π^+-Proton Scattering at 990 Mev*

J. K. Kopp†
Brookhaven National Laboratory, Upton, New York

A. M. Shapiro
Brown University, Providence, Rhode Island

AND

A. R. Erwin
University of Wisconsin, Madison, Wisconsin

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Positive-pion scattering at 990±30 Mev has been examined in a 6×3×2 in. hydrogen bubble chamber without a magnetic field. The cross sections for elastic and inelastic scattering were found to be 15.3±1.5 mb and 12.6±3.3 mb, respectively. The inelastic scattering cross section includes $0.19_{-0.07}^{+0.09}$ mb of Σ^+-K^+ production and 0.78±0.14 mb of $\pi^+\pi^+\pi^-p$ production. A simple pion-pion model which predicts the branching ratios for double pion production in π^--p collisions is found to be inconsistent with the double pion production observed in this experiment. The relation of the experiment to π^--p experiments in the region of the second and third resonances is discussed.

INTRODUCTION

There recently has been a renewed intensity of interest in pion-nucleon scattering for pion energies between 550 and 950 Mev. A strong impetus for this was provided by the counter experiment of Burrowes *et al.*,[1] which succeeded in resolving the "bump" of the π^--p total cross section into two distinct peaks. Other experimenters have since corroborated the existence of these maxima and established their locations at 600 and 900 Mev.[2,3]

Detailed experimental investigation of the pion-nucleon partial cross sections in the energy interval including the two new peaks has not been very extensive. Photoproduction experiments[4–8] have indicated that the 600-Mev peak is consistent with a resonance in a state having isotopic spin $T=\frac{1}{2}$, angular momentum $J=\frac{3}{2}$, and odd parity. Hydrogen bubble chamber experiments with π^- mesons at 950 Mev[9] and 1.0 Bev[10] have demonstrated what appears to be the very strong influence of the $T=\frac{3}{2}$, $J=\frac{3}{2}$ isobar on single pion production at these energies.

Theoretical discussion of the two peaks has either been in terms of the $T=\frac{3}{2}$, $J=\frac{3}{2}$ isobar[11] or in terms of models combining this isobar with a strong pion-pion interaction.[12–15] Present experimental knowledge does not lend overwhelming support to any particular model.

Experiments with π^--p interactions have not yet thoroughly covered the energy range of interest. Because they are chiefly derived from data designed for studying strange particles, their energy locations have not been optimum for studying the new peaks. Their interpretation has been further complicated by the absence of corresponding π^+-p experiments to provide data on the $T=\frac{3}{2}$ component of the π^--p scattering.

It is the purpose of this paper to report data on π^+-p interactions at 990 Mev and, where it is possible, to relate these results to the π^--p experiments previously performed.

PROCEDURE AND ANALYSIS

The experiment was performed at the Brookhaven Cosmotron in a 6×3×2 in. hydrogen bubble chamber without a magnetic field.[16] The beam arrangement and measuring procedure are described in a previous report.[17]

While the strange particle production events were easily recognized, analysis and enumeration of almost all the other events was complicated by the presence of protons in the beam. As the kinematics charts (Fig. 1) show, it becomes difficult to separate $\pi-p$ and $p-p$ elastic scatterings for small pion angles and to separate $\pi-p$ scatterings for large pion angles from deuteron productions by protons. Further, it was not generally possible to determine whether a given two-prong, inelastic event was caused by a proton or pion. Hence, it

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† This paper is based on a dissertation to be submitted by one of us (JKK) to Harvard University in partial fulfillment of the requirements for the Ph.D. degree.

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2 J. C. Brisson, J. Detoef, P. Falk Vairant, L. van Rossum, G. Valladas, and L. C. L. Yuan, Phys. Rev. Letters **3**, 561 (1959).

