30% contribution to the fission cross section from this kind of reaction at 95 and 124 Mev. This value has to be regarded as an upper limit due to the possible increase of K_0^2 with I.

The fission cross sections observed agree well with calculated cross sections for compound-nucleus formation based on the square-well nuclear potential with radius parameter $r_0=1.5\times10^{-13}$ cm. From our observations, it would appear that the calculated cross sections would be more aptly termed the "interaction cross section for reactions leading to deposition of excitation energies of more than 5 Mev."

It is evident that only charged-particle-fission-fragment-coincidence experiments can give a clearer picture of the reactions occurring prior to the fission process.

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Studies of Stripping and Pickup Reactions on the Basis of the Pairing Plus Quadrupole-Quadrupole Interaction Model*

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Cross sections for (d,p) and (d,t) reactions in units of the single-particle cross sections (the spectroscopic factors) are calculated for spherical nuclei. It is assumed that the protons fill a closed shell and that only neutrons in an unfilled shell interact with each other through the pairing and quadrupole-quadrupole interactions. First the pairing interactions problem is solved by introducing quasi-particles according to Belyaev. Next the quadrupole-quadrupole interaction is diagonalized, taking into accout two quasi-particle states for the first excited state of even-even nuclei. Using these wave functions the spectroscopic factors are obtained in simple form, and are evaluated numerically for the case of the Sn isotopes. Comparison is made with experiments for the transitions to the ground states of even-even and even-odd isotopes as well as to the vibrational states of even-even isotopes. Agreement in both cases is fairly good.

1. INTRODUCTION

THE importance of deuteron stripping and pickup reactions as a tool of nuclear spectroscopy has been emphasized by many authors. These reactions provide rather direct information on the wave functions of low-lying nuclear states. Macfarlane and French[1] gave the most elaborate and complete reviews of these reactions, mainly based on the shell model, while Satchler[2] summarized studies of the stripping reactions based on the collective model. For deformed nuclei with rotational spectra Satchler gave a straightforward prescription for analysis of experimental data and work has been published along this line.[3] However for the vibrational spectra further detailed calculations like the intermediate coupling theory may be necessary to analyze experimental data.

Recently another aspect of nuclear structure was revealed by the Copenhagen group[4] in analogy with superconductivity in solid state physics. The pairing force is responsible for this new aspect of structure and the existence of an energy gap in the intrinsic spectra of deformed even-even nuclei was the first experimental support for it. The powerful mathematical method of superconductivity[5] was applied in the nuclear case by Belyaev[6] and further detailed comparisons with experiments have been carried out successfully by Kisslinger and Sorensen.[7] The latter authors treated single closed-shell nuclei and calculated the energy spectra, electromagnetic moments, and transition rates. To study the

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vibrational states they assumed the cranking model.[8] These states also have been studied by Baranger and other people[9] using more refined methods.

The pairing forces cause a strong configuration mixing among the nucleon states in the unfilled shell, which is difficult to treat by usual shell model calculations. This configuration mixing gives rise to the energy gap, and to secondary effects, like the deviation of the moment of inertia from the rigid-body value.[10] However its effects may be seen directly by studying the stripping and pickup process. In fact Cohen and Price[11] have made experiments with (d,p) and (d,t) reactions in a wide range of atomic numbers, and found many facts which seem difficult to explain by the simple shell model. This paper was inspired by their work and will give theoretical considerations of the (d,p) and (d,t) reactions based on the superconductive nature of nuclei. For the vibrational state the Tamm-Dancoff method will be applied and rather simple and explicit formulas for the reduced width will be presented.

The (d,p) and (d,t) cross sections may be expressed as[1]

$$d\sigma_{d,x}/d\Omega = R_{d,x}\sum_l S_l\phi_{d,x}(l,Q,\theta), \quad x=p,t \qquad (1.1)$$

where $R_{d,x}$ is the statistical factor and is given by

$$R_{d,p}=(2J_f+1)/(2J_i+1), \quad R_{d,t}=1. \qquad (1.2)$$

J_i and J_f are the spin of the target and residual nucleus respectively. The second factor in Eq. (1.1), S_l, is called the spectroscopic factor, which gives the probability of the appearance of the single particle state in the parent nuclear wave function. The last factor is the single-particle cross section and is considered as a function of orbital angular momentum l of the stripped or captured neutron, the Q value, and the angle of the outgoing particle θ.

The spectroscopic factor S_l may be expressed as a sum of overlap integrals between the parent nucleus and a free state composed of the daughter nucleus and a captured or stripped neutron with angular momentum j. That is,

$$S_l=\sum_{j=l\pm\frac{1}{2}} S_j, \qquad (1.3)$$

where

$$S_j=A\langle\Psi_{JM}|\Phi_{JM}(j,J_0)\rangle^2. \qquad (1.4)$$

In the last equation, A is the mass number of the parent nucleus and Ψ_{JM} is its wave function, J and M being its spin and the Z component. $\Phi_{JM}(j,J_0)$ is given by

$$\Phi_{JM}(j,J_0)=\sum_{m,M}(jmJ_0M_0|JM)\phi_{jm}\Psi_{J_0M_0}, \qquad (1.5)$$

where $(jmJ_0M_0|JM)$ is the Clebsch-Gordan coefficient, ϕ_{jm} is the spin-angle part of the captured or stripped neutron, and $\Psi_{J_0M_0}$ is the wave function of the daughter nucleus with spin J_0 and Z component M_0. This spectroscopic factor corresponds also to the reduced width in units of the single-particle reduced width.

As for the single-particle cross section $\phi(l,Q,\theta)$, the dependences on l, Q, and θ are known empirically.[11] Therefore the knowledge of the spectroscopic factor is sufficient to discuss the cross section. The spectroscopic factor may be calculated easily once the wave functions are given. In Sec. 2 the necessary formulas for the nuclear wave function based on the pairing interaction model will be summarized. Then the spectroscopic factor will be calculated in Sec. 3 for the ground states or single-particle states. Section 4 will be devoted to the construction of the wave function of the vibrational states and in the following section (Sec. 5), these wave functions will be applied to the calculation of the spectroscopic factor for vibrational states. These results will be compared with experiments in Sec. 6 and discussions will be presented.

