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s-Wave Pion-Nucleon Scattering

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The Mandelstam relations for pion-nucleon scattering are used to obtain equations for the s-wave partial wave amplitudes in the two isotopic spin states. The solutions of these equations are investigated in the approximation where only the one-nucleon contributions and the unitarity integral are kept. It is found that there are no solutions of the form N/D without complex zeros, and that this is a consequence of the large size of the one-nucleon terms. A comparison with experiment is made which suggests that the dominant contribution to the $T=\frac{3}{2}$ s-wave amplitude (other than the one-nucleon contribution) comes from a region of the complex energy plane that is outside the physical region for the related processes ($\pi-\pi$ into NN and "crossed" $\pi-N$ scattering). An Appendix is devoted to discussing the available experimental data and they are found to be consistent with a scattering length (δ/k at threshold) of 0.098 ± 0.004 in the $T=\frac{3}{2}$ state.

I. INTRODUCTION

THIS paper may be considered as an extension of the recent work by Frautschi and Walecka[1] investigating p-wave ($J=\frac{3}{2}$) pion-nucleon scattering from the viewpoint of the Mandelstam relations. These authors were able to show that the simplest possible equations for phase-shifts deduced from the Mandelstam relations implied the qualitative results of the static model.[2] In particular it was shown that a $\frac{3}{2}$, $\frac{3}{2}$ resonance could be obtained without the benefit of an empirical cutoff, although the position of this resonance was not in agreement with experiment.

It seems remarkable that the qualitative features of the experimental results are consequences of the simplest possible version of the theory. The p-wave solutions take their essential features from the statement that the partial scattering amplitudes are analytic functions of the total center-of-mass energy except for certain cuts whose discontinuities are known. One set of cuts is derived from the "one-nucleon term,"[3] and the discontinuities merely incorporate the knowledge of the spins, masses, and parities of the interacting particles. The discontinuity across the other cut (the "physical" cut) is prescribed by unitarity. The inclusion of other features implied by the Mandelstam relations (e.g., crossing symmetry) affects the details but not the gross features of Frautschi and Walecka's results.

We shall now proceed to carry out an investigation of the s-wave scattering to see whether a similar situation holds. To put it differently, we shall calculate the s-wave scattering in the approximation where only the one-nucleon term and unitarity are kept in order to obtain a measure of the importance of the processes that are ignored. These are: "crossed" pion-nucleon scattering, two pions annhilating to make a nucleon anti-nucleon pair, and all processes involving more than two simultaneous particles (so-called "anelastic" processes[4]).

Analysis of pion-nucleon scattering experiments suggests that the s-wave phase shifts are small over a considerable range of energy above threshold. The phase shifts for the two isotopic spin states are of opposite sign with the isotopic spin $\frac{3}{2}$ phase shift being negative (see Appendix II). Lowest-order relativistic perturbation theory, or, to use a different language, calculations using only the one-nucleon contribution to the scattering, predict very strong s-wave scattering with negative phase shifts for both isotopic spin states. The one nucleon contribution to the s-wave scattering amplitude is, in fact, much larger than is permitted by unitarity of the scattering matrix and there seems to be little sense in attaching any significance to an estimate made in this way.

The Frautschi and Walecka analysis of p-wave scattering suggests that the disagreement may be a consequence of the particular way in which the one-nucleon contribution has been identified with experiment. The suggestion is that this contribution may, in fact, dominate the s-wave scattering, but it is necessary to take unitarity into account also when calculating the partial-wave scattering amplitude. The manner in which this is to be done is outlined in Sec. II, where it is shown that the inclusion of the unitarity requirement leads to an integral equation for the partial wave amplitude.

At this point we seem to be on the verge of having a well-defined theory for the scattering amplitude. The theory is made unique by imposing the formal requirement that the amplitude not have complex zeros (otherwise the solution would contain undetermined parameters), and it becomes possible to formulate a procedure for obtaining numerical solutions. Section III is devoted to a description of this procedure.

We now discover that we have so overdetermined the system that it has no solutions. The one-nucleon contribution turns out to be too large to be consistent with unitarity and the aforementioned uniqueness

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[1] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

[2] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[3] This is modern parlance for the field-theoretic Born approximation.

