Photodisintegration of Be^9†

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The reaction $Be^9(\gamma,n)Be^8$ is discussed for those transitions in which the odd neutron goes from an initial p state, assumed to be strongly coupled to a deformed Be^8 core, to an s state in the continuum. The relation between this strong-coupling model and the single-nucleon calculations of Francis, Goldman, and Guth is given. The model is applied to experiments recently reported by Jakobson, and accounts for the observed increase in cross section observed at an excitation energy of 4.6 Mev, the isotropic distribution of the neutrons associated with this rise, and the ratio of the integrated cross section for this rise to that for the threshold peak.

THE present paper has been motivated by two recent contributions to an old problem. The first is an experimental study of the photodisintegration of Be^9 for gamma rays of energy up to 5 Mev carried out by Jakobson.[1] The salient features of this work are: (a) The peak in the cross section for gamma ray energies just above the threshold energy (1.67 Mev), is in accord with the results of many previous investigations.[2–5] (b) There is a much broader peak in the cross section centered around $E_\gamma = 4.6$ Mev with the area, $\int\sigma_\gamma dE_\gamma$, attributed to this peak roughly a factor 1.9 times that of the threshold peak. Both the neutron angular distributions due to the 4.6 Mev rise and the threshold peak appear isotropic. (c) A sharp and rather large peak occurs at 2.9 Mev. The associated neutron angular distribution is anisotropic. (d) There is weak but observable excitation of the well-known 2.43-Mev level of Be^9. [The strong excitation of this level in the (e,e') experiments of Barber *et al.*,[6] which particularly favored $M1$ transitions, and the absence of corresponding excitation of the 2.9-Mev level, require that the parity of the 2.9-Mev level be even and the 2.43-Mev level be odd.]

The second contribution is a calculation of the photodisintegration cross section by Francis *et al.*[7] In this work the photo process in the energy range slightly above threshold [comprising peak (a) above] is attributed to the excitation of a "valence" p-state neutron, which is coupled to a spherical Be^8 core, into an s state in the continuum. The final continuum state is assumed to have a resonance for scattering from the Be^8 core. This calculation is an extension of earlier calculations with this model by Guth and Mullin[8] where realistic diffuse-edge nuclear potentials are now employed.

When the plot of experimental $Be^9(\gamma,n)$ cross sections versus energy in the region just above threshold is compared to these recent calculations, it is found that the experimental shape is adequately represented by several theoretical curves corresponding to different values for a, the diffuseness parameter of the Saxon potential affecting the final-state neutrons. The magnitudes of the experimental cross sections however, are always less than the calculated values. For a canonical value of the diffuseness parameter, $a=0.6$ f, the ratio of theoretical to experimental cross section is 1.8, for $a=0.9$ f the ratio is 1.59, while for a rather large value of the diffuseness parameter, $a=1.2$ f, this ratio is still 1.2. In view of the information concerning the electric charge distribution in Be^9, the value $a=1.2$ f in the nuclear potential of the n-Be^8 system seems to us to be unrealistically large. In an analysis of high-energy electron-scattering data which includes quadrupole scattering, Meyer-Berkhout *et al.*[9] find $a\cong 0.79$ f in the Saxon form of the electric-charge distribution. Further, the successful analysis of these data in terms of the alpha-particle model[10,11] also implies that the tail of the charge distribution is not abnormally long-ranged.

The purpose of this note is to point out that both the above discrepancies between the experimental magnitudes of the cross section and calculations in which a typical value is used for the diffuseness parameter, and features (a) and (b) of Jakobson's experiments, may be accounted for when a model for the ground state of Be^9 is adopted in which there is substantial coupling of the "valence" neutron to both the ground and first excited states of Be^8. More specifically, we find that for the nearly equivalent alpha particle or Nilsson strong-coupling models of Be^9, there is semiquantitive agreement with experiment.

