetic field, but no electric field, would be at the point **P**. The actual path in **P** space of the carrier in the magnetic field is the curve formed by the intersection of the constant-energy surface and a plane perpendicular to **H**. This path is called the hodograph. Using the fact that the velocity **v** is a periodic function of s, he makes a Fourier expansion of **v** in terms of s:

$$\mathbf{v}=\sum_{m=-\infty}^{+\infty}\mathbf{v}(m)\exp[im\omega s]. \tag{3}$$

With this, ϕ is expressed as a function of s. In the usual expression for the current density,[7]

$$\mathbf{j}=(-2e/h^3)\int d^3P\ \mathbf{v}\phi(-\partial f_0/\partial\epsilon), \tag{4}$$

where the integration is over the basic Brillouin zone. McClure replaces the integrand at each point in **P** space by its average over the hodograph that passes through the point. With this he obtains the magnetoconductivity tensor in the form

$$\boldsymbol{\sigma}=(2e^2/h^3)\int d^3P(-\partial f_0/\partial\epsilon)\tau\mathbf{S}, \tag{5}$$

where **S** is a tensor, the components of which are given by

$$S_{\alpha\beta}=\sum_{m=-\infty}^{+\infty}\frac{v_\alpha(-m)v_\beta(m)}{1+im\omega\tau}, \tag{6}$$

ω being the cyclotron frequency. The quantities $v_\alpha(-m)$ and $v_\beta(m)$ are the Fourier coefficients in the expansion of the α and β components of **v**. If the Z axis is chosen as the magnetic-field direction, this expression for **S** can be

[3] E. O. Kane, J. Phys. Chem. Solids **1**, 82 (1956).

[4] W. Sasaki, M. Shibuya, K. Mizuguchi, and Hatoyama, J. Phys. Chem. Solids **8**, 250 (1959). This also gives reference to earlier work.

[5] J. W. McClure, Phys. Rev. **101**, 1642 (1956).

[6] See, for example, A. H. Wilson, *The Theory of Metals* (University Press, Cambridge, England, 1953), 2nd ed.

[7] Note that McClure (reference 5) in his expression (2.9) omits the factor 2 for summation over both spin directions.

written

$$S_{\alpha\beta}=v_\alpha(0)v_\beta(0)\delta_{\alpha z}\delta_{\beta z}+\sum_{m=1}^{\infty}\left[\frac{v_\alpha(-m)v_\beta(m)+v_\alpha(m)v_\beta(-m)}{1+m^2\omega^2\tau^2}+\frac{im\omega\tau\{v_\alpha(m)v_\beta(-m)-v_\alpha(-m)v_\beta(m)\}}{1+m^2\omega^2\tau^2}\right], \quad (7)$$

where $\delta_{\gamma z}=1$ for $\gamma=z$, and zero otherwise. Thus, **S** depends on the shape of the constant-energy surfaces, and on the magnetic-field strength through the factor $\omega\tau$.

The feature that limits this treatment to low electric field is the assumption that the distribution function is given by (1) with $f_0(\epsilon)$ the Maxwell-Boltzmann distribution and the condition $\phi\partial f_0/\partial\epsilon\ll f_0$ satisfied. As a consequence of the latter condition, the distribution function has, to a good approximation, the same symmetry in low electric field as in the absence of field, which is the symmetry of $\epsilon(\mathbf{P})$. If this symmetry were still approximately maintained by the distribution function at high electric fields, it could be written in the form (1) for that case also, with f_0 a function of ϵ only and the condition (2) still satisfied. Of course, in high fields, f_0 would no longer be a Maxwell-Boltzmann distribution at the lattice temperature.[8] For Eqs. (1) and (2) to be applicable at high electric fields, the scattering processes must be such as to produce the required randomization of the velocity gained from the field. To do this, they must have the following properties. First, they must be predominantly elastic,[9] and they must not give rise to predominantly forward or backward, i.e. 180°, scattering. Satisfaction of this pair of conditions should produce the required small variation in f over connected portions of a constant-energy surface, i.e., within a valley. It will also be sufficient to produce small variation in f over the entire constant-energy surface in a many-valley semiconductor for the special case that the fields are so oriented as to supply energy to all valleys at the same rate. A second condition must be added for arbitrary field orientations in a many-valley semiconductor, e.g., the scattering must be such as to afford sufficient communication between unconnected parts of a constant energy surface to maintain approximately the same distribution function on all parts. The first pair of conditions should be satisfied in germanium and silicon, for example, when scattering is mainly by acoustical lattice modes[10] and/or optical modes with phonon energy much smaller than the energy of the carriers. This is the case over a considerable range of fields for both materials. The second condition on the scattering processes, as remarked earlier, may not be satisfied in n-germanium over a considerable range of fields.

In the remainder of this section we shall consider only the case where f_0 in high fields varies little over the entire constant-energy surface. The case of the many-valley semiconductor in which the first condition is satisfied but the second is not will be taken up in the next section.

When the distribution function in high fields can be written in the form of Eq. (1) with $\phi\partial f_0/\partial\epsilon\ll f_0$, the solution for ϕ is the same in high electric fields as in low. The treatment for high fields can then be carried out in the same way as that for low fields, and the result (5) is still valid except that f_0 now represents the distribution appropriate to high electric fields. In this case, f_0 may also depend on magnetic field. The **S** tensor is still given by (6) or (7) and is independent of electric-field intensity provided the shape of the constant-energy surfaces does not change with increasing energy. It should be noted, however, that the $\boldsymbol{\sigma}$ of (5) is no longer a tensor, i.e., a linear vector operator, for high electric fields because f_0 depends on electric-field intensity.

If the band structure is such that contributions to the current come from more than one band, as in p-germanium for example, the total $\mathbf{j}$ and $\boldsymbol{\sigma}$ are made up of the sums of expressions (4) and (5), respectively, over the contributing bands.

III. MAGNETOCONDUCTIVITY FOR THE MANY-VALLEY BAND STRUCTURE

As indicated earlier, the treatment of the last section will be valid for a many-valley semiconductor when all valleys have approximately the same distribution function, whether through special orientation of the applied fields, or sufficient intervalley scattering. In this section, we shall take up the case where the first pair of conditions on the scattering is satisfied, so that the distribution function does not vary much over a constant-energy surface within a valley, but does vary substantially from valley to valley. For this case, we assign to the ith valley a distribution function $f^{(i)}(\epsilon)$. The over-all dis-

[8] For discussions of $f_0(\epsilon)$ in high electric field but no magnetic field see W. Shockley, Bell System Tech. J. **XXX**, 990 (1951); J. Yamashita and M. Watanabe, Progr. Theoret. Phys. (Kyoto) **12**, 443 (1954); and H. G. Reik, H. Risken, and G. Finger, Phys. Rev. Letters **5**, 423 (1960).