3 T. J. Devlin, B. C. Barish, W. N. Hess, V. Perez-Mendez, and J. Solomon, Phys. Rev. Letters **4**, 242 (1960).

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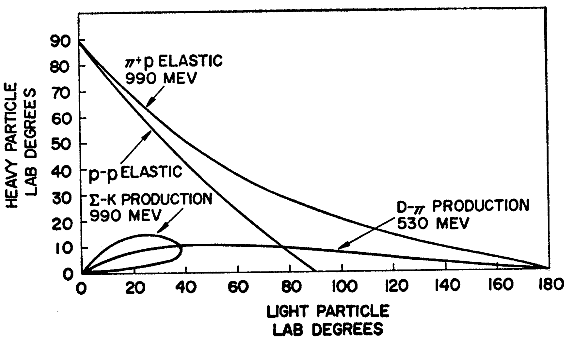

FIG. 1. Two-body kinematics for $\pi-p$ and $p-p$ collisions in the momentum range of this experiment. The laboratory scattering angle of the heavier particle is plotted on the ordinate vs the corresponding angle of the lighter particle on the abscissa.

was necessary to use statistical methods to divide such events between pions and protons.

Two-prong events were classified largely on the basis of whether they were coplanar and fitted kinematics requirements. The categories used were (a) elastic $p-p$ or $\pi-p$ scatterings above 26°, which could be classified unambiguously (1154 events), (b) deuteron productions by protons (159 events), (c) two-prong inelastic events of necessarily unknown parentage (655 events), and (d) small-angle elastic scattering (1409 events), again of unknown parentage. The four-prong and strange particle events (27 events) were ascribed to the pions. Large-angle $\pi-p$ scatters which could be confused with deuteron production events were possible only in a negligible amount of solid angle in the $\pi-p$ center-of-mass system and thus were not classified separately.

It was necessary to determine how many of the group (d) were $\pi-p$ events so as to extend the differential cross section to as small an angle as possible. It was also necessary to divide the events (c) in a similar manner so as to determine the total number of $\pi-p$ events in our sample. Given that number, absolute cross sections could be calculated from the known π^+-p total cross section. The following separation scheme was employed.

First, all elastic events with scattering planes less than 30° from the horizontal were rejected. These events were seen "edge on" by the cameras, and in reconstructing them in space measurement errors were greatly magnified. The observed azimuthal distributions of the remaining elastic events suggested that no scanning-loss corrections were necessary for the sample except in the most forward interval, as mentioned below.

Next those events accepted above which comprised group (d) were sorted by the smaller scattering angle into groups corresponding to pion scattering angles with (center-of-mass) cosine of 0.6–0.7, 0.7–0.8, 0.8–0.9, 0.9–0.95, and 0.95–1.0. The last group was rejected because of the large scanning losses to be expected. Each event was, in effect, plotted as a point on the kinematics chart, on which the coordinate axes represented the two scattering angles. The point was, therefore, a certain "distance" x from the $p-p$ elastic kinematics curve, measured in degrees. For each group of events, a frequency histogram in x was made. In the limit of small angular intervals, one would expect the number of events, $N(x)$, to approximate two superposed normal distributions, of the form

$$N(x)=\{[\sigma_1^{-1}\exp(-x^2/2\sigma_1^2)+A\sigma_2^{-1}\exp(-(x-s)^2/2\sigma_2^2)]/(2\pi)^{\frac{1}{2}}(A+1)\},\quad (1)$$

where the first, centered on zero, corresponds to the $p-p$ elastic events clustered about the $p-p$ kinematics curve. The second, s degrees from zero, represents the $\pi-p$ elastic events grouped about their kinematics curve. The standard deviations would in our case be $\sim 1°$.

Figure 2 shows one of the histograms fitted to the distribution (1), the object being to determine the relative areas of the pion and proton contributions, given by the parameter A. Of the four groups analyzed, only those with cosine 0.6–0.7 and 0.7–0.8 gave results that could be considered significant. For the other two groups, the pion and proton parts were too close together, and probably too nongaussian. The criterion of a significant fit was largely the sensitivity of the χ^2 parameter and the appearance of the fitted curve to variations in A. The errors assigned to the number of events in the two groups for which the method was satisfactory were computed in such a way that they tend to overestimate the actual uncertainty involved.