[4] This term was introduced by M. Cini and S. Fubini, Ann. Phys. **3**, 352 (1960).

requirement, and we are led to the suspicion that the processes that were ignored must be equally important as the one nucleon terms. It would seem desirable to seek confirmation of this suspicion by making a numerical deduction from experimental data. Section IV is devoted to establishing a basis for performing such a deduction, and the comparison with experiment is discussed in Sec. V. Section VI contains the conclusions.

The notation follows that of reference one and is summarized in Appendix I. Appendix II is an analysis of the s-wave scattering data.

II. s-WAVE EQUATIONS

Following MacDowell[5] we shall work with the "helicity" amplitudes[6] ϕ_l ϕ_l' defined in terms of the partial-wave amplitudes $f_{l\pm}$ by

$$\phi_l=(f_{l+}+f_{l-})S^{-\frac{1}{2}}, \tag{1a}$$

$$\phi_l'=(f_{l+}-f_{l-}). \tag{1b}$$

These amplitudes depend only upon the square of the center-of-mass energy, S. It will be convenient to define a new, dimensionless, energy variable, x, by

$$x+1=S(M+\mu)^{-2}. \tag{2}$$

It is clear that x ranges over all positive values for physical pion-nucleon scattering and is proportional to q^2 for small q^2.

Neglecting for the moment the "one-nucleon" terms, which may be treated exactly, we know that ϕ_l and ϕ_l' are analytic functions of the complex variable, S (and, it follows, x) except for the following cuts[1,5]:

$x>0$, the unitarity cut;

$x<-4M\mu(M+\mu)^{-2}$, the "crossing" cut;

$$\left.\begin{aligned} &x+1=(M-\mu)(M+\mu)^{-1}e^{i\phi},\ 0<\phi<2\pi\\ &x<-1.\end{aligned}\right\}\text{the } \pi\pi\to NN \text{ cut.} \tag{3}$$

It follows immediately that if we keep only the "unitarity" cut we may write

$$\phi_l=B_l+\frac{1}{\pi}\int_0^\infty dx'\,\frac{\mathrm{Im}\phi_l(x')}{x'-x-i}, \tag{4}$$

where B_l is the appropriate projection of the one-nucleon terms.[5] A similar equation may be written for ϕ_l' in terms of the projection B_l'.

We may now combine the ϕ_l and ϕ_l' expressions and set l equal to zero in order to obtain an equation for the s-wave amplitude in terms of the imaginary parts of the $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ partial amplitudes. This is easily seen to be

$$f_s(x)=\mathcal{B}_s(x)+\frac{1}{2\pi}\int_0^\infty\left\{\left[1+\left(\frac{x'+1}{x+1}\right)^{\frac{1}{2}}\right]\mathrm{Im}f_s(x)\right.$$
$$\left.+\left[1-\left(\frac{x'+1}{x+1}\right)^{\frac{1}{2}}\right]\mathrm{Im}f_{p\frac{1}{2}}(x)\right\}\frac{dx'}{x'-x-i\epsilon}, \tag{5}$$

where

$$\mathcal{B}_s=\tfrac{1}{2}[S^{\frac{1}{2}}B_s+B_s']. \tag{6}$$

The contribution of the $p_{\frac{1}{2}}$ amplitude to the s-wave amplitude is readily bounded by setting $\mathrm{Im}fp_{\frac{1}{2}}$ equal to its "resonance" value, $1/q(x)$. The result is negligible when compared with $\mathcal{B}_s$, and the p-wave contribution will, accordingly, be ignored in the remainder of this work. This is actually a minor convenience; if the p wave were important it could be retained in the following formulation and Eq. (5) solved to obtain both the s-wave and p-wave amplitudes.

To simplify the writing of equations in the remainder of this work we will also set

$$\tfrac{1}{2}\{1+[(x+1)/(x'+1)]^{\frac{1}{2}}\},$$

equal to unity. Explicit numerical calculation shows that this approximation has very small influence upon the results.