[9] This condition has been discussed for spherical constant-energy surfaces by E. M. Conwell, Phys. Rev. **88**, 1379 (1952). See also S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, New York, 1939).

[10] Sasaki *et al.*, reference 4, in attempting to account for the anisotropy voltage observed at high electric field for p-germanium, postulate that this pair of conditions is not satisfied for acoustical phonon scattering in that case. Specifically, they suggest that, for the approximately cubic heavy-hole constant-energy surfaces, transitions between opposite sides are more frequent than transitions between adjacent sides. To account for this predominantly forward or backward scattering, they further postulate highly anisotropic constant-energy surfaces for the acoustical phonons in $\mathbf{q}$ space. Such anisotropy would require a correspondingly anisotropic sound velocity. In germanium, however, the difference in sound velocity between the [001] and [110] directions is only about 10%. This seems too small to give much of an anisotropy in scattering probability for the hole temperatures that are involved.

tribution $f(\epsilon)=\sum_i f^{(i)}(\epsilon)$. Since $f^{(i)}$ does not vary much over a constant-energy surface in the ith valley, it can be written in the form

$$f^{(i)}=f_0^{(i)}(\epsilon)-\phi^{(i)}\partial f_0^{(i)}/\partial\epsilon, \tag{8}$$

where

$$\phi^{(i)}\partial f_0^{(i)}/\partial\epsilon \ll f_0^{(i)}(\epsilon). \tag{9}$$

It is readily seen that, when $f_0^{(i)}$ is the same for all valleys, the over-all distribution function can be written

$$f=f_0(\epsilon)-(\sum_i\phi^{(i)})\partial f_0/\partial\epsilon, \tag{10}$$

so that ϕ of Sec. II is to be identified with $\sum_i\phi^{(i)}$ for this case.

With (8) and (9) valid for $f^{(i)}$, the solution of the Boltzmann equation can be carried out for each valley in the same fashion as set forth in Sec. II. The resulting magnetoconductivity can be written

$$\boldsymbol{\sigma}=\sum_{i=1}^{N}(2e^2/h^3)\int_i d^3P(-\partial f_0^{(i)}/\partial\epsilon)\tau\mathbf{S}^{(i)}, \tag{11}$$

where the integration is to be taken over the states in the ith valley, and N is the number of valleys. The tensor $\mathbf{S}^{(i)}$ is given by (6) and (7), where, of course, the Fourier analysis of $\mathbf{v}$ is to be carried out for the constant-energy surfaces in the ith valley.

For calculations, it is more convenient to have $\boldsymbol{\sigma}$ expressed in terms of the number of carriers. Since

$$n^{(i)}=(2/h^3)\int_i d^3P\, f_0^{(i)},$$

Eq. (11) for $\boldsymbol{\sigma}$ can be transformed into

$$\boldsymbol{\sigma}=\sum_{i=1}^{N}n^{(i)}e^2\langle-(1/f_0^{(i)})(\partial f_0^{(i)}/\partial\epsilon)\tau\mathbf{S}^{(i)}\rangle, \tag{12}$$

where the average indicated is to be taken over all the carriers of the ith valley. In general, the components of the matrix $\mathbf{S}^{(i)}$ will vary with position on a constant-energy surface. Since f_0 and τ are functions of energy only in the present treatment, the correct values of the average in (12) will be obtained if $S_{\alpha\beta}^{(i)}$ is replaced by $\bar{S}_{\alpha\beta}^{(i)}$, its average over a constant-energy shell. We can then write the $\alpha\beta$ component of the magnetoconductivity as

$$\sigma_{\alpha\beta}=\sum_{i=1}^{N}n^{(i)}e^2\langle-(1/f_0^{(i)})(\partial f_0^{(i)}/\partial\epsilon)\tau\bar{S}_{\alpha\beta}^{(i)}\rangle. \tag{13}$$

Calculation of the $\mathbf{S}^{(i)}$ tensor will now be carried out for the case of a many-valley band structure in which the constant-energy surfaces are spheroids with axis of revolution the major axis. The coordinate system in $\mathbf{P}$ space to be used for the ith valley is shown in Fig. 1. The magnetic-field direction is chosen as the Z axis. The angle between the magnetic-field direction and the major axis $\mathbf{Z}^{(i)}$ of a constant-energy ellipsoid is denoted by $\alpha^{(i)}$. The Y axis is taken to coincide with the minor axis of the ellipsoid perpendicular to $\mathbf{H}$, in the sense $\mathbf{Z}^{(i)}\times\mathbf{Z}$. The X axis is perpendicular to Y and Z, forming with them a right-handed system.

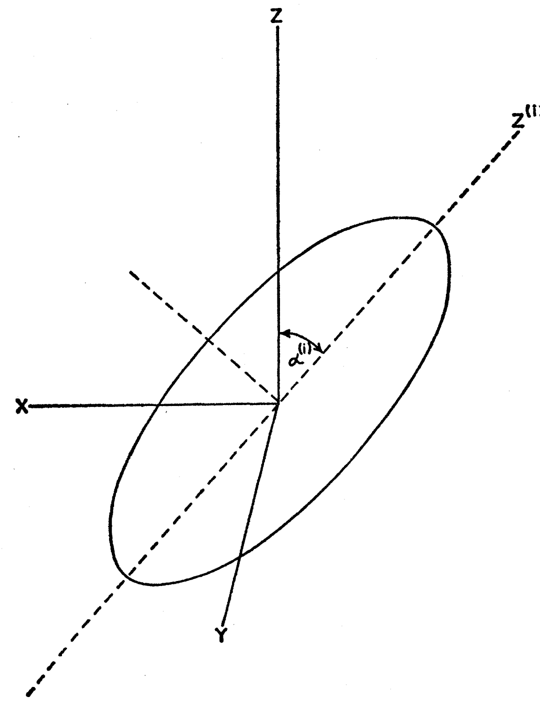

FIG. 1. Constant-energy ellipsoid and coordinate system for the ith valley. Z is the magnetic-field direction, and Y and the two dashed lines are the principal axes of the ellipsoid.