The above separation also yields the number N_{ppe} of $p-p$ elastic scatterings, free of pion contamination, up to a center-of-mass cosine of 0.8. Using the known $p-p$ total cross section, partial inelastic cross sections, and differential cross sections,[18] we then calculate from N_{ppe} how many two-prong inelastic events would result from the protons in our beam, N_{ppi}. Subtracting N_{ppi} from the total of two-prong inelastic events gives the number

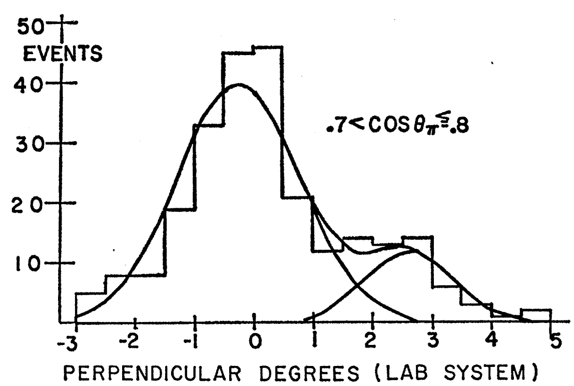

FIG. 2. One histogram of the number of events vs their "distance" in perpendicular degrees from the $p-p$ kinematics curve. The smooth curve is a fit to the data using two superposed normal distributions. Only the two histograms containing π events with center-of-mass $\cos\theta_\pi \leqslant 0.8$ were used in data reduction, as described in the text.

[18] W. N. Hess, Revs. Modern Phys. **30**, 368 (1958).

TABLE I. Partial cross sections determined in this experiment. Absolute values were obtained by normalizing to total cross sections of footnote 3. Errors shown are statistical only.

Event	Cross section (mb)
Elastic $\pi^+ - p$	15.3 ± 1.5
Total inelastic $\pi^+ - p$	12.6 ± 3.3
$\pi^+ + p \rightarrow \Sigma^+ + K^+$	$0.19_{-0.07}^{+0.09}$
$\pi^+ + p \rightarrow \pi^+ + \pi^+ + \pi^- + p$	0.78 ± 0.14

of such events caused by incident pions, though with rather large statistical error. The two-prong inelastic events were first corrected for scanning losses by 7%. This figure, somewhat arbitrary, was a compromise between two calculations of possible scanning losses for inelastic events. One, 11%, was based on the observed losses of small-angle elastic scatterings as a function of their azimuthal orientation. It assumed that scanning loss depended on the smaller of the two projected scattering angles presented to the scanner. The second figure, 3%, was obtained by comparing the laboratory angular distribution of our inelastic events with one obtained by reasonably extrapolating some lower energy $p-p$ data.[19] In the latter estimate it was assumed that all losses were among inelastic $p-p$ events since they are much more confined to forward directions by energy considerations than the corresponding $\pi - p$ events.

In order to extend the differential cross section to zero degrees use was made of the optical theorem and the vanishing real part of the forward scattering amplitude predicted by the dispersion relations.[20] The manner of extrapolating the cross section of course affected the total number of $\pi - p$ events and hence the absolute scale of the differential curve. A few trials, however, gave a consistent fit to the expected differential cross section at zero degrees, indicating that the normalization is not very sensitive to the manner in which the extrapolation is carried out.

ENERGY OF THE BEAM

Analysis of seven strange-particle production events indicated that the beam had a spread in momentum of about ± 30 Mev, centered about 990 ± 10 Mev. Calculations of the momentum from the kinematics of a large sample of definitely indentified $p-p$ and $\pi - p$ events were in substantial agreement. The sample used was selected so as to minimize the effects of measuring errors or possible systematic errors in the stero paramenters.

