PHYSICAL REVIEW VOLUME 123, NUMBER 6 SEPTEMBER 15, 1961

Neutron-Proton Phase-Shift Analysis at 95 Mev

MALCOLM H. MACGREGOR*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

(Received May 12, 1961)

A phase-shift analysis of neutron-proton differential cross section and polarization measurements at 95 Mev has been carried out. With the isotopic-spin-one phase shifts taken from the results of proton-proton phase-shift analyses at the same energy, only one set of isotopic-spin-zero phase shifts was obtained that gave a good least-squares fit to the data. The analysis indicates that the forward and backward peaks in the neutron-proton differential cross section at 95 Mev are predominantly triplet- and singlet-spin scattering states, respectively.

In principle, five kinds of experiments should be required to specify the proton-proton ($T=1$) elastic scattering matrix.[1] However, at 95 Mev the analysis of only three kinds of experiments, together with some plausible physical restrictions on the phase shifts, has apparently yielded a "unique" set of phase shifts.[2,3] If one now analyzes neutron-proton scattering experiments and assumes charge independence (i.e., uses the $T=1$ phase shifts determined from proton-proton scattering analyses at the same energy), then each kind of neutron-proton scattering measurement will give two independent equations relating the scattering amplitudes. These correspond to $T=0$ terms and to $T=0$, $T=1$ interference terms in the scattering amplitudes (T is the isotopic spin). Hence, the phase-shift analysis of neutron-proton differential cross section and polarization measurements at 95 Mev together with the known $T=1$ phase shifts[2,3] will give four independent conditions on the $T=0$ amplitudes, and a unique phase-shift solution might be expected.

A phase-shift analysis of neutron-proton differential cross section and polarization measurements at 95 Mev was carried out on the DASK computer. Harwell measurements[4] of the polarization at 15 angles were used, together with differential cross-section measurements of several groups[5] combined so as to give weighted values for the cross sections at the same angles as those used in the polarization experiments, as well as two additional angles (5.1° and 176°). All phase shifts above H waves were set equal to zero. The $T=1$ phase shifts were taken from reference 2. The grid search method was used, with the Stapp "nuclear bar" parametrization[6] for the phase shifts. Searches were carried out on the $T=0$ phase shifts with all 12 of the phase shifts (S–H) included in the search program, and also with the lower seven phase shifts (S–F) searched, and the G and H waves determined by the one-pion exchange potential.[7,8] Only one solution type giving a reasonable fit to the data was discovered. Starting points near this solution would go to the solution, but starting points that were far away in phase-shift space tended to end in local minima corresponding to very high values for the least-squares sum[8] χ^2. (This may be due to the fact that the $T=1$ phase shifts were held fixed.)

The phase shifts that were obtained are listed in Table I. A comparison of the 7- and 12-parameter searches gives a measure of the accuracy with which the phase shifts are determined. In particular, it indicates the error incurred by setting all phase shifts above H waves equal to zero. The S–D phase shifts are rather accurately determined, while the F–H phase shifts are not. In order to get accurate values for these higher phase shifts, one should substitute the full one-pion exchange amplitude (OPEC) for the higher phase shifts and then simultaneously vary both the $T=0$ and $T=1$ phase shifts.

Included in the search program were 32 values for the differential cross section and polarization, so that χ^2 values of 25 and 20 might be expected for the 7- and 12-parameter searches. The values actually obtained (45.1 and 34.6) are somewhat higher than this. The polarization data contributed about two-thirds of this total in each case. Since the errors on the polarization data (as quoted by Hess[5]) are probably a little small, judging by the scatter of the experimental points, the values for χ^2 actually obtained seem reasonable. The errors used for the differential cross-section data were about 8% for forward angles and 3% for backward angles.

Comparison of the phase shifts shown in Table I with

---

[*] NATO Postdoctoral Fellow; on leave from Lawrence Radiation Laboratory, Livermore, California.

[1] L. Puzikov, R. Ryndin, and J. Smorodinsky, Nuclear Phys. **3**, 436 (1957).