In the coordinate system of Fig. 1, the equation of a constant-energy surface is

$$\epsilon=\frac{m_t}{m_*^2}\frac{P_x^2}{2}+\frac{P_xP_z}{m_1}+\frac{P_z^2}{2m_2}+\frac{P_y^2}{2m_t}, \tag{14}$$

where

$$\frac{m_t}{m_*^2}=\frac{\cos^2\alpha}{m_t}+\frac{\sin^2\alpha}{m_l},$$
$$\frac{1}{m_1}=\frac{\sin\alpha\cos\alpha}{m_t}-\frac{\sin\alpha\cos\alpha}{m_l}, \tag{15}$$
$$\frac{1}{m_2}=\frac{\sin^2\alpha}{m_t}+\frac{\cos^2\alpha}{m_l}.$$

It is, of course, understood that the values of α, m_*, m_1, and m_2 for the ith valley must be used even though the superscripts have been omitted. The quantity m_*, it may be noted, is the cyclotron resonance mass.[11] From (14), the components of the velocity are obtained as

$$\begin{aligned} v_x&=\partial\epsilon/\partial P_x=(m_t/m_*^2)P_x+(1/m_1)P_z,\\ v_y&=\partial\epsilon/\partial P_y=(1/m_t)P_y,\\ v_z&=\partial\epsilon/\partial P_z=(1/m_1)P_x+(1/m_2)P_z.\end{aligned} \tag{16}$$

[11] See, for example, G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. 98, 368 (1955).

To express the velocity components as functions of s, we make use of the Lorentz force equation to obtain

$$s(\mathbf{P})=(1/eH)\int_{\mathbf{P}=Px}^{\mathbf{P}} v_\perp^{-1}dP, \tag{17}$$

where dP is a line element along the hodograph, and $v_\perp$ is the component of velocity perpendicular to $\mathbf{H}$. Since $\mathbf{H}$ is in the Z direction, $v_\perp=(v_x^2+v_y^2)^{\frac{1}{2}}$ for this case. The limits of integration have been chosen to give $s=0$ when $\mathbf{P}$ is in the X direction, or when v_x has its maximum value. Using the fact that (14) with P_z constant, as well as ϵ, is the equation of the hodograph, we can express dP of (17) in terms of P_x and P_y. Integration of (17) then leads to an expression for s as a function of v_x that can be rewritten to give the desired Fourier expansion of v_x

$$v_x(s)=\frac{1}{2}\frac{m_t}{m_*}\left(\frac{2\epsilon}{m_t}-\frac{m_*^2P_z^2}{m_lm_t^3}\right)^{\frac{1}{2}}(e^{i\omega s}+e^{-i\omega s}). \tag{18}$$

To find $v_y(s)$ it is convenient to rewrite the equation of the hodograph in terms of v_x and v_y instead of P_x and P_y. We then have

$$\frac{m_*^2v_x^2}{2m_t}+\frac{m_tv_y^2}{2}=\epsilon-\frac{m_*^2P_z^2}{2m_lm_t^2}. \tag{19}$$

Using (18) for $v_x(s)$ in (19), we obtain

$$v_y(s)=\frac{1}{2i}\left(\frac{2\epsilon}{m_t}-\frac{m_*^2P_z^2}{m_lm_t^3}\right)^{\frac{1}{2}}(e^{i\omega s}-e^{-i\omega s}). \tag{20}$$

The Fourier expansion for $v_z(s)$, calculated by use of (16) and (18), is

$$v_z(s)=\frac{m_*^2P_z}{m_lm_t^2}+\frac{1}{2}\frac{m_*}{m_1}\left(\frac{2\epsilon}{m_t}-\frac{m_*^2P_z^2}{m_lm_t^3}\right)^{\frac{1}{2}}(e^{i\omega s}+e^{-i\omega s}). \tag{21}$$

With the Fourier components of $\mathbf{v}$ determined, the expression (7) can be used to calculate the components of the tensor $\mathbf{S}^{(i)}$ in the system of axes of Fig. 1. In this system, the Z axis is the same for every valley, but the X and Y axes are not. We now choose one set of X and Y axes such that the X and Y axes of the ith valley must be rotated through an angle $\beta^{(i)}$ clockwise to be brought into coincidence with it. The tensor $\mathbf{S}^{(i)}$, obtained in the first system of axes, must then be transformed in the usual way to the new set of axes. Referred to the new axes, which are the same for every valley, the $\mathbf{S}$ tensor for the ith valley is

$$\mathbf{S}^{(i)}=V_H^2\begin{bmatrix} \dfrac{m_t^2}{m_*^2}\cos^2\beta+\sin^2\beta & \left(1-\dfrac{m_t^2}{m_*^2}\right)\sin\beta\cos\beta-\dfrac{m_t^2}{m_*^2}\omega_t\tau & \dfrac{m_t}{m_1}\cos\beta+\dfrac{m_t}{m_1}\omega_t\tau\sin\beta \\ \left(1-\dfrac{m_t^2}{m_*^2}\right)\sin\beta\cos\beta+\dfrac{m_t^2}{m_*^2}\omega_t\tau & \cos^2\beta+\dfrac{m_t^2}{m_*^2}\sin^2\beta & -\dfrac{m_t}{m_1}\sin\beta+\dfrac{m_t}{m_1}\omega_t\tau\cos\beta \\ \dfrac{m_t}{m_1}\cos\beta-\dfrac{m_t}{m_1}\omega_t\tau\sin\beta & -\dfrac{m_t}{m_1}\sin\beta-\dfrac{m_t}{m_1}\omega_t\tau\cos\beta & \dfrac{m_*^4}{m_l^2m_t^4}\dfrac{P_z^2}{V_H^2}+\dfrac{m_*^2}{m_1^2} \end{bmatrix}, \tag{22}$$

where

$$V_H^2=\left(\frac{\epsilon}{m_t}-\frac{m_*^2P_z^2}{2m_lm_t^3}\right)(1+\omega^2\tau^2)^{-1}=\left(\frac{m_*^2v_x^2}{2m_t^2}+\frac{v_y^2}{2}\right)(1+\omega^2\tau^2)^{-1}, \tag{23}$$

and $\omega_t=H_e/m_t=\omega m_*/m_t$. Superscripts (i) are, of course, to be understood on β, and on m_* and m_1, which are functions of $\alpha^{(i)}$ defined in (15). To recapitulate, the angle $\alpha^{(i)}$ is the angle between $\mathbf{H}$ and the major axis of the spheroids in the ith valley, and $\beta^{(i)}$ is the angle through which that minor axis of the spheroids in the ith valley that is perpendicular to $\mathbf{H}$ must be rotated clockwise to be brought into coincidence with the Y axis. It is to be noted that the components of $\mathbf{S}$ obey the symmetry requirement $S_{\alpha\beta}(\mathbf{H})=S_{\beta\alpha}(-\mathbf{H})$.[5]