Since the chain of calculations described in the previous section made frequent use of $p-p$ and $\pi^+ - p$ cross sections obtained by other groups, it is conceivable that the results could depend critically on the beam momentum. To test this possibility the calculations were repeated assuming the two limiting energies 960 and 1020 Mev. The results were:

960 Mev, $\sigma(\text{elastic}) = 13.4$ mb, $\sigma(\text{inelastic}) = 13.4$ mb;

990 Mev, $\sigma(\text{elastic}) = 15.3$ mb, $\sigma(\text{inelastic}) = 12.6$ mb;

1020 Mev, $\sigma(\text{elastic}) = 17.6$ mb, $\sigma(\text{inelastic}) = 11.0$ mb.

One can thus conclude that a systematic error of this type is certainly no larger than the statistical errors quoted for the cross sections in Table I.

CROSS SECTIONS

Table I summarizes the partial cross sections obtained in this experiment. All values are normalized to a total $\pi^+ - p$ cross section of 27.9 mb, and the errors given do not include the uncertainty in this number. The total elastic and total inelastic cross sections were computed from a sample of data containing, after scanning corrections, approximately 3200 $p-p$ events and 900 $\pi^+ - p$ events. Since the four-prong events do not require detailed kinematical analysis, the cross section for these events was based on a somewhat larger sample of data containing approximately 8300 $\pi - p$ and $p-p$ scatterings. The Σ^+ production cross section was obtained using a sample of data described in a previous paper[21] and has been adjusted here to the total $\pi^+ - p$ cross section of 27.9 mb.

Figure 3 is a histogram of the $\pi^+ - p$ elastic scattering in the center-of-mass system. The point marked on the

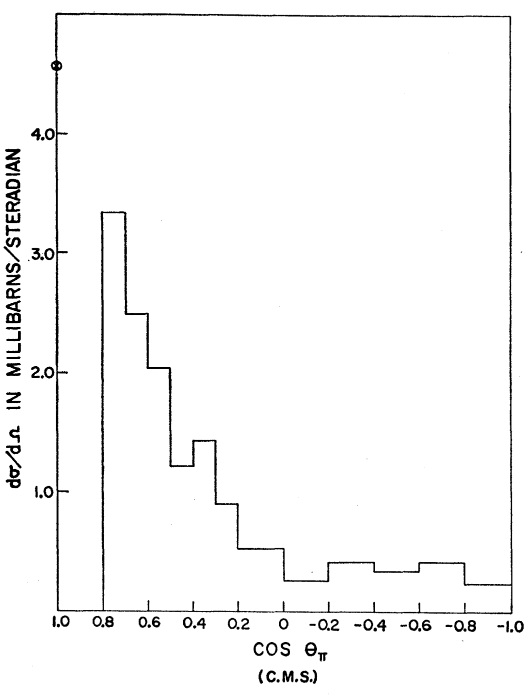

FIG. 3. Center-of-mass differential cross section for $\pi^+ - p$ elastic scattering vs the cosine of the pion scattering angle in the center-of-mass system. The point marked on the axis at $\cos\theta_\pi = 1.0$ was obtained from the total $\pi^+ - p$ cross section using the optical theorem.

[19] T. H. Fields, J. G. Fox, J. A. Kane, R. A. Stallwood, and R. B. Sutton, Phys. Rev. **109**, 1713 (1958).

[20] See R. Cool, O. Piccioni, and D. Clark, Phys. Rev. **103**, 1082 (1956) for dispersion relations by R. M. Sternheimer.

[21] A. R. Erwin, J. K. Kopp, and A. M. Shapiro, Phys. Rev. **115**, 669 (1959).

axis at $\cos\theta_\pi = 1.0$ was computed from the optical theorem using a total cross section of 27.9 mb and does not represent data obtained in this experiment.

