Integrated Cross Section for a Velocity-Dependent Potential*

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For a possible distinction between a velocity-dependent two-nucleon potential, and a static potential with an infinite repulsive core, we study their contributions to the integrated cross section (σ_{int}) for the deuteron photoeffect. Both potentials considered are central, with square shapes, and have Serber mixtures for the attractive parts. (They have Wigner character for the repulsive core, and for the velocity-dependent term, respectively.) These potentials are adjusted to give (1) the observed binding energy of the deuteron, (2) the same effective range $\rho(-\epsilon, -\epsilon)=1.76$ fermis; and (3) the same value (260 Mev) at which the ^3S phase shift changes sign. Using sum-rule calculations, in the electric-dipole approximation, we find that σ_{int} for the static case is 37.7 Mev-mb, while for the velocity-dependent case it is very nearly the same: namely, 38.8 Mev-mb.

INTRODUCTION

Scattering measurements on the proton-proton system show that the ^1S phase shift changes sign. If the two-body potential is assumed static, it should contain a very strong repulsion at short distance; e.g., the Gammel-Thaler[1] infinite repulsive core. On the other hand, a well-behaved potential would be easier to deal with in solving the nuclear many-body problem.[2] This potential needs to be velocity-dependent to reproduce the ^1S phase shift curve.[3,4] With a suitable choice of parameters it can also reproduce the other singlet-even phase shifts.

Since phase shifts alone have not served to verify the existence of a static repulsive core,[5] it is desirable to consider properties of the neutron-proton system,[6] such as electric dipole transitions in the deuteron photoeffect.[7] Here a static potential of Wigner character gives a model-independent value for the integrated cross section; $\sigma_{\text{int}}=30$ Mev-mb. This value is increased by our velocity-dependent potential, but it is also increased by the Majorana exchange term in the Gammel-Thaler static potential. This paper gives an oversimplified calculation of σ_{int} for velocity-dependent and static cases.

The first application of a velocity-dependent potential to find σ_{int} for the deuteron was made by Way[8] for a separable potential. Our present numerical results are not very different from her values.

Our potentials are oversimplified in that we assume pure central forces, and square-well shapes. Our calculations should not be compared seriously with the experimental value of σ_{int} for the deuteron photoeffect; it may be significant to compare the calculated values of σ_{int}. The velocity-dependent and static potentials are chosen to agree with each other on three other properties of the neutron-proton system (in triplet-even states); the binding energy, the energy at which the calculated ^3S phase shift passes through zero, and the triplet effective range. In this example, treated below, the two types of potentials give very nearly identical values for σ_{int}, so that this means of confirming the existence of the repulsive core proves to be unsuccessful.

CALCULATION

For the deuteron, the dipole oscillator strength[9] f_{0n} is given by

$$f_{0n}=(M/4\hbar^2)(E_n-E_0)(z_{0n})^2. \quad (1)$$

Here z is the neutron-proton distance; the subscripts 0 and n refer to ground and excited states, respectively. The summed oscillator strength $\sum_n f_{0n}$, is proportional to the expectation value in the ground state of the double-commutator of the Hamiltonian H with the coordinate z:

$$\sum_n f_{0n}=-(M/8\hbar^2)\{[[H,z],z]\}_{00}. \quad (2)$$

The integrated cross section is proportional to the summed oscillator strength:

$$\sigma_{\text{int}}=\int_0^\infty \sigma(W)dW=(4\pi^2e^2\hbar/Mc)\sum_n f_{0n}$$
$$=120\sum_n f_{0n} \quad \text{Mev-mb.} \quad (3)$$

To evaluate the double-commutator in Eq. (2) we write H as the sum of kinetic energy T and potential energy V. The double-commutator using T is model-

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TABLE I. Parameters for potentials.[a]

	Range in f	Depth in Mev	Wave number in f^{-1}	Core, or velocity-dependence
Velocity-dependent potential	$b=2.0$	$V_0=37.5$	$k'=0.836$	$\lambda=-0.25$
Static potential, with core	$b'=1.71$	$V_s=47.6$	$k''=1.046$	$c=0.24f$

[a] The velocity-dependent potential is given in Eqs. (5) and (9); the static potential is given in Eq. (14). The wave number k', or k'', is given for the ground state, for the region inside the attractive square well.

ndependent[9]:

$$-(M/8\hbar^2)\{[[T,z],z]\}_{00}=\tfrac{1}{4}. \quad (4)$$

We assume a velocity-dependent potential of the form used by Razavy[3,4]:

$$\hat{v}=-V_0J_1(r)\{1-x+xP^M\}-(\lambda/M)\mathbf{p}\cdot J_2(r)\mathbf{p}. \quad (5)$$

That is, the static term of shape $J_1(r)$ contains a fraction x of the Majorana exchange operator P^M; the velocity-dependent term of shape $J_2(r)$ is assumed to have Wigner character. [In Eq. (5), $\mathbf{p}$ is the quantum-mechanical operator $-i\hbar$ grad.]