As stated earlier, in evaluating $\sigma_{\alpha\beta}$ the quantity $S_{\alpha\beta}^{(i)}$ may be replaced by its average over a constant-energy shell, $\bar{S}_{\alpha\beta}^{(i)}$. The only quantities in the matrix (22) that vary with position on a constant-energy surface are P_z and, as a result, V_H^2. To obtain $\bar{S}_{\alpha\beta}^{(i)}$, it is thus necessary to evaluate averages of V_H^2 and $[(m_*^4P_z^2/m_l^2m_t^4)+(m_*^2V_H^2/m_1^2)]$ over a constant-energy shell. Both quantities can be expressed in terms of v_x^2 and v_y^2 by use of Eq. (19), this being shown for V_H^2 in Eq. (23). The velocity components can, in turn, be expressed as functions of the components v_x', v_y', and v_z' in the principal-axis system of the ellipsoids by $v_x=v_x'\cos\alpha-v_z'\sin\alpha$ and $v_y=v_y'$ since Y is already a principal axis in the coordinate system for which Eq. (19) is written. Averages of quantities $v_\alpha'v_\beta'$ can be evaluated very simply by transforming the constant-energy ellipsoids to spheres as suggested by Herring.[12] Thus, $\langle(v_x')^2\rangle=\langle(v_y')^2\rangle=2\epsilon/3m_t$, $\langle(v_z')^2\rangle=2\epsilon/3m_l$, and $\langle v_x'v_z'\rangle=0$. After aver-

[12] C. Herring, Bell System Tech. J. **XXXIV**, 237 (1955).

aging, $\mathbf{S}^{(i)}$ becomes

$$(\bar{S}^{(i)})=\frac{2\epsilon}{3m_t(1+\omega^2\tau^2)}\begin{bmatrix} \frac{m_t^2}{m_*^2}\cos^2\beta+\sin^2\beta & \left(1-\frac{m_t^2}{m_*^2}\right)\sin\beta\cos\beta-\frac{m_t^2}{m_*^2}\omega_t\tau & \frac{m_t}{m_1}\cos\beta+\frac{m_t}{m_1}\omega_t\tau\sin\beta \\ \left(1-\frac{m_t^2}{m_*^2}\right)\sin\beta\cos\beta+\frac{m_t^2}{m_*^2}\omega_t\tau & \cos^2\beta+\frac{m_t^2}{m_*^2}\sin^2\beta & -\frac{m_t}{m_1}\sin\beta+\frac{m_t}{m_1}\omega_t\tau\cos\beta \\ \frac{m_t}{m_1}\cos\beta-\frac{m_t}{m_1}\omega_t\tau\sin\beta & -\frac{m_t}{m_1}\sin\beta-\frac{m_t}{m_1}\omega_t\tau\cos\beta & \frac{m_*^2}{m_1^2}+\frac{m_*^2}{m_l m_t}(1+\omega^2\tau^2) \end{bmatrix}. \quad (24)$$

For cases in which $n^{(i)}$ and $f_0^{(i)}$ are the same for all valleys, the integration required to obtain $\sigma_{\alpha\beta}$ need not be carried out over each valley separately. Equation (13) can be simplified by replacing $n^{(i)}$ by the total carrier concentration n_{tot}, and $\bar{S}_{\alpha\beta}^{(i)}$ by its average over the valleys, to give

$$\sigma_{\alpha\beta}=n_{\text{tot}}e^2\langle-(1/f_0)(\partial f_0/\partial\epsilon)\tau\sum_{i=1}^{N}\bar{S}_{\alpha\beta}^{(i)}/N\rangle. \quad (25)$$

The average of $\bar{S}_{\alpha\beta}^{(i)}$ over the valleys has been evaluated for the case of a cubic crystal in weak magnetic fields. If we retain only terms linear in $\mathbf{H}$ the required summations can be carried out in straightforward fashion to give for a cubic crystal

$$\sum_{i=1}^{N}\frac{(\bar{S}^{(i)})}{N}=\frac{2\epsilon}{3m_t}\begin{bmatrix} \frac{1}{3}\left(2+\frac{m_t}{m_l}\right) & -\frac{1}{3}\left(1+\frac{2m_t}{m_l}\right)\omega_t\tau & 0 \\ \frac{1}{3}\left(1+\frac{2m_t}{m_l}\right)\omega_t\tau & \frac{1}{3}\left(2+\frac{m_t}{m_l}\right) & 0 \\ 0 & 0 & \frac{1}{3}\left(2+\frac{m_t}{m_l}\right) \end{bmatrix}. \quad (26)$$

It is readily seen that this matrix leads to the correct low-field conductivity. For low electric field,

$$-(1/f_0)(\partial f_0/\partial\epsilon)=1/kT. \quad (27)$$

Using this and (26) in (25), we obtain

$$\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=\frac{n_{\text{tot}}e^2}{(3/2)kT}\times\frac{1}{3}\left(\frac{2}{m_t}+\frac{1}{m_l}\right)\langle\tau\epsilon\rangle =\frac{n_{\text{tot}}e^2}{m^{(I)}}\frac{\langle\tau\epsilon\rangle}{\langle\epsilon\rangle}, \quad (28)$$

in the form obtained by Herring.[12] In the absence of magnetic field, the off-diagonal components all vanish, as is correct for low field. It will be shown in the next section that (25) and (26) lead to the correct low-field Hall coefficient also.

IV. ANISOTROPY VOLTAGE AND HALL EFFECT AT HIGH ELECTRIC FIELD

We shall consider first the situation in which an electric field is applied but there is no magnetic field. Whether or not there is anisotropy in the conductivity depends on the $\mathbf{S}$ tensor, or, more basically, on the symmetry of the constant-energy surfaces.[5] We shall confine the discussion of the anisotropy to the many-valley band structure, for which we have derived the $\mathbf{S}$ tensor, and to cubic crystals.

In a cubic crystal, when Ohm's law is obeyed, $\sigma_{\alpha\beta}$ must vanish for $\alpha\neq\beta$. Although the $\bar{S}_{\alpha\beta}^{(i)}$, and therefore $\sigma_{\alpha\beta}^{(i)}$, do not vanish in general, when the contributions of the different valleys to the off-diagonal elements are added they cancel to produce, in the absence of magnetic field, a scalar. [See (26).] When a high electric field is applied, not in a symmetry direction, the valleys may have different $f_0^{(i)}$, however, and then the off-diagonal contributions to $\boldsymbol{\sigma}$ would no longer cancel. When they do not cancel, the current flows initially at an angle with the applied field, until it produces a transverse electric field sufficient to buck out the transverse flow. We shall now obtain an expression for this transverse field, the anisotropy field.