[2] M. H. MacGregor, M. J. Moravcsik, and H. P. Noyes (to be published).

[3] J. K. Perring, Atomic Energy Research Establishment, Harwell, England (to be published).

[4] G. H. Stafford, C. Whitehead, and P. Hillman, Nuovo cimento **5**, 1589 (1957).

[5] O. Chamberlain and J. W. Easley, Phys. Rev. **94**, 208 (1954); C. Y. Chih, University of California Radiation Laboratory Report UCRL-2575, May, 1954 (unpublished); T. C. Griffith, A. P. Banford, M. Y. Uppal, and W. S. C. Williams, Proc. Phys. Soc. (London) **A71**, 305 (1958); W. Selove, K. Strauch, and F. Titus, Phys. Rev. **92**, 724 (1953); R. H. Stahl and N. F. Ramsey, *ibid.* **96**, 1310 (1954). All data here and in reference 4 were taken from the compilation of W. N. Hess, Revs. Modern Phys. **30**, 368 (1958).

[6] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[7] A. F. Grashin, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1717 (1959) [Soviet Phys.—JETP **36**(9), 1223 (1959)].

[8] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).

TABLE I. *n-p* phase-shift solutions at 95 Mev.[a]

$T=1$		$T=0$	7-parameter	12-parameter
$^1S_0$	(22.18)	$^1P_1$	−23.79	−21.35
$^1D_2$	(3.87)	$^1F_3$	−0.24	−2.10
$^1G_4$	(0.36)	$^1H_5$	(−0.56)	0.66
$^3P_0$	(14.24)	$^3S_1$	41.69	40.24
$^3P_1$	(−11.98)	ϵ_1	7.09	9.46
$^3P_2$	(11.17)	$^3D_1$	−11.12	−11.73
ϵ_2	(−2.78)	$^3D_2$	13.50	14.73
$^3F_2$	(0.88)	$^3D_3$	2.32	1.25
$^3F_3$	(−1.70)	ϵ_3	(3.87)	4.55
$^3F_4$	(0.22)	$^3G_3$	(−0.90)	−0.04
ϵ_4	(−0.52)	$^3G_4$	(2.15)	2.24
$^3H_4$	(0.10)	$^3G_5$	(−0.26)	−0.06
$^3H_5$	(−0.30)			
$^3H_6$	(0.03)	χ^2	45.1	34.6

[a] Nuclear bar phase shifts in degrees. The phase shifts in parentheses were held fixed during the search. χ^2 is the least-squares sum. $T=1$ phase shifts are from reference 2.

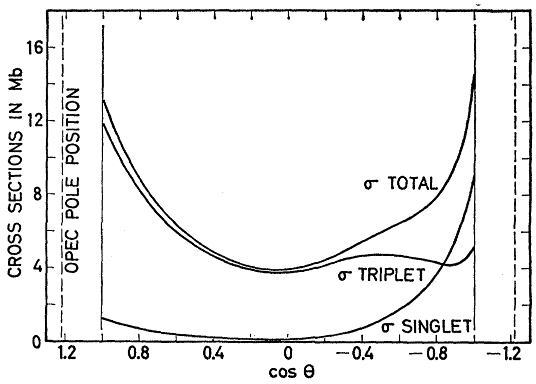

FIG. 1. *n-p* differential cross sections at 95 Mev.

those obtained by the Yale group,[9] using an energy-dependent code, shows qualitative similarity. These phase shifts also agree quite well with the $T=0$ (and $T=1$) phase shifts calculated from the potential models of Gammel and Thaler[10] and Hamada.[11] Recent calculations of properties of nuclei in terms of nucleon-nucleon scattering have generally been based on the Gammel-Thaler phase shifts[12]; hence, the present results give additional support to the validity of these calculations.