ISOBAR MODEL

Perhaps the most useful number which one might hope to extract from the $\pi^+ - p$ data is the parameter ρ used in the isobar model of Sternheimer and Lindenbaum[11]:

$$\rho = \sigma^+ / 2\sigma^-.$$

The partial cross sections σ^+ and σ^- refer to the single pion production in $\pi^+ - p$ and $\pi^- - p$ scattering, respectively. By consulting the curve sketched in Fig. 4, it is possible to find an upper limit to σ^+ and thereby fix an upper limit for ρ in any experiment which determines σ^-. At present no experiment has completely determined σ^- because of the experimental difficulties associated with separating the three prevalent charge exchange processes:

$$(-p) \rightarrow (0n), \quad \text{elastic charge exchange;}$$
$$(-p) \rightarrow (00n), \quad \text{single pion production;}$$
$$(-p) \rightarrow (000n), \quad \text{double pion production.}$$

It is possible, however, to set a lower limit on σ^- in some experiments by assuming the second process above has a vanishing cross section.

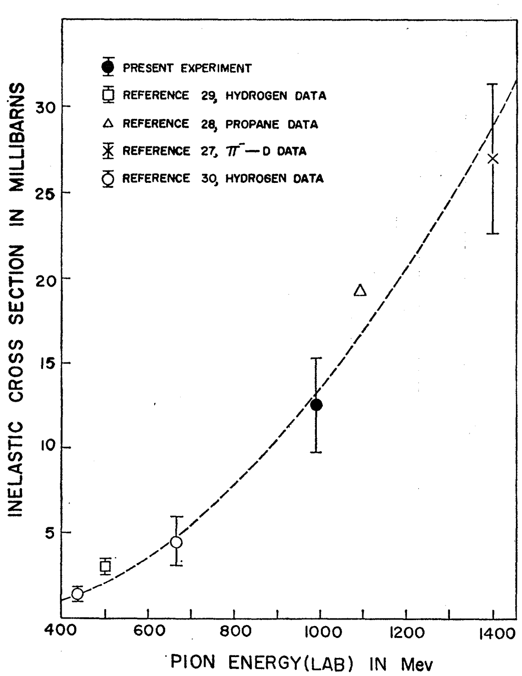

Fig. 4. Plot of available data on total inelastic cross sections in $\pi^+ - p$ scattering vs energy of the incident pion in the laboratory system. The three low-energy points were normalized to total cross sections of the work cited in footnote 2. The three high-energy points were normalized to total cross sections of the work cited in footnote 3.

Of particular interest is the $\pi^- - p$ experiment of Alles-Borelli *et al.*[9] at 960 Mev. Normalizing their data to the total cross sections of Devlin *et al.*,[3] we find $\sigma^- \gtrsim 16.3 \pm 0.6$ mb. By using the curve in Fig. 4, one obtains 12.3 ± 3.0 mb for the total $\pi^+ - p$ inelastic cross section. About 0.9 ± 0.2 mb of this is due to the reactions $(+p) \rightarrow (++-p)$ and $(+p) \rightarrow (\Sigma^+ K^+)$ identified in the present experiment, leaving $\sigma^+ \lesssim 11.4 \pm 3.0$ mb. Thus,

$$\rho = \sigma^+ / 2\sigma^- \lesssim 0.35 \pm 0.09.$$

It is of interest to note that these authors obtain their best fit to the Sternheimer-Lindenbaum model with $\rho = 0.1$, corresponding to $\sigma^+ = 6.3$ mb. They point out that reasonable agreement can still be obtained for σ^+ as high as 10 mb, a value which is still easily in agreement with our upper limit of 11.4 mb.

PION-PION SCATTERING

In an effort to make a more precise estimate of σ^+ we have studied the branching ratios obtained by Alles-Borelli *et al.*[9] for double pion production. An understanding of these ratios might make possible the use of our measured cross section for $(+p) \rightarrow (++-p)$ to infer the total amount of double production in our present experiment. Subtracting this from the total inelastic cross section would yield an improved estimate for σ^+.

The branching ratios obtained by Alles-Borelli *et al.* are given in Table II. These can be most simply accounted for by the model diagrammed in Fig. 5.[22] It may be described by saying that the incoming proton emits a $\frac{3}{2}$, $\frac{3}{2}$ isobar, I, and a pion. The pion interacts with the incident pion, and the isobar decays into another pion plus a nucleon, N.