For the initially applied field in a general crystallographic direction, to be taken as the X direction, all nine components of $\boldsymbol{\sigma}$ might be nonvanishing. If this were the case, transverse fields in both Y and Z directions would result. We shall simplify the situation by choosing the applied field direction such that $\sigma_{xz}=\sigma_{yz}=0$. Then there will be no component of current, and no transverse field developed, in the Z direction. This will be the case, for example, in n-germanium with the Z direction taken as [110], and the X direction anywhere in the (110) plane, as shown in the Appendix. For such an orientation we can write

$$j_x=\sigma_{xx}(0)E_x+\sigma_{xy}(0)E_y(0), \tag{29a}$$

$$j_y=\sigma_{yx}(0)E_x+\sigma_{yy}(0)E_y(0), \tag{29b}$$

where 0 indicates the value of the quantity in the absence of magnetic field, and the σ's are given by (13). In the steady state $j_y=0$, from which we deduce for the anisotropy field

$$E_y(0)=-\frac{\sigma_{yx}(0)}{\sigma_{yy}(0)}E_x, \tag{30a}$$

or

$$\frac{E_y(0)}{E_x}=\tan\chi=-\frac{\sigma_{yx}(0)}{\sigma_{yy}(0)}. \tag{30b}$$

Eliminating $E_y(0)$ from (29) with the use of (30), we obtain

$$j_x=\left[\sigma_{xx}(0)-\frac{\sigma_{xy}(0)^2}{\sigma_{yy}(0)}\right]E_x, \tag{31}$$

since σ is symmetric in the absence of magnetic field. It is worth noting that, when measurements of j_x/E_x in the hot-carrier range are made in the usual way on n-germanium samples, the quantity measured is the one in brackets in (31).

With the use of (13) we can write $\tan\chi$ in terms of the population and distribution functions of the individual valleys as

$$\tan\chi=-\frac{\sum_{i=1}^{N} n^{(i)}e^2\langle-(1/f_0^{(i)})(\partial f_0^{(i)}/\partial\epsilon)\tau\bar{S}_{yx}^{(i)}(0)\rangle}{\sum_{i=1}^{N} n^{(i)}e^2\langle-(1/f_0^{(i)})(\partial f_0^{(i)}/\partial\epsilon)\tau\bar{S}_{yy}^{(i)}(0)\rangle}. \tag{32}$$

If $f_0^{(i)}$ is a Maxwell-Boltzmann distribution at $T^{(i)}$, this can be simplified to

$$\tan\chi=-\frac{\sum_{i=1}^{N}(n^{(i)}/T^{(i)})\langle\tau\bar{S}_{yx}^{(i)}(0)\rangle}{\sum_{i=1}^{N}(n^{(i)}/T^{(i)})\langle\tau\bar{S}_{yy}^{(i)}(0)\rangle}$$

$$=-\frac{\sum_{i=1}^{N}(n^{(i)}/T^{(i)})\langle\tau\epsilon\rangle^{(i)}(1-m_t^2/m_*^{(i)2})\sin\beta^{(i)}\cos\beta^{(i)}}{\sum_{i=1}^{N}(n^{(i)}/T^{(i)})\langle\tau\epsilon\rangle^{(i)}[\cos^2\beta^{(i)}+(m_t/m_*^{(i)2})\sin^2\beta^{(i)}]}. \tag{33}$$

Use of (32) and (33) can be simplified in practice by combining terms for different valleys that have the same $f_0^{(i)}$. It is also useful to have some idea of the relative "temperatures" of different valleys when they are not the same. A quick way of obtaining this information is to compare the power absorbed by the carriers in different valleys at low electric fields. For the electric field in the X direction, the power absorbed in the ith valley is

$$P^{(i)}=\mathbf{j}^{(i)}\cdot\mathbf{E}=\sigma_{xx}^{(i)}E_x^2=(n^{(i)}e^2/kT^{(i)})\langle\tau\bar{S}_{xx}^{(i)}\rangle. \tag{34}$$

At low electric field, $n^{(i)}$ and $T^{(i)}$ are the same for all the valleys, so the ratio of power absorbed in the ith valley to that absorbed in the jth valley is

$$\frac{P^{(i)}}{P^{(j)}}=\frac{\bar{S}_{xx}^{(i)}}{\bar{S}_{xx}^{(j)}}=\frac{(m_t/m_*^{(i)})^2\cos^2\beta^{(i)}+\sin^2\beta^{(i)}}{(m_t/m_*^{(j)})^2\cos^2\beta^{(j)}+\sin^2\beta^{(j)}}. \tag{35}$$

It is expected that if $P^{(i)}/P^{(j)}$ is unity for a particular pair of valleys, they have the same $f_0^{(i)}$ in high field; while if $P^{(i)}/P^{(j)}>1$, the ith valley is hotter than the jth in high field.

Although it is beyond the scope of the present paper to evaluate expressions (31) and (32) for any particular case,[13] the $\bar{S}_{\alpha\beta}^{(i)}$ tensors have been calculated for n-germanium with current in the (110) plane and are presented in the Appendix. Also in the Appendix is a plot for each of the four valleys in n-germanium of the quantity $(m_t/m_*^{(i)})^2\cos^2\beta^{(i)}+\sin^2\beta^{(i)}$, which appears in (35), as a function of the angle between the current direction and the [001] direction, for current in the (110) plane.

With the results (13) and (24) it is possible to obtain readily expressions for magnetoresistance and Hall effect of a many-valley semiconductor at arbitrary magnetic- and electric-field strengths.[14] In this paper, we shall confine ourselves to the Hall effect. For the case of small magnetic field, or $\omega\tau\ll 1$, the expression for the Hall

[13] This has been done for a range of fields in n-Ge by H. G. Reik, H. Risken, and G. Finger, Phys. Rev. Letters **5**, 423 (1960). See also E. G. S. Paige, Proc. Phys. Soc. (London) **75**, 174 (1960).

