Symmetry Properties of Warm Electron Effects in Cubic Semiconductors

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(Received May 12, 1961)

The constant β which describes the field dependence of the mobility μ of warm electrons according to $\mu=\mu_0(1+\beta E^2)$ is found to be anisotropic even for crystals of cubic symmetry. Furthermore, the directions of field strength and current density include an angle the tangent of which is given by $E^2(\gamma_0/2)\sin\varphi\cos\varphi \times(1-3\cos^2\varphi)$, where φ is the angle between the field situated in the (110) plane and the cubic axis, and γ_0 is a constant. For silicon and germanium one finds the following symmetry relations: $\beta_{110}=\beta_{100}-(\gamma_0/2)$, $\beta_{111}=\beta_{100}-(2\gamma_0/3)$, where the subscripts denote the direction of field strength. The theoretical results have been confirmed experimentally with n-type germanium of different impurity content.

THIS paper is concerned with a group-theoretical analysis of the quadratic deviations from Ohm's law in cubic semiconductors, the so-called warm electron problem. Two effects have been observed:

(1) *Longitudinal effect.*[1] This is a variation of carrier mobility according to

$$[\mu_l(E)-\mu_0]/\mu_0=E^2\beta(T,e_1,e_2,e_3), \tag{1}$$

where μ_0 is the zero-field mobility, E the field strength, $\mu_l(E)$ the mobility associated with the current component j_l in field direction, and β a function, for a given sample, of lattice temperature T and the direction cosines e_i of the electric field[2] referring to the cubic axes.

(2) *Transverse effect.*[3] This is the appearance of a current component j_t perpendicular to the direction of field strength:

$$j_t/j_l=E^2\gamma(T,e_1,e_2,e_3). \tag{2}$$

The functions β and γ can be calculated when the distribution function of the carriers in momentum space is known, which is obtained in principle by solving a Boltzmann equation. This meets, however, tremendous difficulties when the real band structure and the various kinds of scattering mechanisms are taken into account. Even the most elaborate treatments available[4] yield β values which agree with experiments only in the range of pure lattice scattering and give, for γ and for β of more impure samples, only the correct order of magnitude.

It seems worthwhile, therefore, to look into the symmetry properties of the problem, since these admit of considerable simplifications of Eqs. (1) and (2). The results of the subsequent calculations which are valid for arbitrary types of band structure, scattering mechanisms, and electron statistics may be summarized as follows:

(1) Longitudinal and transverse effects in cubic semiconductors of crystal class 43, $\bar{4}3m$, and $m3m$ can completely be described, for a given sample, by two functions of lattice temperature only: $\beta_0(T)$ and $\gamma_0(T)$.

(2) The angular dependence of β is given by

$$\beta(T,e_1,e_2,e_3)=\beta_0+\gamma_0(\textstyle\sum_i e_i^4-1), \tag{3}$$

or, in particular cases,

$$\beta_{100}=\beta_0;\quad \beta_{110}=\beta_0-(\gamma_0/2);\quad \beta_{111}=\beta_0-(2\gamma_0/3). \tag{4}$$

(3) The angular dependence of γ turns out to be

$$\gamma(T,e_1,e_2,e_3)=\gamma_0[\textstyle\sum_i e_i^6-(\sum_i e_i^4)^2]^{\frac{1}{2}}. \tag{5}$$

For the special arrangement used by Sasaki *et al.*[3]: $e_1=e_2=(\frac{1}{2})^{\frac{1}{2}}\sin\varphi$, $e_3=\cos\varphi$, we obtain

$$\gamma(\varphi)=(\gamma_0/2)\sin\varphi\cos\varphi(1-3\cos^2\varphi). \tag{6}$$

The functions β_0 and γ_0 must still be calculated from a Boltzmann equation, but the numerical task has been reduced considerably since the field direction need not be varied. Furthermore, the function $\gamma_0(T)$ can be derived from two of the functions β_{ijk} mentioned in Eq. (4). This means that, for example, in the case of n-type Ge only a one-valley model (for β_{100}) and a two-valley model (for β_{110} or β_{111}) need be considered, thus avoiding the full treatment of a four-valley model.

The interconnection of longitudinal and transverse effects as embodied in Eq. (4) has been checked experimentally by direct measurements of β_{100}, β_{110}, β_{111}, and γ for two samples of different impurity content; see Table I.