To conclude this section we record the reduced form of (5) which we propose to solve, and the explicit forms of $\mathcal{B}_s$ for the two possible isotopic spins. These are (using unitarity)

$$f_s(x)=\mathcal{B}_s(x)+\frac{1}{\pi}\int_0^\infty\frac{q(x')dx'}{x'-x-i\epsilon}|f_s(x')|^2, \tag{7}$$

$$\mathcal{B}_s^{\frac{1}{2}}=-g^2(M+\mu)^{-1}$$
$$\times\left\{\frac{3}{4}\left[\frac{1-\beta/\alpha}{x+1}+\frac{1}{(x+1)^{\frac{1}{2}}}\left(1-\frac{\beta^2/\alpha}{\alpha+(x+1)^{\frac{1}{2}}}\right)\right.\right.$$
$$\left.\left.-X(\alpha,\beta)I_0(x)-X(-\alpha,-\beta)I_1(x)\right]\right\}, \tag{8}$$

$$\mathcal{B}_s^{\frac{3}{2}}=-2g^2(M+\mu)^{-1}$$
$$\times[X(\alpha,\beta)I_0(x)+X(-\alpha,-\beta)I_1(x)], \tag{9}$$

with the definitions

$$X(\alpha,\beta)=[(x+1)^{\frac{1}{2}}-\alpha]$$
$$\times\{[(x+1)^{\frac{1}{2}}+\alpha]^2-\beta^2\}/4x(x+4\alpha\beta), \tag{10a}$$

$$I_0(x)=\ln\left\{(x+1)\frac{x+\beta(2\alpha-\beta)}{\alpha^2x+\beta(2\alpha-\beta)}\right\}, \tag{10b}$$

$$I_1(x)=2-\left[2\left(\frac{x+1}{x}\right)\frac{x+\beta(2\alpha-\beta)}{x+4\alpha\beta}\right]I_0(x), \tag{10c}$$

$$\alpha=M/(M+\mu)=1-\beta.$$

[5] W. W. MacDowell, Phys. Rev. **116**, 774 (1959).
[6] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

The coupling constant g^2 is the rationalized (and renormalized) pseudo-scalar coupling constant which we take to have the value 14.5.

III. SOLUTION OF THE INTEGRAL EQUATIONS

It shall now be shown that a particular solution of the nonlinear integral equation (7) may be obtained in terms of the solution of a linear integral equation that will be derived. The following discussion applies explicitly to the amplitude for isotopic spin equal to one-half.

It is clear from Eq. (8) that $\mathcal{B}_s^{\frac{1}{2}}$ has poles in the variable x at points (-1) and $(-1-\alpha)$, as well as certain cuts on the negative real x axis. We suppose that all of these cuts are to the left of the point $(-x_0)$ and that the total discontinuity across the left-hand cuts is $\bar{\alpha}(x)/(x+1)(x+1+\alpha)$. These quantities need not be further specified. Next we dispose of the poles by multiplying through by $(x+1)(x+1+\alpha)$ and defining a new function,

$$h_s^{\frac{1}{2}}(x)=(x+1)(x+1+\alpha)f_s^{\frac{1}{2}}=n(x)/D(x). \quad (11)$$

In the last equation we have introduced the Chew-Mandelstam ansatz[7] that $h(x)$ may be written as a ratio of two functions with specified analytical properties. The conventional assumptions then lead to the relations (the reader is referred to reference 1 for the detailed argument)

$$\text{Im}n(x)=\bar{\alpha}(x)D(x) \text{ for } x<x_0, \quad (12a)$$

$$\text{Im}D(x)=-q(x)n(x)/(x+1)(x+1+\alpha) \text{ for } x>0. \quad (12b)$$

It is an obvious consequence of unitarity and the definition of h_s that in the limit of large x the asymptotic behavior of h is not stronger than x^2. We shall then write "dispersion" relations for n and D under the assumption that n has the same behavior as h_s and that D approaches a constant for large x. The two functions are further defined by their values at the poles. At each of these points we require that n equals the residue of the pole and D is equal to unity. The result is (the $-i\epsilon$ in the singular integrand is understood)

$$n(x)=(x+1)(x+\alpha+1)\left[p(x)+\frac{1}{\pi}\int_{-\infty}^{-x_0}dx' \times\frac{\bar{\alpha}(x')D(x')}{(x'-x)(x'+1)(x'+\alpha+1)}\right], \quad (13a)$$

$$D(x)=1-(x+\alpha+1)\frac{(x+1)}{\pi}\int_0^{\infty}dx' \times\frac{q(x')n(x')}{(x'-x)(x'+1)^2(x'+\alpha+1)^2}. \quad (13b)$$

[7] G. F. Chew and S. Mandelstam Phys. Rev. **119**, 467 (1960).