The term $-xV_0J_1(r)P^M$ of the static potential contributes the following to the summed oscillator strength:

$$-x(M/8\hbar^2)V_0\{[[J_1(r)P^M,z],z]\}_{00} = x(MV_0/6\hbar^2)\langle r^2J_1(r)P^M\rangle_{00}. \quad (6)$$

The velocity-dependent term contributes

$$(M/8\hbar^2)\{[[(\lambda/M)\mathbf{p}\cdot J_2(r)\mathbf{p},z],z]\}_{00} = -(\lambda/4)\langle J_2(r)\rangle_{00}. \quad (7)$$

Combining these results, the integrated cross section has the value

$$\sigma_{\text{int}}=(4\pi^2e^2\hbar/Mc)\{\tfrac{1}{4}+(xMV_0/6\hbar^2)\langle r^2J_1(r)P^M\rangle_{00} -(\lambda/4)\langle J_2(r)\rangle_{00}\}. \quad (8)$$

We see that σ_{int} is increased for negative values of λ.

We shall apply Eq. (8) to the case of a well-behaved velocity-dependent central potential, and also to a static square well with an infinite repulsive core. For each potential we adjust three parameters to fit the following data on the neutron-proton system:

(a) The binding energy ϵ of the deuteron is 2.226 Mev.

(b) The ^3S phase shift goes through zero at 260 Mev. (This value is chosen between 200 and 300 Mev to give qualitative agreement with the known value for the ^1S phase shift.)

(c) The effective range $\rho(-\epsilon,-\epsilon)=1.76$ fermis.

We assume the velocity-dependent potential of Eq. (5), with square shapes of range b:

$$J_1(r)=J_2(r)=\begin{cases}1, & r<b\\ \tfrac{1}{2}, & r=b\\ 0, & r>b.\end{cases} \quad (9)$$

Following Razavy,[4] we solve the Schrödinger equation for the ground state in the inside region ($r<b$) and the outside region ($r>b$):

$$u_1''+k'^2u_1=0;\quad u_1=A\sin k'r,\quad r<b \quad (10)$$

$$u_2''-\gamma^2u_2=0;\quad u_2=C\exp(-\gamma r),\quad r>b$$

$$k'^2=M(V_0-\epsilon)/\hbar^2(1-\lambda);\quad\text{and}\quad \gamma^2=M\epsilon/\hbar^2. \quad (11)$$

The wave functions $u_1(r)$ and $u_2(r)$ are continuous at $r=b$; but their first derivatives [i.e., left-hand derivative $u_1'(b)$ and right-hand derivative $u_2'(b)$] are not equal:

$$u_2'(b)-(1-\lambda)u_1'(b)=(\lambda/b)u(b). \quad (12)$$

These boundary conditions give the following relation among the three parameters λ, k', and b:

$$(1-\lambda)k'\cot k'b+\lambda/b=-\gamma. \quad (13)$$

The static potential, with infinite repulsive core, is assumed to be[10]

$$v_s=\begin{cases}\infty, & r<c\\ -V_S, & c<r<b'+c\\ 0, & b'+c<r.\end{cases} \quad (14)$$

We obtain the usual transcendental relation

$$k''\cot k''b'=-\gamma. \quad (15)$$

Here $k''^2=(M/\hbar^2)(V_S-\epsilon)$.

The ^3S phase shift δ for the velocity-dependent potential is given by[4]

$$\tan(kb+\delta)=k/[(1-\lambda)k'\cot k'b+\lambda/b]. \quad (16)$$

Here k is the wave number at large distances, and k' is the wave number for $r<b$. [Use Eq. (11), replacing $-\epsilon$ by E, the energy available in the center-of-mass system.] For the static potential v_s, we have phase shift δ_s:

$$\tan(kb'+kc+\delta_s)=(k/k'')\tan k''b'. \quad (17)$$

Here k'' is the wave-number in the region $c<r<b'+c$.

The effective range $\rho(-\epsilon,-\epsilon)$ for the velocity-dependent potential is found[11] using the wave function

[10] L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. 3, 241 (1958).

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of Eq. (10). We use $C=1$, and $A=\csc k'b \exp(-\gamma b)$.

$$\rho(-\epsilon, -\epsilon)=2\int_0^b (v^2-u^2)dr$$
$$=\gamma^{-1}-\exp(-2\gamma b)(\gamma^{-1}+b\csc^2 k'b - k'^{-1}\cot k'b). \quad (18)$$

For the static well with repulsive core, the effective range $\rho_s(-\epsilon, -\epsilon)$ is given by

$$\rho_s(-\epsilon, -\epsilon)=\gamma^{-1}-\exp(-2\gamma b'-2\gamma c) \times(\gamma^{-1}+b'\csc^2 k''b'-k''^{-1}\cot k''b'). \quad (19)$$

For the velocity-dependent potential of Eqs. (5) and (9) we have adjusted the parameters b, V_0, and λ using Eqs. (13), (16), and (18). For the static potential of Eq. (14) we have adjusted the parameters b', V_S, and c using Eqs. (15), (17), and (19). (The values for $\delta(^3S)$ at 260 Mev are -0.02 and $+0.03$ rad for the velocity-dependent and static cases, respectively.) The values obtained for these parameters (and also for k' and k'') are given in Table I.