TABLE I. Warm electron mobility coefficients of n-type germanium at $T=84°$K, in units of 10^{-6} cm^2 v^{-2}. Calculated values are obtained from $\beta_{110}^{\text{meas}}$ and γ_0^{meas} by means of Eq. (4). The accuracy of measurement is about 20% and 5% for the low- and high-resistivity samples, respectively.[a]

N_i (cm^{-3})	$\beta_{110}^{\text{meas}}$	γ_0^{meas}	$\beta_{111}^{\text{meas}}$	$\beta_{111}^{\text{calc}}$	$\beta_{100}^{\text{meas}}$	$\beta_{100}^{\text{calc}}$
1.87×10^{15}	−5.9	7.2	−8.0	−7.1	−3.8[b]	− 2.4
1.25×10^{14}	−64	35	−70	−69.8	−46	−46.5

[a] Gunn[1] finds at $T=78.5°$K for $N_i=1.8\times10^{15}$ and 1.25×10^{14} cm^{-3} that $\beta\approx0\times10^{-6}$ and 80×10^{-6} cm^2 v^{-2}, respectively. Sample orientation is not reported.
[b] In this case the E^2 region is very ill-defined and the value may be regarded as an estimate of the maximum value of $|\beta_{100}|$.

[1] J. B. Gunn, Progress in Semiconductors (John Wiley & Sons, Inc., New York, 1957), Vol. 2, p. 213; J. Phys. Chem. Solids **8**, 239 (1959).
[2] M. I. Nathan, Bull. Am. Phys. Soc. **5**, 194 (1960).
[3] W. Sasaki, M. Shibuya, K. Mizuguchi, and G. M. Hatoyama, J. Phys. Chem. Solids **8**, 250 (1959); S. H. Koenig, Proc. Phys. Soc. (London) **B73**, 959 (1959).
[4] I. Adawi, Phys. Rev. **120**, 118 (1960); J. Yamashita and K. Inoue, J. Phys. Chem. Solids **12**, 1 (1959).

A brief outline of the calculations is as follows. The current vector may be expanded into a power series of the electric field components:

$$j_i=\sigma_0 E[e_i+E^2\sum_{j,k,l} h_{ijkl}e_je_ke_l+\cdots]. \quad (7)$$

The linear term is scalar, and only odd powers occur in the series. Warm electron effects are described by the four-dimensional tensor h_{ijkl}, whose 81 components depend on the nature of the sample and the lattice temperature. The tensor is symmetric with respect to the last three subscripts. Crystal symmetry requires that the tensor (h) is invariant under the transformations of the point group of the lattice. This reduces the number of independent components to three or two for the cubic crystal classes 23, $m3$ or 43, $\bar{4}3m$, and $m3m$, respectively.

The reduction can easily be accomplished by contemplating the conditions which are imposed on the tensor components by the generating elements of the group.[5] Invariance under the point group T corresponding to class 23 reduces the tensor to seven independent, non-necessarily vanishing components[6]:

(1)	(2)	(3)
h_{1111}	h_{1122}	h_{2211}
	h_{1221}	h_{2121}
	h_{1212}	h_{2112}

[5] See, for example, H. Jagodzinski, *Encyclopedia of Physics* (Springer-Verlag, Berlin, Germany, 1955), Vol. 7, p. 1.

[6] A detailed calculation based on tensor transformation rules has been given by E. Poindexter, Am. Mineralogist **40**, 1032 (1955).

The additional symmetry with respect to the last three subscripts, which is a consequence not of crystal symmetry but of the physical meaning of the tensor, makes the constants arranged in the same column equal to each other, thus leaving three independent components:

$$h_{1111},\ h_{1122},\ \text{and}\ h_{2211}.$$

This is the minimum number of constants for the crystal classes 23 and $m3$, since the inversion which is additionally present in the group T_h corresponding to class $m3$ does not imply further reduction.

The cubic classes 43, $\bar{4}3m$, and $m3m$ of higher symmetry contain a fourfold axis which makes the coordinate axes completely equivalent. This leads to $h_{1122}=h_{2211}$. For this case, which applies to silicon and germanium, we define

$$\beta_0=h_{1111};\quad \gamma_0=h_{1111}-3h_{1122}, \quad (9)$$

and obtain, by simplifying Eq. (7),

$$j_i=\sigma_0 E\{e_i+E^2[(\beta_0-\gamma_0)e_i+\gamma_0 e_i^3]\}. \quad (10)$$

From here the calculation of Eqs. (3) to (6) is straightforward.

ACKNOWLEDGMENTS

The author is indebted to Professor D. Polder, Dr. H. J. G. Meyer (Eindhoven), Dr. H. G. Reik (Aachen), and Dr. B. K. Ridley (Salfords) for stimulating discussions, and especially to D. Restorff for invaluable experimental assistance.