2. PAIRING INTERACTION

In this section the nuclear model based on the pairing interaction will be summarized in order to give necessary wave functions for the calculation of the spectroscopic factor. We follow the work of Belyaev[6] but use the Condon and Shortley[12] phase for the wave function. It is also assumed that only one kind of nucleons (neutrons) are active while the other kind of nucleons (protons) form a closed shell and will not be taken into account in the calculation. The vacuum state $|0\rangle$ will be understood to stand for the state where all states in filled shells are occupied and none of the unfilled shell is occupied. The Hamiltonian for the pairing interaction then is given by

$$H_0=\sum_{j,m}\epsilon_j a_{jm}^\dagger a_{jm}-\tfrac{1}{4}G\sum_{j'm'}(-)^{j'-m'}a_{j'm'}^\dagger a_{j'-m'}^\dagger \times\sum_{jm}(-)^{-j+m}a_{j-m}a_{jm}, \qquad (2.1)$$

where the first term of the right-hand side represents the sum of the single-particle energies (of the shell model states) while the second term represents the pairing interactions among nucleons in the unfilled shell. $a_{jm}^\dagger$ and a_{jm} are the creation and annihilation operators of the shell model state with spin j and Z component m. Also ϵ_j is the single-particle energy of the shell model state j, and G is the strength of the pairing interaction. To specify a shell model state, quantum numbers other than j and m will be necessary, but they will be suppressed unless they are needed.

Following the procedure of Bogoliubov and Valatin[5] the operators $a_{jm}^\dagger$ and a_{jm} now will be transformed into new operators by a canonical transformation. As this transformation mixes states with different mass

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number, it is necessary to introduce the auxiliary Hamiltonian

$$H_0'=H_0-\lambda\sum_{j,m}a_{jm}^{\dagger}a_{jm}, \tag{2.2}$$

where λ is the chemical potential, serving as a Lagrange multiplier to take into account the constraint that for the solution Ψ the average occupation number equals the number n of nucleons in the unfilled shell; $\langle\Psi|\sum a_{jm}^{\dagger}a_{jm}|\Psi\rangle=n$. The Bogoliubov and Valatin[5] transformation is given by

$$a_{jm}=U_j\alpha_{jm}+V_j(-)^{j-m}\alpha_{j-m}^{\dagger}, \tag{2.3}$$

where U_j and V_j satisfy

$$U_j^2+V_j^2=1. \tag{2.4}$$

The coefficients U_j and V_j are chosen so that the new Hamiltonian in terms of α_{jm} and $\alpha_{jm}^{\dagger}$ will not contain terms like $\alpha^{\dagger}\alpha^{\dagger}$ and $\alpha\alpha$. Therefore the following equations are obtained

$$\tfrac{1}{4}G\sum_j\frac{2j+1}{[(\epsilon_j-\lambda)^2+\Delta^2]^{\frac{1}{2}}}=1, \tag{2.5}$$

where

$$\Delta=\tfrac{1}{2}G\sum_j(2j+1)U_jV_j, \tag{2.6}$$

and

$$U_j^2=\tfrac{1}{2}\left[1+\frac{\epsilon_j-\lambda}{[(\epsilon_j-\lambda)^2+\Delta^2]^{\frac{1}{2}}}\right], \tag{2.7}$$

$$V_j^2=\tfrac{1}{2}\left[1-\frac{\epsilon_j-\lambda}{[(\epsilon_j-\lambda)^2+\Delta^2]^{\frac{1}{2}}}\right]. \tag{2.8}$$

The transformed Hamiltonian takes the form

$$H_0'=\text{const}+\sum E_j\alpha_{jm}^{\dagger}\alpha_{jm}+\text{terms containing four }\alpha\text{'s}, \tag{2.9}$$

where the second term represents the transformed single-particle energy, and

$$E_j=[(\epsilon_j-\lambda)^2+\Delta^2]^{\frac{1}{2}}. \tag{2.10}$$

In this new representation the nuclear state is specified by the various occupation numbers of the new particle states which are called the quasi-particle states. The last term of (2.9) will be neglected. The quasi-particle states are the elementary excitations with respect to a new vacuum, which is the ground state of an even-even nucleus. In terms of the old representation, the wave function of the new vacuum is expressed as

$$\Psi_0=\prod_{j,m>0}(U_j+V_j(-)^{j-m}a_{jm}^{\dagger}a_{j-m}^{\dagger})|0\rangle. \tag{2.11}$$

It is easily shown that

$$\alpha_{jm}\Psi_0=0. \tag{2.12}$$

The odd nucleus then is the state with one quasi-particle and the wave function is given by

$$\alpha_{JM}^{\dagger}\Psi_0. \tag{2.13}$$

The number of nucleons is obtained as

$$n=\sum_j\frac{2j+1}{2}\left[1-\frac{\epsilon_j-\lambda}{[(\epsilon_j-\lambda)^2+\Delta^2]^{\frac{1}{2}}}\right]+\sum_{i=1}^{r}\frac{\epsilon_i-\lambda}{[(\epsilon_i-\lambda)^2+\Delta^2]^{\frac{1}{2}}}, \tag{2.14}$$

where the first term represents the contribution from the new vacuum and the second term is the contribution from the quasi-particles.

Equations (2.5) and (2.14) are the basic equations, and Δ and λ will be obtained if ϵ_j, G, and n are given, Δ is the lowest energy of the quasi-particle as seen from (2.10). After Δ and λ are derived U_j and V_j may be calculated from (2.7) and (2.8) and accordingly the energy and the wave function will be obtained. Numerical values of Δ and λ are tabulated by Kisslinger and Sorensen[7] for the single closed shell nuclei.