By assuming that the pion-pion interaction occurs exclusively in a single state of isotopic spin, $T=0$, $T=1$, or $T=2$, it is possible to calculate Clebsch-Gordon coefficients at each vertex. These predict the $\pi^- - p$ branching ratios listed in Table II. A similar set of branching ratios for $\pi^+ - p$ is also listed in Table III. It

Table II. Branching ratios for double pion production in $\pi^- - p$ collisions. Column 2 lists experimental results at 960 Mev. Columns 3, 4, and 5 list the relative number of events predicted by the model in Fig. 5 provided the $\pi - \pi$ scattering takes place only for $T=0$, $T=1$, or $T=2$, respectively.

Final charge state	Events footnote 9	Relative numbers $T=0$	$T=1$	$T=2$
$(+--p)$	23	4	2	326
$(+-0n)$	43	8	8	76
$(00-p)$	8 probable	2	8	40
$(000n)$	⋯	4	0	8

[22] F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960). These authors have used the diagram of Fig. 5 in calculating the ratio of single to double pion production in $\pi^- - p$ collisions at energies above 5 Bev.

is evident that a $T=0$ pion-pion interaction agrees very well with the $\pi^- - p$ experimental data, whereas the other two possibilities show little promise.

The prospect of a $T=0$, $\pi-\pi$ interaction is not at all surprising since several authors have already called attention to experimental evidence for such a process. Carruthers and Bethe[14] have suggested that a $T=0$, S-wave interaction could most easily account for the quantum numbers assigned to the 600-Mev maximum. Rodberg[23] has shown that the unexpectedly large cross section[24] for $(-p) \to (+-n)$ near threshold can be simply described by using a $T=0$ and $T=2$ S-wave, $\pi-\pi$ interaction in an impulse approximation. More recently Abashian, Booth, and Crowe[25] have measured what could be interpreted as the mass of the $\pi-\pi$ pair in the reaction $p+d \to He^3+\pi+\pi$ and found it to be about 310 Mev for a bump in the He^3 momentum spectrum. These three cases have in common a low relative energy for the pion-pion pair. Similarly, for double pion production at 960 Mev the energy available after producing an isobar in the $\pi-p$ center of mass system is so low that the two scattered pions must be predominantly S-wave with respect to each other.

TABLE III. Branching ratios for double pion production in $\pi^+ - p$ collisions as predicted by the model in Fig. 5. Columns 2, 3, and 4 list the relative number of events provided the $\pi-\pi$ scattering takes place only for $T=0$, $T=1$, or $T=2$, respectively.

Final charge state from $\pi^+ - p$ collisions	Relative numbers $T=0$	$T=1$	$T=2$
$(++-p)$	36	81	27
$(00+p)$	18	36	54
$(++0n)$	0	18	54

If one now applies the diagram of Fig. (5) to $\pi^+ - p$ collisions, assuming only $T=0$ for the $\pi-\pi$ vertex, the results are rather disappointing. Used in this way, the model predicts a cross section ratio

$$\sigma[(+p) \to (++-p)]:\sigma[(-p) \to (--+p)] = 9:1.$$

The actual ratio is more nearly 1:1. Hence, it does not appear possible to reconcile such a simple model with the experimental data at this energy.

INELASTIC SCATTERING

Using photoproduction results, Peierls has attributed the 900 Mev $\pi^- - p$ maximum to F-wave scattering with total angular momentum $J=\frac{5}{2}$.[15] The inelastic $\pi-p$ scattering results also tend to support his choice of $F_{\frac{5}{2}}$ rather than $D_{\frac{5}{2}}$. To see this it is only necessary to compute the absorption cross section for the $T=\frac{1}{2}$ wave

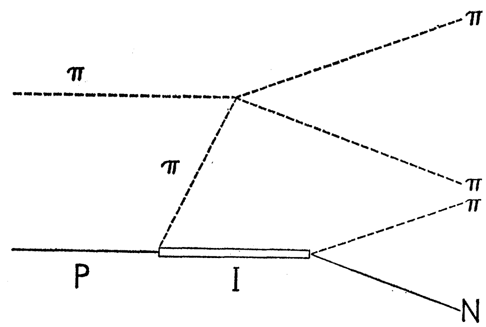

FIG. 5. Diagram of a simple model for double pion production in low-energy $\pi-p$ collisions. I refers to the $T=\frac{3}{2}$, $J=\frac{3}{2}$ isobar.