The physical basis for the statements above may be

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[4] W. John, F. J. Lombard, E. T. Moore, and J. M. Prosser, Bull. Am. Phys. Soc. 5, 44 (1960).
[5] R. L. Walter, M. F. Shea, and W. C. Miller, Bull. Am. Phys. Soc. 5, 229 (1960).
[6] W. C. Barber, F. Berthold, G. Fricke, and F. E. Gudden, Phys. Rev. 120, 2081 (1960).
[7] N. C. Francis, D. T. Goldman, and E. Guth, Phys. Rev. 120, 2175 (1960).
[8] E. Guth and C. J. Mullin, Phys. Rev. 74, 832 (1948); 76, 234 (1949).
[9] U. Meyer-Berkhout, K. Ford, and A. E. S. Green, Ann. Phys. 8, 119 (1959).
[10] E. V. Inopin and B. I. Tischenko, Soviet Phys.—JETP 11, 840 (1960).
[11] P. D. Kunz (to be published).

simply stated: When there is strong coupling between the "valence" neutron and the core, the ground-state wave function contains a sizable admixture of the 2^+ first-excited core state as well as the 0^+ core state. Thus when the "valence" neutron is lifted into an s-wave continuum state via absorption of electric dipole radiation, there is a large probability that the core will be left in its excited state which lies 2.9 Mev above the ground state. This would account for the peak at a gamma-ray energy of $4.6(\cong 1.67+2.9)$ Mev seen by Jakobson. Further, as the probability for core excitation is enhanced, the probability that the core remains in the ground state is diminished. The single-nucleon excitation calculations of Francis *et al.* should then be multiplied by this reduction factor. These arguments will now be developed in detail.

The differential cross section for photodisintegration via an electric dipole interaction[12] is

$$\sigma_d(\gamma,n)=\frac{kp\mu e'^2}{2\pi\hbar^2 c}|\langle f|\mathbf{r}_n\cdot\boldsymbol{\varepsilon}|i\rangle|^2, \tag{1}$$

summed over different final-spin states and averaged over initial-spin states. The initial state will be denoted $\psi_{I',M'}(\xi,\mathbf{r}_n)$, where ξ and $\mathbf{r}_n$ refer to core and neutron coordinates, respectively. We consider only those final states in which the neutron is in an s state when it is well separated from the Be^8 core. The crucial assumption in the derivation leading to Eq. (6) is now made that in such states there is no coupling between the final-neutron and core angular momenta. This is equivalent to the statement that the final-state wave function over all space may be written in the product form

$$|f\rangle=\psi_{I,M}(\xi)R_{0,\frac{1}{2}}(r_n)\chi_{\frac{1}{2},\mu}. \tag{2}$$

Here $\psi_{I,M}(\xi)$ describes the core, $R_{0,\frac{1}{2}}(r_n)$ is the continuum s state whose form is independent of I, and $\chi_{\frac{1}{2},\mu}$ is the neutron-spin function. In justification of this assumption we note that for the spherically symmetric wave function of Eq. (2), the interaction potential seen by the neutron due to a deformed core is independent of I. (Such is not the case when the neutron is in states of higher orbital angular momentum and indeed we will later assume that p-wave neutrons are strongly coupled to the core.) We recognize that the assumption is not trivial however, and that only for a spherical core is the wave function of Eq. (2), in which there is no admixture at small r_n of neutron states with higher orbital angular momentum, strictly valid.