It is noted that the values of U_j and V_j differ from one nucleus to another and change smoothly with increasing mass number. In the next section it will become necessary to treat the even-even nucleus with nucleon number n and the even-odd nucleus with nucleon number $n\pm1$ in calculating the overlap integral. If the former wave function is denoted by Ψ_0, in which U_j and V_j are adjusted to give the average nucleon number n', then the latter wave function given by Eq. (2.13) $\alpha_{JM}^{\dagger}\Psi_0$ will no longer correspond to the state with a nucleon number $n\pm1$. Instead for this nucleus Eq. (2.14) once again gives the average nucleon number n, because the contribution from the one quasi-particle JM is very small if the odd nucleus is in the ground or low excited state, where $\epsilon_J\approx\lambda$. In order to get the odd-nuclear wave function with nucleon number $n\pm1$, it is necessary to use slightly different U_j' and V_j' from those used in the even-even nucleus. Let that vacuum state be denoted by Ψ_0' which gives the average nucleon number $n\pm1$, and those quasi-particle operators by $\alpha_{jm}'^{\dagger}$ and α_{jm}' which are obtained by replacing U_j and V_j by U_j' and V_j'. Then the vacuum state with prime may be expressed in terms of the original vacuum state and its operators as

$$\Psi_0'=\prod_{j,m>0}(V_jV_j'+U_jU_j')[1-(U_jV_j'-U_j'V_j)\times(V_jV_j'+U_jU_j')^{-1}(-)^{j-m}\alpha_{j-m}^{\dagger}\alpha_{jm}^{\dagger}]\Psi_0. \tag{2.16}$$

Neglecting the higher order terms, the last equation is expressed approximately

$$\Psi_0'\simeq\left[1+\tfrac{1}{2}\sum_{j,m}\frac{\Delta U_j}{V_j}(-)^{j-m}\alpha_{j-m}^{\dagger}\alpha_{jm}^{\dagger}\right]\Psi_0, \tag{2.17}$$

where

$$\Delta U_j = U_j' - U_j. \tag{2.18}$$

For the one quasi-particle state it is easy to prove that

$$\alpha_{jm}'^{\dagger}\Psi_0' \simeq \alpha_{jm}^{\dagger}\Psi_0', \tag{2.19}$$

where higher order terms are neglected.

3. GROUND-STATE SPECTROSCOPIC FACTOR

The spectroscopic factor is expressed in the second quantization form as

$$S_j = \langle \Psi_{JM} | \Phi_{JM}(j,J_0) \rangle^2, \tag{3.1}$$

where Ψ_{JM} is the wave function of the parent nucleus with spin J and Z component M, while

$$\Phi_{JM}(j,J_0) = \sum_{m,M_0} (jmJ_0M_0|JM) a_{jm}^{\dagger}\Psi_{J_0M_0}. \tag{3.2}$$

In the last equation $\Psi_{J_0M_0}$ is the wave function of the daughter nucleus with spin J_0 and Z component M_0, and j and m are the spin and Z component of the captured or stripped nucleon.

We first consider the case of the odd parent nucleus. If the ground state of the even-even nucleus is denoted by Ψ, and the parent even-odd nucleus by $\alpha_{JM}'^{\dagger}\Psi_0'$, then the spectroscopic factor is expressed as

$$S_j = \langle \alpha_{JM}'^{\dagger}\Psi_0' | a_{jm}^{\dagger}\Psi_0 \rangle^2. \tag{3.3}$$

By using (2.17), (2.19), and (2.3), the last equation gives the result

$$S_j = \delta_{Jj} U_J^2, \tag{3.4}$$

where U_J^2 is the probability of the orbit with spin J in the even-even nucleus being empty. The correction factor given by (2.17) plays no role in this expression, in which U_j and V_j apply to the daughter nucleus, but would if the U_j and V_j of the parent nucleus were used. If we use the U_j and V_j for the parent nucleus, then we have

$$S_j = \langle \alpha_{JM}^{\dagger}\Psi_0 | a_{jm}^{\dagger}\Psi_0' \rangle^2 = \delta_{Jj}(U_J + \Delta U_J)^2,$$

which agrees with (3.4) because of the correction term ΔU_J. Equation (3.4) may be given the interpretation that one nucleon is captured by the even-even nucleus into the orbit j and that the probability is proportional to the probability of the orbit j being empty.

Next let us consider the case of an even-even parent nucleus. If the wave function of the parent nucleus is denoted by Ψ_0 and that of the odd nucleus by $\alpha_{J_0M_0}'^{\dagger}\Psi_0'$, then the spectroscopic factor is obtained

$$S_j = \delta_{J_0 j}(2J_0+1) V_{J_0}^2, \tag{3.5}$$

where $V_{J_0}^2$ is again the probability of the orbit with spin J_0 in the even-even nucleus being occupied. The interpretation is similar to the previous case; one nucleon at the level j is stripped from the even-even nucleus and the probability is proportional to the number of nucleons occupying the orbit j, that is $(2J_0+1)V_{J_0}^2$ [see (2.14)]. It is noted that the spectroscopic factor is described by U and V of the even-even nucleus irrespective of which nucleus actually is the parent. In our treatment the state of the odd nucleus has been described as being the ground state, but in fact it may be any state of that nucleus. For any single quasi-particle state the formulas (3.4) and (3.5) should be valid. These two facts make it possible to obtain all the values of U_j and V_j for the even-even nucleus from the set of experimental cross section of (d,p) and (d,t) reaction for single quasi-particle states.[13]

Here we have to remember that the number of nucleons has not been taken into account exactly, due to the introduction of the Bogoliubov and Valatin[5] transformation. It is expected that the error in the nucleon number n may be proportional to $\sqrt{n}$, and our results also may contain some errors. To test this it is convenient to compare our results with the shell model results in a simple case. Let us assume that the configuration is pure j^n, then the equations (2.5)–(2.8) and (2.14) become very simple and the following solution may be obtained easily

$$\epsilon - \lambda = G(2j+1-2n)/4, \quad \Delta = G[n(2j+1-n)]^{\frac{1}{2}}/2, \tag{3.6}$$

where ϵ is the single particle energy of the orbit j. From (2.7) and (2.8) we obtain

$$U_j^2 = 1 - \frac{n}{2j+1}, \quad V_j^2 = \frac{n}{2j+1}. \tag{3.7}$$

If these are put into (3.4) and (3.5), then

$$\begin{aligned} S_j &= \delta_{jJ}\left[1 - \frac{n}{2j+1}\right] \quad \text{for odd parent nucleus,} \\ &= \delta_{jJ_0} n \quad \text{for even-even parent nucleus,} \end{aligned} \tag{3.8}$$

where n is the number of nucleons j in the even-even nuclei. These results agree exactly with those obtained from the usual shell model calculation.[1] From this we may expect that our results should be very accurate despite the inaccurate treatment of nucleon number.