at 960 Mev using

$$\sigma_a(\pi^- - p) = \tfrac{1}{3}\sigma_a(\pi^+ - p) + \tfrac{2}{3}\sigma_a(T=\tfrac{1}{2}).$$

The $\pi^- - p$ absorption, $\sigma_a(\pi^- - p)$, does not include elastic charge exchange. Taking our $\pi^+ - p$ data from the curve of Fig. (4) and our $\pi^- - p$ data from the work cited in footnote 17, normalized to the total cross sections of Devlin *et al.*,[3] we find

$$\sigma_a(\pi^+ - p) = 12.3 \text{ mb},$$
$$\sigma_a(\pi^- - p) = 24.5 \text{ mb},$$
$$\sigma_a(T=\tfrac{1}{2}) = 30.6 \text{ mb}.$$

The amount of $\pi^- - p$ elastic charge exchange has been estimated using results from the fit of Alles-Borelli *et al.*[9] to the isobar model.

The $T=\frac{1}{2}$ absorption is also given by an expression of the form

$$\sigma_a(T=\tfrac{1}{2}) = \frac{\pi\lambda^2}{2}\Big[\sum_{J=l+\frac{1}{2}} (2J+1)(1-|\eta_J^+|^2) + \sum_{J=l-\frac{1}{2}} (2J+1)(1-|\eta_J^-|^2)\Big], \quad |\eta_J^\pm| \leqslant 1.$$

Thus, it is possible to account for the observed absorption by completely absorbing $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, $P_{\frac{3}{2}}$, $D_{\frac{3}{2}}$, and $D_{\frac{5}{2}}$ waves to obtain $\sigma_a(T=\frac{1}{2}) = 9\pi\lambda^2 = 31.2$ mb. Since such a situation seems very unlikely, one is forced to assume that F-wave scattering plays a very prominent role at this energy. This conclusion was pointed out earlier by Walker[26] using preliminary results from the present experiment.

ACKNOWLEDGMENTS

We would like to thank Dr. R. K. Adair and Dr. L. B. Leipuner for running their 6-in. hydrogen chamber for this experiment and for the use of their scanning and measuring facilities.

[23] K. S. Rodberg, Phys. Rev. Letters **3**, 58 (1959).
[24] W. A. Perkins, J. C. Caris, R. W. Kenney, E. A. Knapp, and V. Perez-Mendez, Phys. Rev. Letters **3**, 56 (1959).
[25] A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters **5**, 258 (1960).
[26] R. R. Crittenden, J. H. Scandrett, W. D. Shephard, W. D. Walker, and J. Ballam, Phys. Rev. Letters **2**, 121 (1959).
[27] V. P. Kenney, Phys. Rev. **104**, 784 (1956).
[28] D. A. Glaser and L. O. Roellig, Phys. Rev. **116**, 1001 (1959).
[29] W. J. Willis, Phys. Rev. **116**, 753 (1959).
[30] M. E. Blevins, M. M. Block, and J. Leitner, Phys. Rev. **112**, 1287 (1958).

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Nucleon-Antinucleon Mechanism for Pion-Pion Scattering Resonances

JOHAN G. BELINFANTE*
Palmer Physical Laboratory, Princeton, New Jersey
(Received February 27, 1961)

The Chew-Mandelstam N/D equations for pion-pion scattering were modified to include contributions from the nucleon-antinucleon intermediate state, which was estimated in perturbation theory as well as by limitations imposed by unitarity alone. It was found that the Frazer resonance could not be obtained by such a simple mechanism, starting from S-wave dominant solutions.