It is advantageous to employ the concept of the "reduced width"[13] (or fractional parentage coefficient), $(\theta_{j,l}{}^{I,I'})^2$, defined by

$$\int d\xi\psi_{I',M'}{}^*(\xi,\mathbf{r}_n)\psi_{I,M}(\xi)=\sum_{l,j,m}(I,j,M,m|I',M')R_{l,j}(r_n)\mathcal{Y}_{jl\frac{1}{2}}{}^{m*}\theta_{j,l}{}^{I,I'}, \tag{3}$$

where $\mathcal{Y}_{jl\frac{1}{2}}{}^m$ is a function of neutron angular and spin coordinates,

$$\mathcal{Y}_{jl\frac{1}{2}}{}^m=\sum_{m'}(l,\tfrac{1}{2},m',m-m'|j,m)Y_{l,m'}\chi_{\frac{1}{2},m-m'},$$

and $R_{l,j}(r_n)$ is the radial-neutron wave function. The coefficient, $\theta_{j,l}{}^{I,I'}$, is thus a measure of the various core states with angular momentum I, present in the initial wave function. This coefficient occurs naturally in the expression for dipole matrix element,

$$\langle f|\mathbf{r}_n\cdot\boldsymbol{\varepsilon}|i\rangle^*=\sum_{l,j,m}(I,j,M,m|I',M')\theta_{j,l}{}^{I,I'}\times\int d\mathbf{r}_nR_{0,\frac{1}{2}}(r_n)\chi_{\frac{1}{2},\mu}(\mathbf{r}_n\cdot\boldsymbol{\varepsilon})R_{l,j}(r_n)\mathcal{Y}_{jl\frac{1}{2}}{}^{m*}. \tag{4}$$

Then, because of the orthogonality of the Clebsch-Gordan coefficients, the appropriate sum and average of the square of the dipole matrix element becomes

$$\frac{1}{2I'+1}\sum_{M',M,\mu}|\langle f|\mathbf{r}_n\cdot\boldsymbol{\varepsilon}|i\rangle|^2=\sum_j(\theta_{j,l}{}^{I,I'})^2\left\{\sum_{m,\mu}\frac{1}{(2j+1)}\times\left|\int d\mathbf{r}_nR_{0,\frac{1}{2}}(r_n)\chi_{\frac{1}{2},\mu}(\mathbf{r}_n\cdot\boldsymbol{\varepsilon})R_{l,j}(r_n)\mathcal{Y}_{jl\frac{1}{2}}{}^{m*}\right|^2\right\}, \tag{5}$$

where we have assumed for convenience that but one value of l characterizes the initial-neutron orbital. The quantity in brackets is simply the square of the matrix element in the single-nucleon approximation, suitably summed over final spin projections μ, and averaged over projections of the initial angular momentum m.

When the core states are characterized by sharp energies, the photo cross section leading to final core state I is

$$\sigma_I(\gamma,n)=\sum_j(\theta_{j,l}{}^{I,I'})^2\sigma_j(\gamma,n), \tag{6}$$

where $\sigma_j(\gamma,n)$ is the single-nucleon cross section, a calculation of which is provided by Francis *et al.*[7] The momentum of the outgoing neutron $\hbar p$, occurring in σ_j, is related to k by $\hbar k=\hbar^2p^2/2\mu+\text{B.E.}+(E_I-E_0)$ where E_I is the energy of the core state. In actuality, excited core states do not have sharp energy and indeed, in the Be^9 problem, it would be anticipated that the breadth of the peak in the photo cross section around 4.6 Mev is determined by the moderately large width ($\sim$1 Mev) of the 2.9-Mev ($I=2$) state of Be^8. The integrated cross sections may be estimated independ-

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[12] We generally follow the notation of reference 7. The relative coordinate between the neutron and the core is $\mathbf{r}_n$.

[13] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955).

ently of the widths, however, with the result that

$$\frac{\int \sigma_{I_1} dE_\gamma}{\int \sigma_{I_2} dE_\gamma} \cong \frac{k_{I_1} \sum_j (\theta_{j,l}{}^{I_1,I'})^2}{k_{I_2} \sum_j (\theta_{j,l}{}^{I_2,I'})^2}, \tag{7}$$

where $\hbar k_I$ is the photon energy corresponding to the peak in cross section for excitation of the core state with angular momentum I.