The normalized wave function for the velocity-dependent potential is found using Eq. (10), with $C^2=2\gamma/[1-\gamma\rho(-\epsilon, -\epsilon)]$. The coefficient A is determined from the continuity of the wave function at $r=b$. The squared normalized wave function is plotted as a solid line in Fig. 1 and is substituted in Eq. (8) to calculate the integrated cross section. (The arrow shows the edge of the well, where the wave function has a discontinuity in its derivative.) We find

$$\sigma_{\text{int}}=30(1+0.210+0.083)=38.8 \quad \text{Mev-mb.} \quad (20)$$

Inside the parentheses, the term unity comes from the kinetic energy operator. It is model-independent, as shown in Eq. (4). The term 0.210 comes from the Serber exchange mixture ($x=\frac{1}{2}$) chosen for the attractive static potential. The last term 0.083 comes from the explicit velocity dependence.

The squared, normalized wave function for the static potential is plotted as a dashed line in Fig. 1. Since the two potentials v and v_s give the same binding energy and effective range, the wave functions are identical outside the range of the force ($b<r$ or $b'+c<r$). We see from the figure that the squared wave functions are similar for $r\approx 1$ f, though of course they disagree inside the static repulsive core. The wave function for the static potential is used in Eq. (8), assuming a Serber mixture for the attractive term, and putting $\lambda=0$. We find

$$\sigma_{\text{int}}=30(1+0.257)=37.7 \quad \text{Mev-mb.} \quad (21)$$

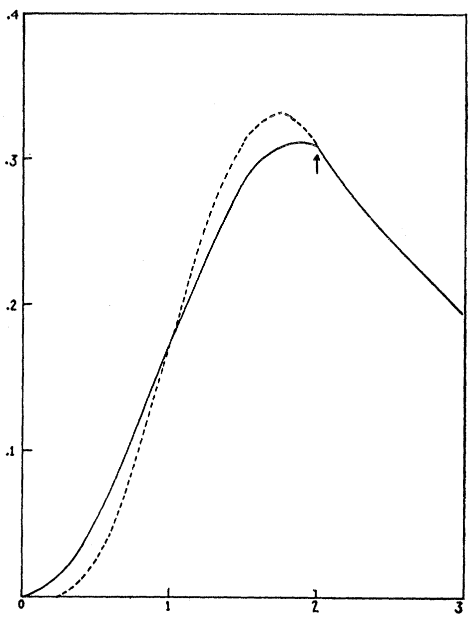

FIG. 1. The solid line gives the squared radial wave function (Eq. 10) in f^{-1} for the velocity-dependent potential of Eqs. (5) and (9) *versus* r in f. The arrow shows the edge of the square well, where there is a discontinuity in the slope of the wave function. The dashed line gives the squared wave function for the static potential, with repulsive core, of Eq. (14). The two wave functions are identical for $r\geq 2.0$ f.

DISCUSSION

Equations (20) and (21) show that our two choices of (grossly oversimplified) potentials give very nearly the same values for the integrated cross section for the deuteron photoeffect. We note that our present calculations of σ_{int} are, within 3 Mev-mb, in agreement both with earlier calculations[8,9] and with an experimental value[9] for the cross section integrated to the threshold for photoproduction of mesons. These agreements may be accidental. The conclusion which we can make from our present preliminary study is that the integrated cross section is insensitive to the presence of either a velocity-dependent term or a static core in the potential. The possible values of σ_{int} are confined to the region $30<\sigma_{\text{int}}\lesssim 30/[1-\gamma\rho(-\epsilon, -\epsilon)]=50$ Mev-mb. Any exchange force or velocity dependence of reasonable signs raise the value of σ_{int} above the model-independent value of 30 Mev-mb. The approximate upper limit of 50 Mev-mb comes from the effective-range formula for the electric dipole deuteron photoeffect. (The effective-range formula for electric dipole transitions is valid at low energies,[11] and provides a plausible but not rigorous upper limit for the cross section at high energies.) For square-well shapes, and a Serber mixture, a static well-behaved potential gives 36 Mev-mb, a static potential with infinite core gives 37.7 Mev-mb, and a velocity-dependent well-behaved potential gives 38.8 Mev-mb. Thus, our three values all fall in $\frac{1}{6}$ of the allowed region.

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