The fact that different kinds of analyses (present work and references 9–11) yield substantially the same set of *n-p* phase shifts (at least at 100 Mev) emphasizes the statement made at the beginning of this paper to the effect that the *n-p* differential cross-section and polarization data, when combined with analyses of *p-p* scattering, are more restrictive than they might at first appear. One of the reasons for this is the fact that the *n-p* differential cross section at 100 Mev (as discussed later) consists of a triplet-spin forward peak and a singlet-spin backward peak (Fig. 1). Hence, the singlet- and triplet-spin amplitudes are effectively decoupled. This fact, together with the presence of singlet and triplet $T=0$, $T=1$ interference terms, means that the *n-p* differential cross section contains more unambiguous information concerning the higher phase shifts than does the isotropic *p-p* differential cross section.

When the *n-p* differential cross section is separated into singlet and triplet spin components by means of the phase shifts (Table I), the interesting result emerges that the backward peak is due to singlet scattering while the forward peak is predominantly triplet (Fig. 1). The origin of the singlet peak can be described in two ways. The first way is to say that the singlet phase shifts have opposite signs in the $T=0$ and $T=1$ states (as expected from OPEC), and thus add constructively at the back angles and destructively at the forward angles. The second way is to separate the amplitudes into OPEC and non-OPEC (MPEC) parts. For example, we can write for the singlet amplitude near $\cos\theta=-1$,

$$M_{ss}=A(1+x)/(x_0+x)+B, \tag{1}$$

where M_{ss} is the real part of the singlet amplitude (the imaginary part is small), $A=-g^2/2E$ for the singlet charge exchange OPEC pole, B is the MPEC amplitude (plus the forward OPEC pole amplitude), and $(1+x)/(x_0+x)$ is the OPEC angular dependence[7,8] ($x=\cos\theta$ and $x_0=$position of the OPEC pole). B will be slowly varying near the OPEC pole, since it is due to more distant singularities than OPEC.[13] Since the OPEC amplitude vanishes at $x=-1$, B is determined by the value of the singlet differential cross section at 180°. A is determined by the known value of the coupling constant ($g^2\sim 14$). At 100 Mev it turns out that A and B are of almost equal magnitude and of opposite sign. Hence, the sharp singlet rise is due to the vanishing of OPEC-MPEC interference near 180°. Near 0° the OPEC singlet amplitude is only half as large as at 180°, and the MPEC amplitude B is very much smaller. Hence, the singlet contribution to the forward *n-p* peak is small.

The behavior of the triplet amplitudes near the poles is best studied by using Wolfenstein parametrization.[14] In the Wolfenstein notation, only the amplitudes B (singlet), G, and H contain OPEC pole terms. In the *n-p* case at 95 Mev, the G (H) OPEC-MPEC interference is destructive (constructive) near 180°, while near 0° both G and H have destructive interference. Hence, there is a flat triplet cross section at 180° and a sharp triplet rise at 0° (Fig. 1).

In the case of proton-proton scattering at 95 Mev,[2,3] the singlet OPEC-MPEC interference is destructive near 0°, while the triplet amplitudes G and H both have

---

[9] M. Hull, K. Lassila, H. Ruppel, F. McDonald, and G. Breit, Phys. Rev. **122**, 1606 (1961).

[10] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 1337 (1957).

[11] T. Hamada, Progr. Theoret. Phys. (Kyoto) **25**, 247 (1961).

[12] See, for example, H. A. Bethe, Ann. Phys. NY **3**, 190 (1958); A. K. Kerman, H. McManus, and R. M. Thaler, *ibid.* **8**, 551 (1959).

[13] G. F. Chew, Phys. Rev. **112**, 1380 (1958).

[14] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954), Eq. (3.4).

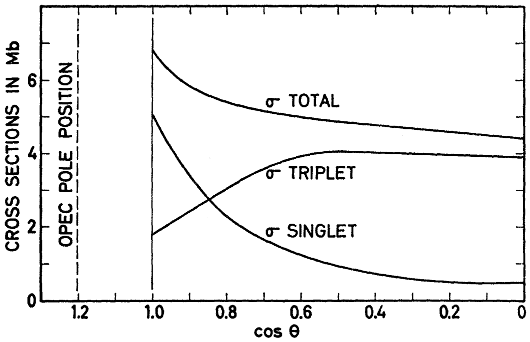

FIG. 2. *p-p* "nuclear" differential cross sections at 95 Mev.

constructive interference (Fig. 2). Hence, the resulting *p-p* "nuclear" cross section is flat near the OPEC poles, in contrast to the *n-p* case.