4. VIBRATIONAL STATE

In considering the vibrational state of even-even nuclei, the quadrupole-quadrupole interaction,[14]

$$\begin{aligned} H_{Q-Q} = -\tfrac{1}{2}x \sum_{\mu j_1 j_1' j_2 j_2'} (-)^{\mu} &\langle j_1' m_1' | Y_{2\mu v} | j_1 m_1 \rangle \\ &\times \langle j_2' m_2' | Y_{2-\mu v} | j_2 m_2 \rangle a_{j_1' m_1'}^{\dagger} a_{j_2' m_2'}^{\dagger} a_{j_2 m_2} a_{j_1 m_1}, \end{aligned} \tag{4.1}$$

is taken into account in addition to the pairing interaction. In the last equation, x is the coupling constant

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[this corresponds to $(\hbar/m\omega_0)^2\chi$ in the Kisslinger and Sorensen paper[7]] and $Y_{2\mu}$ are the second order spherical harmonics. Harmonic oscillator single-particle wave functions are assumed in order to make the calculation easy. The dimensionless oscillator potential v is given by

$$v=(m\omega_0/\hbar)r^2, \tag{4.2}$$

where r is the radial coordinate of the single particle, m is the mass of the nucleon, and ω_0 is the angular frequency of the harmonic oscillator.

After applying the Bogoliubov and Valatin transformation[5] (2.3), the Q–Q interaction is expressed in a normal form:

$$\begin{aligned}H_{Q-Q}=&-(x/10)\sum_{j_1j_1'j_2j_2'}(j_2'\|Y_2v\|j_2)(j_1'\|Y_2v\|j_1)\\
&\times[\tfrac{1}{4}u_{j_2'j_2}u_{j_1'j_1}\sum_\mu(-)^\mu A^\dagger(j_2'j_22-\mu)A^\dagger(j_1'j_12\mu)\\
&+u_{j_2'j_2}v_{j_1'j_1}\sum_\mu(-)^\mu A^\dagger(j_2'j_22-\mu)A^0(j_1'j_12\mu)\\
&+\tfrac{1}{2}u_{j_2'j_2}u_{j_1'j_1}\sum_\mu A^\dagger(j_2'j_22\mu)A(j_1'j_12\mu)\\
&+v_{j_2'j_2}u_{j_1'j_1}\sum_\mu A^0(j_2'j_22\mu)A(j_1'j_12\mu)\\
&+\tfrac{1}{4}u_{j_2'j_2}u_{j_1'j_1}\sum_\mu(-)^\mu A(j_2'j_22\mu)A(j_1'j_12-\mu)\\
&-5(-)^{j_2-j_1'}v_{j_2'j_2}v_{j_1'j_1}\sum_{\nu\kappa}(-)^\nu W(j_2'j_2j_1'j_1;2\nu)\\
&\times A^\dagger(j_1'j_2'\nu\kappa)A(j_1j_2\nu\kappa)],\end{aligned} \tag{4.3}$$

where

$$\begin{aligned}A^\dagger(j_1j_2\lambda\mu)&=\sum_{m_1m_2}(j_1m_1j_2m_2|\lambda\mu)\alpha_{j_1m_1}^\dagger\alpha_{j_2m_2}^\dagger,\\
A(j_1j_2\lambda\mu)&=\sum_{m_1m_2}(j_1m_1j_2m_2|\lambda\mu)\alpha_{j_2m_2}\alpha_{j_1m_1},\\
A^0(j_1j_2\lambda\mu)&=\sum_{m_1m_2}(-)^{j_2+m_2}(j_1m_1j_2m_2|\lambda\mu)\\
&\quad\times(\alpha_{j_1m_1}^\dagger\alpha_{j_2-m_2}-\tfrac{1}{2}\delta_{j_1j_2}\delta_{m_1,-m_2}).\end{aligned} \tag{4.4}$$

The first two lines of (4.4) represent a pair of quasi-particle creation and annihilation operators, respectively, and the last line represents the quadrupole transition operator (for $\lambda=2$). Also $u_{j_1j_2}$ and $v_{j_1j_2}$ are the following combinations of U_j and V_j:

$$u_{j_1j_2}=U_{j_1}V_{j_2}+V_{j_1}U_{j_2},\quad v_{j_1j_2}=U_{j_1}U_{j_2}-V_{j_1}V_{j_2}. \tag{4.5}$$

In Eq. (4.3), $(j'\|Y_2v\|j)$ is the reduced matrix element of the nondimensional quadrupole transition operator and is given by

$$\begin{aligned}(N'l'j'\|Y_2v\|Nlj)=&\frac{1}{(4\pi)^{\frac{1}{2}}}(-)^{j'-j}[5(2j'+1)]^{\frac{1}{2}}\\
&\times(j'\tfrac{1}{2}20|j\tfrac{1}{2})\frac{1+(-)^{l+l'}}{2}(N'l'|v|Nl),\end{aligned} \tag{4.6}$$

TABLE I. The radial integrals $(N'l'|v|Nl)$.