The newly defined function, $p(x)$, is the pole term (the first term in curly brackets) of Eq. (8).

At this point we reverse the usual procedure[1] and substitute the expression for $D(x)$ into the first of the equations (13). The advantage of this approach becomes apparent when the order of integrations is inverted to give "dispersion" integrals over $\bar{\alpha}(x)$. Such integrals will be recognized immediately as integral representations of the "Born" terms, $\mathcal{B}_s$ (less the pole).

$$\mathcal{B}_s^{\frac{1}{2}}(x)=p(x)+\frac{1}{\pi}\int_{-\infty}^{-x_0}\frac{\bar{\alpha}(x')dx'}{(x'-x)(x'+1)(x'+1+\alpha)}. \quad (14)$$

Use of this equation and a little juggling leads us directly to the results[8]

$$N(x)=\frac{n(x)}{(x+1)(x+\alpha+1)} =\mathcal{B}_s(x)+\frac{1}{\pi}\int_0^{\infty}dx'\frac{q(x')N(x')}{x'-x} \times\left[K_s(x')-\frac{(x+1)(x+1+\alpha)}{(x'+1)(x'+1+\alpha)}K_s(x)\right], \quad (15)$$

$$D(x)=1-\frac{(x+1)(x+1+\alpha)}{\pi} \times\int_0^{\infty}dx'\frac{q(x')N(x')}{(x'-x)(x'+1)(x'+1+\alpha)}, \quad (16)$$

where $K_s(x)$ is the "Born" term less the pole.

Equation (15) and its counterpart for the isotopic spin $\frac{3}{2}$ case were solved numerically on the MURA IBM 704 at Madison. The integral equation was replaced by a finite set of algebraic equations, thus reducing the problem to the inversion of a matrix. It was found convenient to transform the independent variable to

$$\nu=x/(x+1), \quad (17a)$$

in order to work in a finite range. The number of mesh points was varied from 10 to 40 without appreciable modification of the results.

The nature of the numerical solutions is roughly characterized by their values at threshold ($x=0$), the scattering lengths. These turned out to be of the same sign as the Born terms but reduced in magnitude by factors of 4 and 7 (for the two different isotopic spins). Of considerably greater importance, however, was the discovery that the N/D solutions, when substituted back into Eq. (7), did not satisfy that equation.

[8] Dr. T. W. B. Kibble has commented upon (private communication) the resemblance of these equations to the determinantal method of Marshall Baker, Ann. of Phys. (N. Y.) 4, 271 (1958). The essential difference lies in the treatment of the denominator function.

IV. DISCUSSION OF THE "SOLUTIONS"

The source of difficulty is probably well understood in the light of the analysis by Castillejo, Dalitz, and Dyson[9] of a comparable, although mathematically more complaisant model. The key point is the assumption, implied in the paragraph preceding Eq. (12), that $f(s)$ is devoid of complex zeros. In the discussion by Castillejo *et al.*, it is shown that this assumption has the consequence that there is a critical value of the coupling constant; when this is exceeded solutions (devoid of complex zeros) of equations such as Eq. (7) no longer exist.

Some additional insight into the matter may be gained from the study of a trivially soluble model that was recently described by Zachariasen.[10] In this case we find that the critical coupling constant is associated with the appearance of a bound state or, depending upon the sign associated with the one-nucleon pole term, a resonance at infinite energy. It is not surprising that the critical coupling constant is just equal to the radius of convergence of the perturbation solution (in powers of the coupling constant). Confirmation for the notion that the Zachariasen model analysis is applicable to the case in hand is derived from the observation that the numerical solutions mentioned in the previous section actually do possess resonances at very high energy. In addition, it is found that if the coupling constant is made sufficiently small, then the N/D solutions satisfy Eq. (7) (and the high-energy resonances disappear).