[14] This does not include fields high enough to give rise to quantum effects.

coefficient takes a relatively simple form which we shall now derive. The $\bar{S}_{\alpha\beta}^{(i)}$ tensor for small $\omega\tau$ is that of (24), with $\omega\tau$ neglected in the multiplying factor and in S_{zz}. When terms in $\omega^2\tau^2$ are neglected, the off-diagonal components $\bar{S}_{\alpha\beta}^{(i)}$ consist of a sum of two terms, one independent of $\mathbf{H}$ and already denoted by $\bar{S}_{\alpha\beta}^{(i)}(0)$, the other linear in $\mathbf{H}$, to be denoted by $\bar{S}_{\alpha\beta}^{(i)}(\mathbf{H})$. As a result of this, $\sigma_{\alpha\beta}^{(i)}$ will consist of a sum of two terms that can be written

$$\sigma_{\alpha\beta}^{(i)}=\sigma_{\alpha\beta}^{(i)}(0)+\sigma_{\alpha\beta}^{(i)}(\mathbf{H}), \tag{36}$$

the second term being linear in $\mathbf{H}$. For the calculation of the Hall coefficient, the directions of magnetic and electric fields will be chosen so that off-diagonal components of the magnetoconductivity involving z (now the magnetic-field direction) again vanish. Inspection of (24) shows that the terms occurring as coefficients of $\omega_t\tau$ in $\bar{S}_{xz}^{(i)}$ and $\bar{S}_{yz}^{(i)}$ also occur in $\bar{S}_{xz}^{(i)}(0)$ and $\bar{S}_{yz}^{(i)}(0)$. Thus, if σ_{xz} and σ_{yz} vanish for some set of directions in the absence of magnetic field, they will do so also in the presence of the magnetic field. We can then write for high electric fields and small magnetic fields, oriented as discussed,

$$j_x=\sigma_{xx}(0)E_x+[\sigma_{yx}(0)-\sigma_{yx}(\mathbf{H})]E_y(\mathbf{H}), \tag{37a}$$

$$j_y=[\sigma_{yx}(0)+\sigma_{yx}(\mathbf{H})]E_x+\sigma_{yy}(0)E_y(\mathbf{H}), \tag{37b}$$

where the σ's are given by (13) and the matrix (24) with the terms in $\omega^2\tau^2$ dropped. In writing (37), we have made use of the symmetry of the $\mathbf{S}$ matrix. In the steady state, $j_y=0$, from which we obtain

$$E_y(\mathbf{H})=-\left[\frac{\sigma_{yx}(0)}{\sigma_{yy}(0)}+\frac{\sigma_{yx}(\mathbf{H})}{\sigma_{yy}(0)}\right]E_x. \tag{38a}$$

The transverse voltage is thus the sum of contributions from the anisotropy and Hall effects. If the magnetic field were reversed, the sign before $\sigma_{yx}(\mathbf{H})$ would change. In that case, the transverse voltage would be

$$E_y(-\mathbf{H})=-\left[\frac{\sigma_{yx}(0)}{\sigma_{yy}(0)}-\frac{\sigma_{yx}(\mathbf{H})}{\sigma_{yy}(0)}\right]E_x. \tag{38b}$$

By subtracting (38b) from (38a), the anisotropy voltage can be eliminated. The Hall field is then

$$\begin{aligned}E_{\mathrm{Hall}}&=\tfrac{1}{2}[E_y(\mathbf{H})-E_y(-\mathbf{H})]\\&=-[\sigma_{yx}(\mathbf{H})/\sigma_{yy}(0)]E_x.\end{aligned} \tag{39}$$

If we now calculate the Hall coefficient from the usual definition

$$R=E_{\mathrm{Hall}}/(j_xH),$$

we obtain, neglecting terms of order H^2,

$$R_{\mathbf{H}\to 0}=-\frac{1}{H}\frac{\sigma_{yx}(\mathbf{H})}{\sigma_{xx}(0)\sigma_{yy}(0)-\sigma_{xy}(0)^2}, \tag{40}$$

where

$$\begin{aligned}
\frac{1}{H}\sigma_{yx}(\mathbf{H})&=\frac{2e^3}{3m_t^2}\sum_{i=1}^{N}n^{(i)}\left(\frac{m_t}{m_*^{(i)}}\right)^2\left\langle-\frac{1}{f_0^{(i)}}\frac{\partial f_0^{(i)}}{\partial\epsilon}\tau^2\epsilon\right\rangle,\\
\sigma_{xx}(0)\sigma_{yy}(0)&=\left[\frac{2e^2}{3m_t}\sum_{i=1}^{N}n^{(i)}\left(\frac{m_t^2}{m_*^{(i)2}}\cos^2\beta^{(i)}+\sin^2\beta^{(i)}\right)\left\langle-\frac{1}{f_0^{(i)}}\frac{\partial f_0^{(i)}}{\partial\epsilon}\tau\epsilon\right\rangle\right]\\
&\quad\times\left[\frac{2e^2}{3m_t}\sum_{j=1}^{N}n^{(j)}\left(\cos^2\beta^{(j)}+\frac{m_t^2}{m_*^{(j)2}}\sin^2\beta^{(j)}\right)\left\langle-\frac{1}{f_0^{(j)}}\frac{\partial f_0^{(j)}}{\partial\epsilon}\tau\epsilon\right\rangle\right],\\
\sigma_{xy}(0)^2&=\left[\frac{2e^2}{3m_t}\sum_{i=1}^{N}n^{(i)}\left(1-\frac{m_t^2}{m_*^{(i)2}}\right)\sin\beta^{(i)}\cos\beta^{(i)}\left\langle-\frac{1}{f_0^{(i)}}\frac{\partial f_0^{(i)}}{\partial\epsilon}\tau\epsilon\right\rangle\right]^2.
\end{aligned} \tag{41}$$

For $f_0^{(i)}$ a Maxwell-Boltzmann distribution, (41) can be simplified somewhat by writing

$$\frac{2}{3}\left\langle-\frac{1}{f_0^{(i)}}\frac{\partial f_0^{(i)}}{\partial\epsilon}\tau^2\epsilon\right\rangle=\frac{\langle\tau^2\epsilon\rangle^{(i)}}{\langle\epsilon\rangle^{(i)}},$$

etc. When $T^{(i)}$ and $n^{(i)}$ are all the same, the summations in (41) can be carried out as they were to obtain the average of $\bar{S}_{\alpha\beta}^{(i)}$ over the valleys. The expression for R then becomes

$$R_{\mathbf{H}\to 0}=\frac{1}{n_{\mathrm{tot}}e}\frac{3(1+2m_t/m_l)}{(2+m_t/m_l)^2}\frac{\langle\tau^2\epsilon\rangle\langle\epsilon\rangle}{\langle\tau\epsilon\rangle^2}. \tag{42}$$

For low electric field, this expression is the same as the one obtained by Herring[12] for that case.