N'	l'	$(N'l'\|v\|Nl)$
N	l	$N+\frac{3}{2}$
$N\pm2$	l	$-\frac{1}{2}[(N+l+2\pm1)(N-l+1\pm1)]^{\frac{1}{2}}$
$N\pm2$	$l\pm2$	$\frac{1}{2}[(N+l+1\pm2)(N+l+3\pm2)]^{\frac{1}{2}}$
N	$l\pm2$	$-[(N+l+2\pm1)(N-l+1\mp1)]^{\frac{1}{2}}$
$N\mp2$	$l\pm2$	$\frac{1}{2}[(N-l\mp2)(N-l+2\mp2)]^{\frac{1}{2}}$

where N is the principal quantum number of the harmonic oscillator wave function, such that the energy is $\hbar\omega_0(N+\frac{3}{2})$. The radial integrals $(N'l'|v|Nl)$ are given in Table I.

The transformed Q–Q interaction has terms like $\alpha^\dagger\alpha^\dagger$, $\alpha\alpha$, and $\alpha^\dagger\alpha$ which were not included in Eq. (4.3). However, these terms need not be carried because they should have been eliminated together with corresponding terms in the pairing interaction by choosing an appropriate transformation (2.3), or should have been included in the single quasi-particle energy given in (2.9). Constant terms were also neglected, because we have no interest in them.

The problem of the pairing plus the Q–Q interaction may be solved in various approximations and is known to give rise to the vibrational spectra in some circumstances. Kisslinger and Sorensen[7] used the cranking model[8] under the adiabatic assumption, but their wave function is not convenient for our purpose. Baranger[9] used the method of the linearized equation of motion, which is believed to be the most refined treatment so far obtained, but with this method it is difficult to give the explicit form of the wave function. The Tamm-Dancoff approximation, which is a further approximation to the latter method,[9] gives the explicit wave function. We will use this approximation. The wave function for the excited state with spin 2 of the even-even nucleus is assumed to be a superposition of two quasi-particle states with resultant angular momentum 2, whose coefficients are chosen so that the Hamiltonian is diagonalized. The first excited state wave function is written as

$$\Psi_{2M}=Q_{2M}^\dagger\Psi_0, \tag{4.7}$$

where

$$Q_{2M}^\dagger=\sum_{j_1j_2}f_{j_1j_2}A^\dagger(j_1j_22M). \tag{4.8}$$

Then the Schrödinger equation is expressed as

$$\begin{aligned}0=(H-\hbar\omega)\Psi_{2M}=&[\sum_{j_1j_2}f_{j_1j_2}(E_{j_1}+E_{j_2}-\hbar\omega)A^\dagger(j_1j_22M)\\
&-(x/20)\sum_{j_3j_4j_3'j_4'}(j_3'\|Y_2v\|j_4')(j_3\|Y_2v\|j_4)\\
&\times u_{j_3'j_4'}u_{j_3j_4}\sum_\mu A^\dagger(j_3'j_4'2\mu)A(j_3j_42\mu)\\
&\times\sum_{j_1j_2}f_{j_1j_2}A^\dagger(j_1j_22M)]\Psi_0,\end{aligned} \tag{4.9}$$

where $\hbar\omega$ is the excitation energy of the state. The term in (4.3) with the Racah coefficient was neglected as it is unimportant and makes the calculation complex.[9] Using the following relation:

$$A(j_1'j_2'\lambda'\mu')A^\dagger(j_1j_2\lambda\mu)\Psi_0=\delta_{\lambda'\lambda}\delta_{\mu'\mu}(\delta_{j_1'j_1}\delta_{j_2'j_2}+(-)^{j_1'-i_2'-\lambda'}\delta_{j_1'j_2}\delta_{j_2'j_1})\Psi_0, \quad (4.10)$$

which is easily verified, the equation becomes

$$f_{j_1j_2}(E_{j_1}+E_{j_2}-\hbar\omega)-(x/10)(j_1\|Y_2v\|j_2)u_{j_1j_2}\times\sum_{j_1'j_2'}f_{j_1'j_2'}(j_1'\|Y_2v\|j_2')u_{j_1'j_2'}=0. \quad (4.11)$$

The solution is easily obtained as

$$f_{j_1j_2}=C\frac{(j_1\|Y_2v\|j_2)u_{j_1j_2}}{E_{j_1}+E_{j_2}-\hbar\omega}, \quad (4.12)$$

where C is a constant to be determined later. The eigenvalue equation is

$$1=(x/10)\sum_{j_1j_2}\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}^2}{E_{j_1}+E_{j_2}-\hbar\omega}, \quad (4.13)$$

from which $\hbar\omega$ may be determined. The normalization of the wave function requires

$$\langle\Psi_0|Q_{2\mu}Q_{2\mu'}^\dagger|\Psi_0\rangle=\delta_{\mu\mu'}, \quad (4.14)$$

from which we obtain

$$\sum_{j_1j_2}f_{j_1j_2}^2=\tfrac{1}{2}. \quad (4.15)$$

The constant C in (4.12) is determined by (4.15), and we obtain

$$f_{j_1j_2}=\frac{1}{\sqrt{2}}\frac{(j_1\|Y_2v\|j_2)u_{j_1j_2}}{E_{j_1}+E_{j_2}-\hbar\omega}\times\left[\sum_{j_1'j_2'}\left\{\frac{(j_1'\|Y_2v\|j_2')u_{j_1'j_2'}}{E_{j_1}+E_{j_2}-\hbar\omega}\right\}^2\right]^{-\frac{1}{2}}. \quad (4.16)$$