For the most extreme form of the alpha-particle model or Nilsson strong-coupling model, the projections of total angular momentum and the angular momentum of the valence nucleon on the body axis are good quantum numbers so that[14,15]

$$\psi_{I',K',M'} = [(2I'+1)/16\pi^2]^{\frac{1}{2}} [\phi_\Omega(\mathbf{r}_n') D_{M',K'}{}^{I'}(\Theta)^* + (-1)^{I'-j} \phi_{-\Omega}(\mathbf{r}_n') D_{M',-K'}{}^{I'}(\Theta)^*]. \tag{8}$$

The reduced widths for the above strong-coupling wave functions are well known[16–18]:

$$(\theta_{j,l}{}^{I,I'}) = \left[\frac{2I+1}{2I'+1}\right]^{\frac{1}{2}} C_{l,j}{}^{\Omega}\ (I,j,K,\Omega|I',K') \times (1+\delta_{K,0}+\delta_{K',0})^{\frac{1}{2}}. \tag{9}$$

where $C_{l,j}{}^{\Omega}$ are the coefficients in the expansion of the neutron orbital in the body frame

$$\phi_\Omega = \sum_{l,j} C_{l,j}{}^{\Omega} R_{l,j} Y_{jl\frac{1}{2}}{}^{\Omega}.$$

For the ground state of Be^9, $I'=j=K'=\Omega=\frac{3}{2}$ and $l=1$ so that

$$(\theta_{\frac{3}{2},1}{}^{0,\frac{3}{2}})^2 = (\tfrac{1}{2}), \quad \text{and} \quad (\theta_{\frac{3}{2},1}{}^{2,\frac{3}{2}})^2 = (\tfrac{1}{2}).$$

[14] J. S. Blair and E. M. Henley, Phys. Rev. **112**, 2029 (1958).
[15] P. D. Kunz, Ann. Phys. **11**, 275 (1960).
[16] S. Yoshida, Progr. Theoret. Phys. (Kyoto) **12**, 141 (1954).
[17] G. R. Satchler, Ann. Phys. **3**, 275 (1958).
[18] A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, Can. J. Phys. **36**, 378 (1958).

Mixing of the $\Omega=\frac{3}{2}^-$ band with the next highest $\Omega=\frac{1}{2}^-$ band due to rotational-particle coupling has been considered by Kunz.[15] Coefficients of the eigenvectors of the neutron-wave function for a reasonable value of the oscillator parameter, $\alpha=0.125\times10^{26}$ cm^{-2}, have been given in Table II of reference 15. The reduced widths resulting from these values are easily computed to be

$$\sum_j (\theta_{j,1}{}^{0,\frac{3}{2}})^2 = 0.60, \quad \text{and} \quad \sum_j (\theta_{j,1}{}^{2,\frac{3}{2}})^2 = 0.40.$$

Thus, with the extreme strong-coupling models for the ground state of Be^9, the photodisintegration cross section just above threshold should be half that computed in the single-nucleon approximation, while the more sophisticated calculation, incorporating band mixing, gives the value 0.60 for this ratio. Accordingly, the magnitudes computed by Francis *et al.* using the values $a=0.6$ or 0.9 f are now in fairly good agreement with the low-energy experiments. Further, the rise in cross section around 4.6 Mev observed by Jakobson and the associated isotropic angular distribution may be interpreted as a transition to the s-wave continuum state with corresponding excitation of the $I=2$-core state. The predicted estimates for the ratio of integrated cross sections, $(4.6/1.75)=2.6$ (for the strong-coupling calculation) or $[(4.6)(0.40)/1.75(0.60)]=1.75$ (for the calculation which includes band mixing) are of the order observed.

The strong transition at $E=2.9$ Mev, presumably to a d state, also may be discussed in terms of a strong-coupling model. Unlike the situation for an s-wave transition however, a quantitative treatment of this transition will depend sensitively on the nature of the coupling between the final-state neutron and the core.

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