It is clear from (40) and (41) that, as indicated in the Introduction, even though the total carrier concentration remains constant, changes in R will occur as the electric field is changed due to change in any of the $n^{(i)}$ or $f_0^{(i)}$.

When $\omega\tau$ is not small, it is possible to go through a rather similar analysis to obtain R. It is seen from (24) that $\bar{S}_{yx}^{(i)}$, and therefore σ_{yx}, can again be separated into two terms, one even in $\mathbf{H}$ and the other odd. The term even in $\mathbf{H}$ becomes $\sigma_{yx}(0)$ in the limit of small $\mathbf{H}$, while the term odd in $\mathbf{H}$ becomes $\sigma_{yx}(\mathbf{H})$ in the limit of small **H**. Thus, the former still gives rise to an anisotropy voltage, the latter to a Hall voltage, both now modified by the presence of H^2 terms. The expedient of reversing the magnetic field will again permit separation of the two voltages. By the same procedure carried out in Eqs. (37)–(40), we then obtain

$$E_{\mathrm{Hall}}=-\frac{(\sigma_{yx}-\sigma_{xy})/2}{\sigma_{yy}}E_x, \tag{44}$$

and

$$R=\frac{1}{H}\frac{(\sigma_{yx}-\sigma_{xy})/2}{\sigma_{xx}\sigma_{yy}-\sigma_{xy}\sigma_{yx}}. \tag{45}$$

These expressions are valid for any magnetic-field strength. The product $\sigma_{xy}\sigma_{yx}$ is the square of the even term in σ_{yx} minus the square of the odd term in σ_{yx}. In the limit of small **H**, the square of the odd term can be neglected, as was done previously, and (45) becomes identical with (40).

The expression (45) for R is also valid for materials to which the treatment of Sec. II applies. This is the case because, as can be seen from the general expression (7) for $S_{\alpha\beta}$, $\sigma_{\alpha\beta}$ can always be broken up into a set of terms even in **H** and a set odd in **H**. Of course, the validity of (45) requires also that the material have sufficient symmetry that off-diagonal components of $\boldsymbol{\sigma}$ involving Z vanish. It must also be remembered here that, if carriers from more than one band participate in conduction, the matrix element $\sigma_{\alpha\beta}$ consists of the sum of $\sigma_{\alpha\beta}$'s from each band.

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APPENDIX

In this Appendix we carry out the evaluation of the $\mathbf{S}^{(i)}$ tensors for the case of current in the $(1\bar{1}0)$ plane in n-germanium. The current direction, which is chosen as the X axis, will be specified by the angle θ it makes with the [001] direction, as shown in Fig. 2. The Z direction is taken as the $[1\bar{1}0]$ direction, perpendicular to the plane of the figure. The $\mathbf{S}^{(i)}$ tensors of the four valleys will be expressed as a function of θ, which puts them into a convenient form for further calculation. Also, the quantity $(m_t/m_*^{(i)})^2\cos^2\beta^{(i)}+\sin^2\beta^{(i)}$, which appears in the expression (37) for the ratio of power absorbed in different valleys, will be plotted as a function of θ for each of the valleys.

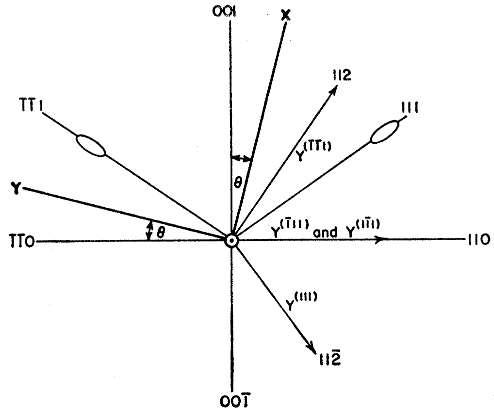

FIG. 2. Plot of $(1\bar{1}0)$ plane in crystal momentum space.

Also shown in Fig. 2 are constant-energy ellipsoids for the two valleys whose major axis lies in the $(1\bar{1}0)$ plane. The locations of the valleys, i.e., the $\mathbf{Z}^{(i)}$ axes, have arbitrarily been chosen along the [111], $[\bar{1}11]$, $[\bar{1}\bar{1}1]$, and $[1\bar{1}1]$ directions. The same $\mathbf{S}^{(i)}$ tensor and conductivity would of course be obtained by replacing any or all of these directions with its negative, i.e., choosing the valley location at the opposite end of the body diagonal. Also indicated in Fig. 2 are the Y axes for the four valleys. These lie in the [112], [110], and $[11\bar{2}]$ directions according to the convention specified, that the sense of Y be that of $\mathbf{Z}^{(i)}\times\mathbf{Z}$. Thus, the angle $\beta^{(111)}$, which is the angle through which $Y^{(111)}$ must be rotated clockwise to be brought into coincidence with the Y axis, is $(\cos^{-1}0.816+90°+\theta)$, $\beta^{(\bar{1}11)}=\beta^{(1\bar{1}1)}=(180°+\theta)$, and $\beta^{(\bar{1}\bar{1}1)}=(\cos^{-1}0.577+180°+\theta)$.

For the [111] and $[\bar{1}\bar{1}1]$ valleys, $\alpha^{(i)}=90°$; while for $[\bar{1}11]$ it is $\cos^{-1}(-0.816)$, and for $[1\bar{1}1]$ it is $\cos^{-1}(+0.816)$.