The operators $Q_{\lambda\mu}^\dagger$ and $Q_{\lambda\mu}$ satisfy the following approximate boson commutation relations

$$[Q_{\lambda'\mu'},Q_{\lambda\mu}^\dagger]=\delta_{\lambda'\lambda}\delta_{\mu'\mu},\quad [Q_{\lambda'\mu'},Q_{\lambda\mu}]=[Q_{\lambda'\mu'}^\dagger,Q_{\lambda\mu}^\dagger]=0, \quad (4.17)$$

whose vacuum expectation values are exact, and which are expected to be approximately valid for the case of a small number of quasi-particles. From the Schrödinger equation (4.9) we obtain

$$HQ_{2\mu}^\dagger\Psi_0\simeq\hbar\omega Q_{2\mu}^\dagger\Psi_0,\quad Q_{2\mu}^\dagger H\Psi_0\simeq 0,$$

from which the approximate commutation relation,

$$[H,Q_{2\mu}^\dagger]\simeq\hbar\omega Q_{2\mu}^\dagger, \quad (4.18)$$

follows. The second excited state may be described as

$$\Psi_{JM}^{(2)}=\frac{1}{\sqrt{2}}\sum_{\mu\mu'}(2\mu'2\mu|JM)Q_{2\mu'}^\dagger Q_{2\mu}^\dagger\Psi_0, \quad (4.19)$$

both because this wave function satisfies the following approximate Schrödinger equation

$$H\Psi_{JM}^{(2)}\simeq 2\hbar\omega\Psi_{JM}^{(2)}, \quad (4.20)$$

which is easily verified by using (4.18), and because it satisfies the normalization equation

$$\langle\Psi_{JM}^{(2)}|\Psi_{JM}^{(2)}\rangle=1. \quad (4.21)$$

Therefore it is seen that the choice of the wave functions (4.7) and (4.8) give rise in fact to the vibrational spectra if the commutation relations (4.17) and (4.18) are satisfied in a good approximation.

When the second-excited vibrational state of an even-even nucleus is considered in a stripping calculation, an improved wave function for the odd nucleus becomes necessary. The three quasi-particle states should be included in addition to the one quasi-particle state. The simplest way to get the improved wave function is to take the Q–Q force as a perturbation and to make a first order calculation. The result is shown as

$$\Psi_{JM}=[\alpha_{JM}^\dagger+\sum_j C_j\sum_{\mu m}(2\mu jm|JM)Q_{2\mu}^\dagger\alpha_{jm}^\dagger]\Psi_0, \quad (4.22)$$

where

$$C_j=-\frac{x}{[5(2J+1)]^{\frac{1}{2}}}(j\|Y_2v\|J)v_{jJ}\times\sum_{j_1j_2}(j_1\|Y_2v\|j_2)u_{j_1j_2}f_{j_1j_2}/(E_J-E_j-\hbar\omega). \quad (4.23)$$

5. SPECTROSCOPIC FACTOR FOR VIBRATIONAL STATES

We are now ready to calculate the spectroscopic factor for the vibrational states using the wave functions that were obtained in the last section. Let us begin with the first vibrational state. First the parent nucleus is assumed to be even-odd, for which the notations without prime will be used. The notations with prime will be used for the even-even daughter nucleus. Then the spectroscopic factor (3.1) may be expressed by using (3.2), (4.7), and (4.23) as

$$S_j=\langle[\alpha_{JM}^\dagger+\sum_{j'}C_{j'}\sum_{\mu'm'}(2\mu'j'm'|JM)Q_{2\mu'}^\dagger\alpha_{j'm'}^\dagger]\Psi_0\times|\sum_{mM_0}(jmJ_0M_0|JM)(U_j\alpha_{jm}^\dagger+V_j(-)^{j-m}\alpha_{j-m})Q_{J_0M_0}'^\dagger\Psi_0'\rangle^2, \quad (5.1)$$

where JM and J_0M_0 are the spin and its Z component for the parent and daughter nucleus, respectively.

Using the following relation:

$$A'^{\dagger}(j_1 j_2 2\mu)\Psi_0' \simeq A^{\dagger}(j_1 j_2 2\mu)\Psi_0', \tag{5.2}$$

which may be proved easily, Eq. (5.1) is simplified as

$$S_j = \left| 2\left(\frac{2J_0+1}{2J+1}\right)^{\frac{1}{2}} V_j f_{jJ}' - C_j U_j \right|^2, \tag{5.3}$$

where U_j and V_j are referred to the even-odd nucleus while f_{jJ}' is referred to the even-even nucleus. In obtaining the first term in the absolute square of the last equation, Eq. (2.17) was utilized, but the second term is calculated neglecting the difference between Ψ_0 and Ψ_0'.

The treatment of the case of the even-even parent nucleus is similar to the previous case. Only the result is presented:

$$S_j = \left| 2U_j f_{jJ_0}' + \left(\frac{2J_0+1}{2J+1}\right)^{\frac{1}{2}} C_j V_j \right|^2, \tag{5.4}$$

where U_j and V_j are referred to the even-odd nucleus again and f_{jJ}' is referred to the even-even nucleus. J and J_0 are spin of the parent and daughter nuclei, respectively, just as in the previous case.

Next the second-excited state is considered. The wave function (4.19) is used instead of (4.7) which was used in the previous cases, and the commutation relation (4.17) is fully utilized to simplify the expression. The calculation is not so much different from the previous case, so the results only will be presented. If the parent nucleus is even-odd, the spectroscopic factor is given by

$$S_j = 40(2J_0+1)V_j^2 |\sum_{j'} C_{j'} W(2j'J_0 j; J2) f_{j'j}'|^2, \tag{5.5}$$

while if the parent nucleus is even-even, then

$$S_j = 40(2J_0+1)U_j^2 |\sum_{j'} C_{j'} W(2j'Jj; J_0 2) f_{j'j}'|^2. \tag{5.6}$$

In the above two equations U_j and V_j are referred to the odd nucleus and $f_{j'j}'$ is referred to the even-even nucleus. J and J_0 are the spin of the parent and the daughter nucleus, respectively. The correction factor for the difference between Ψ_0 and Ψ_0' is omitted. It should be mentioned that the results (5.5) and (5.6) are only given in crude approximation as the approximate commutation relations (4.18) were used, and the correction term given by (2.17) is omitted. The wave function (4.19) itself is also approximate because the wave functions of the second-excited states with 2 and 0 may be expected to be mixed with the first-excited state and the ground state, respectively, even if the number of the quasi-particles is restricted to 4. It may be very interesting to study the second- or higher-excited states of even-even nuclei through the stripping and pickup process by using improved wave functions.