It is possible to obtain a value for the pion-nuclear coupling constant g^2 by extrapolating the "nuclear" differential cross section (multiplied by the OPEC pole denominator) to the position of the OPEC pole.[13] This extrapolation procedure has turned out to be difficult in practice[15,16] and has yielded values for g^2 that tend to be somewhat low. Once a phase-shift analysis of the data has been accomplished, this can be used to separate the scattering amplitude into its various Wolfenstein components. Extrapolation of the amplitudes (or partial cross sections) B, G, H separately to their respective OPEC pole values might prove to be easier than extrapolating the full differential cross section, since we have now removed parts that do not have OPEC poles. As an example of this kind of calculation, the 95-Mev $T=1$ *amplitudes* B, G, H obtained from the phase shifts of reference 2 when extrapolated linearly (real parts only) to the OPEC pole position give values $g^2=18.0$, 15.8, and 16.5, respectively, for the neutral pion-nucleon coupling constant. (In order to assign probable errors to these values, one would have to know the phase-shift error matrix.) Extrapolation of the *n-p* singlet amplitude at 95 Mev to the backward OPEC pole position gives the low value $g^2=10$. This extrapolation procedure is very sensitive to the high angular momentum components in the amplitudes, and the low value in the *n-p* case is probably due primarily to the fact that the waves I, J, $\cdots$ were set equal to zero, whereas in the *p-p* case the I, J, $\cdots$ waves were set equal to OPEC. (Comparison of pre-OPEC[6] and post-OPEC[8] *p-p* phase-shift analyses at 310 Mev shows a similar result when the singlet amplitudes for solutions 1 and 2 are extrapolated to the OPEC pole position.)

Note added in proof. The *n-p* polarization curve at 95 Mev shows large values for the polarization at small scattering angles, and very small values at large (charge-exchange) scattering angles. This is consistent with the result shown above that the charge-exchange scattering occurs predominantly in the singlet-spin state. Since all *n-p* polarization curves from 77 to about 200 Mev show similar behavior, we may infer that the charge-exchange scattering is mainly in the singlet state over this whole energy region. This result has consequences with regard to charge-exchange scattering processes occurring in nuclei. For example, the Be(*p*,*p*) reaction yields large polarizations at small scattering angles, whereas the Be(*p*,*n*) reaction does not, a fact that has made it difficult to produce intense neutron beams of large polarization with a cyclotron. (The author would like to acknowledge a useful discussion about this point with Professor V. P. Dzhelepov and Professor L. I. Lapidus at Dubna.) As another example, the Pb(*p*,*n*) reaction at 150 Mev yields a neutron spectrum at 0° that is dominated by a peak corresponding to an energy loss of 20–25 Mev, and a similar spectrum occurs for Cu. These peaks are probably explained by a process in which a neutron is replaced by a proton in a charge-exchange reaction, with a consequent change in the Coulomb energy of the nucleus. However the Coulomb energy involved in the Pb reaction is only about 15 Mev. In order to account for the remaining energy loss, Mottelson (private communication) points out that the spin-flip part of the process must play an important role. And this is what we expect from a two-body charge-exchange process that tends to occur predominantly in the singlet-spin state.

ACKNOWLEDGMENTS

The author would like to thank Professor Aage Bohr and Professor Ben Mottelson for useful discussions, and Mr. Brian Mayoh for programming and running the *n-p* search problems on the DASK computer. He would also like to thank Professor Niels Bohr for the hospitality extended to him at the Institute for Theoretical Physics, Copenhagen, Denmark.

---

[15] P. Cziffra and M. J. Moravcsik, Phys. Rev. **116**, 226 (1959).

[16] N. S. Amaglobeli, Yu. M. Kazarinov, S. N. Sokolov, and I. N. Silin, *Proceedings of the Tenth Annual International Conference on High-Energy Physics at Rochester, 1960* (Interscience Publishers, Inc., New York, 1960), p. 64.