Because of the similarity in angles, the **S** tensors for the [111] and $[\bar{1}\bar{1}1]$ valleys are identical except for a few signs, and those for the $[\bar{1}11]$ and $[1\bar{1}1]$ valleys are similarly related. The tensors for these pairs of valleys have therefore been displayed together. Only the low-magnetic-field case is shown, with $\bar{S}^{(i)}(0)$ in (46), $\bar{S}^{(i)}(\mathbf{H})$ in (47). For the [111] and $[\bar{1}\bar{1}1]$ valleys, $\bar{S}^{(i)}(0)$ is given by

$$\begin{Bmatrix}(\bar{S}^{111}(0))\\(\bar{S}^{\bar{1}\bar{1}1}(0))\end{Bmatrix}=\frac{2\epsilon}{9m_t}\begin{bmatrix}\frac{m_t}{m_l}[1+\sin^2\theta\pm\sqrt{2}\sin2\theta]+1+\cos^2\theta\mp\sqrt{2}\sin2\theta & \left(1-\frac{m_t}{m_l}\right)[\mp\sqrt{2}\cos2\theta-\frac{1}{2}\sin2\theta] & 0\\ \left(1-\frac{m_t}{m_l}\right)[\mp\sqrt{2}\cos2\theta-\frac{1}{2}\sin2\theta] & 1+\sin^2\theta\pm\sqrt{2}\sin2\theta+\frac{m_t}{m_l}[1+\cos^2\theta\mp\sqrt{2}\sin2\theta] & 0\\ 0 & 0 & 3\end{bmatrix}. \tag{46a}$$

For the $[\bar{1}11]$ and $[1\bar{1}1]$ valleys, $(\bar{S}^{(i)}(0))$ is given by

$$\left\{\begin{matrix}(\bar{S}^{\bar{1}11}(0))\\ (\bar{S}^{1\bar{1}1}(0))\end{matrix}\right\}=\frac{2\epsilon}{9m_t}\begin{bmatrix} \left(2+\dfrac{m_t}{m_l}\right)\cos^2\theta+3\sin^2\theta & \dfrac{1}{2}\left(1-\dfrac{m_t}{m_l}\right)\sin 2\theta & \pm\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\cos\theta \\ \dfrac{1}{2}\left(1-\dfrac{m_t}{m_l}\right)\sin 2\theta & 3\cos^2\theta+\left(2+\dfrac{m_t}{m_l}\right)\sin^2\theta & \mp\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\sin\theta \\ \pm\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\cos\theta & \mp\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\sin\theta & 1+\dfrac{2m_t}{m_l} \end{bmatrix}. \tag{46b}$$

In (46a) and (46b) where there is a difference in sign, the upper sign refers to the valley on top at the left, the lower sign to the valley below on the left. The tensors $(\bar{S}^{(i)}(\mathbf{H}))$ for the pairs of valleys are shown in (47a) and (47b), with the same convention with regard to signs.

$$\left\{\begin{matrix}(\bar{S}^{111}(\mathbf{H}))\\ (\bar{S}^{\bar{1}\bar{1}1}(\mathbf{H}))\end{matrix}\right\}=\frac{2\epsilon}{9m_t}\begin{bmatrix} 0 & -(3m_t/m_l)\omega_t\tau & 0 \\ +(3m_t/m_l)\omega_t\tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{47a}$$

$$\left\{\begin{matrix}(\bar{S}^{\bar{1}11}(\mathbf{H}))\\ (\bar{S}^{1\bar{1}1}(\mathbf{H}))\end{matrix}\right\}=\frac{2\epsilon}{9m_t}\begin{bmatrix} 0 & -\left(2+\dfrac{m_t}{m_l}\right)\omega_t\tau & \pm\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\omega_t\tau\sin\theta \\ +\left(2+\dfrac{m_t}{m_l}\right)\omega_t\tau & 0 & \pm\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\omega_t\tau\cos\theta \\ \mp\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\omega_t\tau\sin\theta & \mp\sqrt{2}\left(1-\dfrac{m_t}{m_l}\right)\omega_t\tau\cos\theta & 0 \end{bmatrix}. \tag{47b}$$

It can readily be seen that if $\bar{S}^{(i)}(0)$ and $\bar{S}^{(i)}(\mathbf{H})$ are added to obtain the full $(\bar{S}^{(i)})$ tensor, and the $\bar{S}^{(i)}$'s summed over the valleys and divided by four, the tensor of (26) is obtained, as it should be. For the purpose of calculating the elements of $\boldsymbol{\sigma}$, however, the $\bar{S}^{(i)}$'s cannot, of course, all be added before integration is performed because of the different temperatures of different valleys.

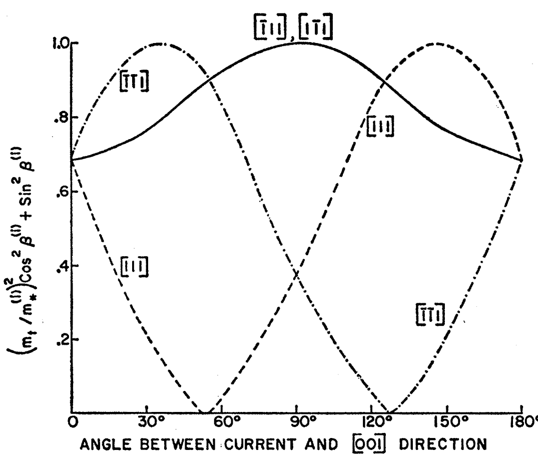

FIG. 3. Plot of $(m_t/m_*^{(i)})^2\cos^2\beta^{(i)}+\sin^2\beta^{(i)}$, which is proportional to the low-field power absorption, as a function of angle between the current and the [001] direction for current in the $(1\bar{1}0)$ plane.

To give an indication of the relative temperature of different valleys for different field orientations, we have plotted, in Fig. 3, the quantity $(m_t/m_*^{(i)})^2\cos^2\beta^{(i)}+\sin^2\beta^{(i)}$, which is proportional to the low-field power absorption, as a function of θ. It is seen that the power absorbed in the $[\bar{1}11]$ and $[1\bar{1}1]$ valleys is the same, independent of θ, as expected by symmetry. Thus, in high field, these two valleys should be at the same temperature. The $[111]$ and $[\bar{1}\bar{1}1]$ valleys, on the other hand, will absorb energy at different rates and thus be at different temperatures, unless $\theta=0$, 90, 180, or 270°. This is also as expected by symmetry. It is seen that the maximum power absorption for a given valley occurs when the electric field is perpendicular to the longitudinal or $\mathbf{Z}^{(i)}$ axis, and the minimum when the field is parallel to the $\mathbf{Z}^{(i)}$ axis.[15]

Since the $[\bar{1}11]$ and $[1\bar{1}1]$ valleys are expected to have the same $f_0^{(i)}$ at all fields, the $\bar{S}_{\alpha\beta}^{(i)}$'s of these two valleys may be added before integration. It is seen then that their off-diagonal components involving z will cancel each other. Since off-diagonal components $\bar{S}_{\alpha\beta}^{(i)}$ involving z are zero for the other two valleys, it is clear that the off-diagonal components of $\boldsymbol{\sigma}$ involving z will vanish for current in the $(1\bar{1}0)$ plane as stated earlier.

[15] For further consideration of the variation of power absorption with direction see L. Gold, Phys. Rev. **104**, 1580 (1956).