6. DISCUSSIONS AND COMPARISON WITH EXPERIMENTS

In this section the theoretical predictions for the reactions of Sn isotopes will be compared with the experimental results of Cohen and Price.[11] Later the validity of the approximations involved in the present theory will be discussed. The cross section is given by (1.1), in which the single-particle cross section $\phi(l,Q,\theta)$ is considered first. Cohen and Price[11] obtained the following empirical rule for $\phi(l,Q,\theta)$ from experimental data of known reactions:

$$\phi_{dp}(l,Q,\theta) = F_p(l,\theta)A^{-Q}, \quad \phi_{dt}(l,Q,\theta) = F_t(l,\theta)A^{Q}, \tag{6.1}$$

where $F_p(l,\theta)$ and $F_t(l,\theta)$ are functions of angle and angular momentum transfer, and at a certain angle near 30°, the dependence on l was found to be

$$F_x(l,\theta)/F_x(l+1,\theta) \simeq 2. \tag{6.2}$$

In Eq. (6.1), A is a constant and its value is around 1.18. Q is the Q value of the reaction, measured in Mev.

The spectroscopic factor may be calculated using the results of the preceding sections. The parameters ϵ_j, λ, and Δ are adopted from Kisslinger and Sorensen,[7] from which U_j and V_j are calculated. For the vibrational state, the strength of the Q–Q interaction x is fixed by (4.13) inserting the experimental value of the $\hbar\omega$ for the even-even nucleus. Then the coefficient C_j for the three quasi-particle state in the odd nucleus wave function is obtained from (4.23). Then formulas (5.3)–(5.6) give the spectroscopic factors for the vibrational state.

The theoretical and experimental[11] cross-section ratios for the ground state transitions are listed in Table II. The ratio is taken of the reaction with odd target nucleus to that with even target nucleus. The agreement is fairly good considering the experimental errors and the theoretical uncertainty concerning the single particle energy spectra ϵ_j.[13] Here it may be instructive to present in contrast the interpretation by the simple shell model.[11] The transitions $Sn^{117}(d,p)Sn^{118}$ and $Sn^{118}(d,p)Sn^{119}$ are considered as $(h_{11/2})^2 s_{1/2} \rightarrow (h_{11/2})^4$ and $(h_{11/2})^4 \rightarrow (h_{11/2})^4 s_{1/2}$, respectively. The former transition is forbidden, so the ratios of these two reactions should be zero. The case of $Sn^{119}(d,p)$ is the same. On the other hand for the (d,t) reaction, $Sn^{118}(d,t)Sn^{117}$ is forbidden and the ratio listed in the table should be infinite. But these do not agree with experiment. In our theory the pairing interaction is taken into account and because of the strong configuration mixing resulting from the pairing interaction, transitions like $Sn^{117}(d,p)Sn^{118}$ no longer are forbidden.

For the first-excited state the strength of the Q–Q interaction x is calculated as explained before and its values are 0.129, 0.125, and 0.127 Mev for Sn^{116}, Sn^{118}, and Sn^{120}, respectively. The calculated results are given in Table III together with the experimental data.[11] The agreement is not bad. It is noted that the correction

TABLE II. The ratio of the ground-state cross sections, (even-odd → even-even)/(even-even → even-odd).

Transition	Theoretical	Experimental[11]
$Sn^{117}(d,p)/Sn^{118}(d,p)$	0.41	0.27
$Sn^{119}(d,p)/Sn^{118}(d,p)$	0.51	0.27
$Sn^{117}(d,t)/Sn^{118}(d,t)$	0.83	0.76
$Sn^{119}(d,t)/Sn^{118}(d,t)$	0.63	0.67

TABLE III. The ratio of the first excited vibrational state cross section to the ground-state cross section.

Transition	Theoretical	Experimental[11]
$Sn^{117}(d,p)$	0.33	0.13
$Sn^{119}(d,p)$	0.14	0.13
$Sn^{117}(d,t)$	0.24	0.26
$Sn^{119}(d,t)$	0.36	0.26

term with C_j in (5.3) and (5.4) is not important and its effect to the spectroscopic factor is less than 10% in the case of Sn. The orbital angular momentum l of the captured or stripped neutron is fixed uniquely by the selection rule in the case of Sn isotopes because the spin of the odd isotopes is $\frac{1}{2}$. However in the general case, many values of l are allowed and the theory may predict each contribution.

As we used many approximations in our calculation, it may be important to discuss their validity. The effect of the inaccurate treatment of the nucleon number was already considered in Sec. 3. In the following discussion approximations concerning the vibrational state will be examined.

First of all we used the Tamm-Dancoff method in obtaining the wave function of the vibrational state. However it is known that the method of linearized equations of motion is superior to the present method.[9] One reason why we did not use the better approximation is that a similar approximation has not yet become available for the odd nucleus and therefore the improvement only of the wave function of the even-even nucleus has little meaning. But it is interesting to compare these two approximations with each other and with other approximations.

The method of linearized equations of motion gives the following eigenvalue equation[9]

$$1=\frac{x}{5}\sum\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2(E_{j_1}+E_{j_2})}{(E_{j_1}+E_{j_2})^2-(\hbar\omega)^2}. \tag{6.3}$$

On the other hand, Kisslinger and Sorensen[7] used the cranking model to evaluate the excitation energy of the vibrational state. The excitation energy is given by

$$\hbar\omega=\left[\sum\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2}{E_{j_1}+E_{j_2}}\Big/\sum\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2}{(E_{j_1}+E_{j_2})^3}\right]^{\frac{1}{2}}\times\left[1-\frac{x}{5}\sum\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2}{E_{j_1}+E_{j_2}}\right]^{\frac{1}{2}}. \tag{6.4}$$