If the coupling constant is very small then the solutions of Eqs. (15) and (16) are well approximated by the "damping approximation":

$$q^{-1}\tan\delta = \mathcal{B}_s, \tag{18}$$

and this circumstance will turn out to be very useful to us. This comes about as a consequence of recognizing that the identification of $\mathcal{B}_s(x)$ with the one-nucleon projection in the derivation of Eqs. (15) and (16) has only been a convention. Explicitly stated, if $\mathcal{B}_s$ were the exact s-wave projection of the contribution from *all* the singularities to the left of the imaginary x axis, then the *form* of Eqs. (15) and (16) would be unchanged. Further, inasmuch as the quantity $\tan\delta$ is known to be small for energies appropriate to a partial-wave decomposition of the scattering (say 0–300 Mev), we conclude that the approximation (18) is applicable (using the new identification of $\mathcal{B}_s$) to the experimentally determined s-wave scattering amplitudes. Consequently, we have available to us an experimental determination of the left-hand contributions from which the one-nucleon contribution may be separated. The energy dependence of the remaining part will then provide us with a clue as to its source. If we are fortunate, we will then be able to assess the relative importance of the pion-pion and crossed pion-nucleon scattering contributions to the s-wave scattering.

V. DEDUCTIONS FROM EXPERIMENT

The comparison technique described at the end of the last section makes use of the experimentally determined s-wave amplitudes. Appendix II contains a brief review of the available information in the region 0–300 Mev and it is concluded there that the two s-wave amplitudes are consistent with the relations:

$$\sin 2\delta_3 = -2q[W/(M+1)](0.098+0.068Z), \tag{19a}$$

$$\sin 2\delta_1 = +2q[W/(M+1)] \times (0.139-0.084Z+0.064Z^2), \tag{19b}$$

in terms of the variable

$$Z = W-(M+1).$$

The meson mass, in the previous equations has been set equal to unity. It is to be emphasized that we have very little confidence in the result obtained for δ_1, and any conclusions based upon Eq. (19b) are to be regarded as tentative. For further discussion of this point see Appendix II.

From Eqs. (19) we now find the experimental quantities $(\tan\delta)/q$ for the two possible isotopic spins and subtract from these the corresponding one-nucleon contributions. We denote the difference by

$$\Delta = (\tan\delta/q) - \mathcal{B}_s, \tag{20}$$

and approximate Δ by a single pole on the negative x axis. The location of the pole is then determined by the ratio of value to slope, taken at threshold $(W=M+\mu)$ of the quantity Δ. We expect, then, that the position of the pole, found in this way, will tell us which portion of the left-hand region (and, in consequence, which physical process) makes the dominant contribution to the scattering. In particular we will learn whether the technique of approximating the left-hand cuts by a resonance in a particular state[1] is likely to prove useful in this problem.

The arithmetic is straightforward and we merely quote the results. If $S_0{}^T$ is the position of the pole in the s plane, we find

$$S_0{}^{\frac{1}{2}} \approx +4\mu^2, \tag{21a}$$

$$S_0{}^{\frac{3}{2}} \approx -1.8(M+\mu)^2. \tag{21b}$$

The second result is extremely noteworthy for it implies that the energy dependence of the s-wave amplitude depends upon singularities in a region that cannot be described by a partial-wave expansion of any physical process.

[9] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956). The amplitude $f(x)$ of the present work does not satisfy the "generalized R-function" conditions of the CDD analysis which is why our understanding of the difficulties in the s-wave problem is described as "qualitative." The reader is also referred to D. B. Fairlie and J. C. Polkinghorne, Nuovo cimento **8**, 345 (1958).

[10] F. Zachariasen, Phys. Rev. **121**, 1851 (1961).

VI. CONCLUSIONS

As stated in the Introduction, the objective of the work described here has been to examine the utility of a very special approximation in s-wave pion-nucleon scattering. The original hope that led to this study was that the coupling of the large one-nucleon contribution with the unitarity condition would lead to sufficient damping so that the predicted scattering amplitude would be small, as required by experiment. What we seem to have found, however, is that the one-nucleon contribution is, in some sense, "too large" to be consistent with the unitarity condition—at least when this condition is joined to the usual uniqueness requirement that there be no complex zeros. At this point we are able to conclude that the missing terms in our approximation are just as important as the one that was kept—the one-nucleon contribution—and we look for further information as to the nature of the neglected terms.