An investigation was made as to whether or not the nucleon-antinucleon intermediate state might provide a simple mechanism for the Frazer $T=1$ P-wave pion-pion resonance proposed to explain the nucleon electromagnetic structure data.[1] Our philosophy is to proceed by iteration, starting with the subtraction constant λ as a first approximation, and retaining only S and P waves. In the elastic approximation of Chew and Mandelstam[2] this corresponds to starting from S-wave dominant solutions. We would expect that, if a mechanism is properly chosen, the P-wave dominant solution would appear instead.

The unitarity condition together with the dispersion relations imply a set of integral equations,

$$\tilde{M}_{Il}(\nu)=\tilde{S}_{Il}(\nu)+\int_0^\infty \frac{d\nu'}{\pi}\left(\frac{\nu'}{\nu'+1}\right)^{\frac{1}{2}}\frac{(\nu')^l|\tilde{M}_{Il}(\nu')|^2}{\nu'-\nu-i\epsilon},$$

where

$$\mathrm{Im}\tilde{S}_{Il}(\nu)=[\mathrm{Im}\tilde{S}]_{\pi\pi}{}^L+[\mathrm{Im}\tilde{S}]_{N\bar{N}}{}^L+[\mathrm{Im}\tilde{S}]_{N\bar{N}}{}^R,$$

$$[\mathrm{Im}\tilde{S}]_{\pi\pi}{}^L=\theta(-\nu-1)\sum_{I'l'}\int_0^{-\nu-1}\frac{d\nu'}{\pi}f_{Il,I'l'}(\nu,\nu')\times|\tilde{M}_{I'l'}(\nu')|^2.$$

We have also made use of crossing symmetry here. The solution of this equation is carried out by transforming to a set of N/D equations in the standard way.

The contribution from the nucleon-antinucleon channel was estimated first on the basis of a pole approximation for the $N\bar{N}\pi\pi$ amplitude which one encounters in the unitarity condition, symbolically,

$$\mathrm{Im}M_{\pi\pi,\pi\pi}=\int M_{\pi\pi,\pi\pi}{}^*M_{\pi\pi,\pi\pi}+\int M_{\pi\pi,N\bar{N}}{}^*M_{\pi\pi,N\bar{N}}.$$

This procedure leads to a resonance in π-π scattering, but the method is not justifiable because of unitarity limitations. The unitarity condition implies, on the right-hand cut,

$$[\mathrm{Im}M_{Il}(\nu)]_{N\bar{N}}\leqslant\tfrac{1}{4}[(\nu+1)/\nu]^{\frac{1}{2}},\quad(\nu>M^2-1).$$

This condition is so severely violated by the pole approximation that proposals to ignore the limitation on the left-hand cut, where the limitation does not rigorously apply, must be treated with some suspicion. What we have done is to look at the limitation imposed by unitarity. Thus we replace the inequality by an equality. We have furthermore approximated the left-hand cut by two poles to be interpreted as the pion-pion self-coupling, and the nucleon-antinucleon pair contribution on the left. The following refers to the $I=l=1$ state:

$$-[\mathrm{Im}\tilde{S}(\nu)]^L=-2\pi(5\lambda^2/9\pi)\delta(\nu+2)+\pi M^2\Gamma\delta(\nu+M^2).$$

The residues were fixed by conditions at threshold. Thus

$$\Gamma=[\tilde{S}(0)]_{N\bar{N}}{}^L.$$

Our numerical results are as follows. The $N\bar{N}$ contribution on the right is

$$[\tilde{S}(0)]_{N\bar{N}}{}^R\approx1/(4\pi M^2).$$

On the left we have many terms, one for each angular momentum. The contributions from the $I'=0$, S wave is

$$[\tilde{S}(0)]_{N\bar{N},I'=l'=0}{}^L\approx1/(144\pi M^4),$$

while the contribution from $I'=l'=1$ is

$$[\tilde{S}(0)]_{N\bar{N},I'=l'=1}{}^L\approx-1/(16\pi M^2).$$

The contributions are increasing, but within our approximations the left-hand contributions are numerically less important than the right-hand contributions. The left-hand contributions are represented

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