The third method we consider is the simplified generator coordinate method used by Ferrell and Visscher.[16] The trial-wave function is constructed by first solving the deformed single-particle potential problem,

$$H_M=H_0-\beta\hbar\omega\textstyle\sum_i v_iY_{20}(\theta_i\varphi_i), \tag{6.5}$$

where H_0 represents the spherical, independent quasiparticle Hamiltonian and the last term represents the deformed part of the independent-particle potential. β is the deformation parameter and r_i, θ_i, and φ_i are the polar coordinates of the ith nucleon. v_i is given by (4.2) with r replaced by r_i. The solution of (6.5) is obtained as a function of β and denoted by $\Psi(\beta)$. Then the wave function for the vibrational state is given by

$$\Psi_{20}=N[(\partial/\partial\beta)\Psi(\beta)]_{\beta=0}, \tag{6.6}$$

with spin 2 and Z component 0. N is the normalization constant. The result is expressed as

$$\Psi_{20}=\sum_{j_1j_2}f_{j_1j_2}A^\dagger(j_1j_220)\Psi_0, \tag{6.7}$$

where $f_{j_1j_2}$ is slightly different from that for the Tamm-Dancoff method, namely

$$f_{j_1j_2}=\frac{(j_1\|Y_2v\|j_2)u_{j_1j_2}}{E_{j_1}+E_{j_2}}\times\left[\sum_{j_1j_2}\left\{\frac{(j_1\|Y_2v\|j_2)u_{j_1j_2}}{E_{j_1}+E_{j_2}}\right\}^2\right]^{-\frac{1}{2}}. \tag{6.8}$$

The excitation energy is obtained as the expectation value of the original Hamiltonian $H=H_0'+H_{Q-Q}$;

$$\hbar\omega=\left\{\sum_{j_1j_2}\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2}{E_{j_1}+E_{j_2}}\Big/\sum_{j_1j_2}\left[\frac{(j_1\|Y_2v\|j_2)u_{j_1j_2}}{E_{j_1}+E_{j_2}}\right]^2\right\}\times\left[1-\frac{x}{10}\sum_{j_1j_2}\frac{(j_1\|Y_2v\|j_2)^2u_{j_1j_2}{}^2}{E_{j_1}+E_{j_2}}\right]. \tag{6.9}$$

Figure 1 shows the excitation energy $\hbar\omega$ as a function of the strength of the Q–Q force, calculated using various of these methods. The calculation was carried through for Sn^{118}, and Kisslinger and Sorensen's U_j and V_j were used again. The experimental value of $\hbar\omega$ for Sn^{118} is indicated in Fig. 1. It is noted that the cranking model gives excellent agreement with the curve obtained for large x by the method of linearized equations of motion. However the Tamm-Dancoff and the simplified generating coordinate methods are poor approximations

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for the strong-coupling case. Therefore an improvement of the wave function by using the method of linearized equations of motion is very desirable, especially for the case of strong coupling. However for the case of the Sn isotopes the results of the Tamm-Dancoff method do not seem so bad.

Next we consider the accuracy of the commutation relations (4.18) which were used in the calculation of the spectroscopic factor. These approximate commutation relations made the calculations simple. For the zero quasi-particle state they are correct, so they are also expected to be a good approximation for states with a small number of quasi-particles. To show this the normalization integrals for the three quasi-particle state and for the second excited state of the even-even nucleus were calculated. They are given by

$$\langle\sum(2\mu' j'm'|JM)Q_{2\mu'}{}^{\dagger}\alpha_{j'm'}{}^{\dagger}\Psi_0 \times|\sum(2\mu jm|JM)Q_{2\mu}{}^{\dagger}\alpha_{jm}{}^{\dagger}\Psi_0\rangle = 1+20\sum_{j'}f_{jj'}{}^{2}(-)^{J-j'}W(j'jjJ;22), \quad (6.10)$$

$$\langle\sum(2\mu_1'2\mu_2'|JM)Q_{2\mu_1'}{}^{\dagger}Q_{2\mu_2'}{}^{\dagger}\Psi_0 \times|\sum(2\mu_12\mu_2|JM)Q_{2\mu_1}{}^{\dagger}Q_{2\mu_2}{}^{\dagger}\Psi_0\rangle = 1-200\sum f_{j_1j_2}f_{j_3j_4}f_{j_1j_3}f_{j_2j_4}X\begin{Bmatrix} j_1 & j_3 & 2 \\ j_2 & j_4 & 2 \\ 2 & 2 & J \end{Bmatrix}, \quad (6.11)$$

where $X(\quad)$ is the Wigner $9-j$ symbol. If the commutation relations (4.17) are used both these normalization constants should be unity. Numerical calculations were carried out for the case of Sn^{118} and the following results are obtained: The three quasi-particle normalization constants are 0.83, 0.99, and 0.99 for $J=1/2$, $j=3/2$; $J=1/2$, $j=5/2$; $J=j=11/2$, respectively. For the second vibrational states the calculation was done only for $J=0$, and the result is 0.70.

In treating the vibrational state the term with the Racah coefficient in the Q–Q interaction (4.3) has not been taken into account, and the interaction has been assumed to take place only among nucleons in the incomplete shell. To examine these two effects it is convenient to use the method of the simplified generating coordinate. An actual calculation was done for Sn^{118} and the contribution to the excitation energy from the term with the Racah coefficient was found to be 5% of the principal contribution. The contribution to the vibrational state wave function from the excitation of two quasi-particles other than the two quasi-particles in the unfilled shell was estimated to be 40% of the total contribution for Sn^{118}. If this effect is taken into account in the calculation of the spectroscopic factor, then the results may be changed. However in this case also the wave function of the odd state should be treated on a similar basis.

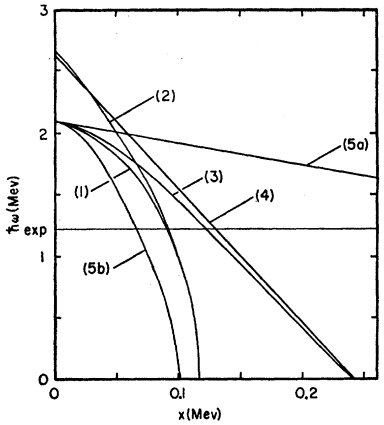

FIG. 1. Excitation energy of the first vibrational state for even-even nuclei as a function of the strength of the Q–Q interaction. The curves are calculated by using the following various approximations for Sn^{118}: (1) method of the linearized equation of motion; (2) cranking model; (3) Tamm-Dancoff method; (4) simplified generator coordinate method; (5a) the first-order perturbation calculation; (5b) the second-order perturbation calculation.

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