The importance of the latter investigation has to do with the technique by which the approximation for the scattering amplitude is to be improved. We recall that we have omitted the two-pion and crossed pion-nucleon scattering processes and we must now ask how these are to be put back into the theory. One popular procedure is to single out resonant states, in particular the $(\frac{3}{2},\frac{3}{2})$ pion-nucleon resonance and a conjectured (1,1) pion-pion resonance, and insert these as poles at appropriate positions on the left-hand cuts.[1,11] The efficacy of this procedure may be examined *a priori* if we note that only limited portions of the cuts (for negative x) of Eq. (3) have discontinuities that are describable in terms of angular momentum decompositions, or, in fact, correspond to physical regions for the two omitted processes. One finds that the additional poles would have to be placed within the regions (returning now to W^2 as the variable)

$$0<W^2<(M-\mu)^2,\ \text{crossed}\ \pi-N\ \text{scattering},$$
$$W^2=(M^2-\mu^2)e^{i\phi},\quad -\phi_0\leqslant\phi\leqslant\phi_0,\quad \pi\pi\rightarrow N\bar{N},\quad (22)$$

where

$$\phi_0\approx 66°.$$

On the other hand, we have deduced from the isotopic spin $\frac{3}{2}$ experimental data, which is the most believable, that if the omitted terms are to be approximated by a *single* pole, that pole must lie far to the left of the origin of the W^2 plane. A pole at such a location cannot correspond to a scattering resonance in any simple sense. It is easy to see that the conclusion only applies to a single pole; a pair of poles whose residues are of opposite sign and that are located in the regions given by Eq. (22) could give the required result. One suspects, however, that the spectral functions that are being approximated by the poles are not sufficiently wildly oscillating functions to give this sort of an effect. The conclusion, then, is that the s-wave scattering is dominated by the "nonphysical" left-hand cuts and that the partial-wave analysis that we have used is not a promising approach for the understanding of low-energy scattering.

[11] This procedure has been applied to the s-wave problem by W. Frazer and M. Goldberger (unpublished). I am indebted to both for valuable discussions of theirs and related work.

ACKNOWLEDGMENTS

The list of those to whom I am indebted for suggestions and comments is unfortunately too long to append here. I am particularly grateful to Professor A. Salam for the hospitality of the Department of Mathematics, Imperial College, where this work was begun in 1959.

APPENDIX I

We summarize here the notation and conventions.

Let p, p' and k, k' be, respectively, the initial and final nucleon and pion momenta, and let M and μ denote the nucleon and pion masses. Choose the metric with

$$p^2=\mathbf{p}^2-p_0{}^2=-M^2.$$

We then denote the square of the total energy in the center-of-momentum system by

$$S=-(p+k)^2,$$

and the square of the spatial momentum of either particle in the same system by

$$q^2=|\mathbf{p}|^2=|\mathbf{k}|^2.$$

The partial wave amplitude for orbital angular momentum, l, and total angular momentum $J=l\pm\frac{1}{2}$ is written

$$f_{l\pm}=\exp(i\delta_{l\pm})\sin\delta_{l\pm}/q.$$

We use a system of units with $\hbar=c=1$.

APPENDIX II

The positive pion-proton s-wave scattering data in the range 0–170 Mev has been analyzed by Hamilton and Woolcock.[12] They find a scattering length (without the "inner" Coulomb correction).

$$f_s{}^{\frac{3}{2}}(0)=-0.087\pm0.005,$$

by extrapolating the quantity

$$F(W)=\text{Re}\left[\frac{W}{M+\mu}\right]f(W)=\frac{W}{M+\mu}\left(\frac{\sin2\delta}{2q}\right).$$

Hamilton's value is in disagreement with the previously accepted scattering length deduced from the low-energy data which in turn is consistent with the assumption that δ/q is constant in the range of 0–40 Mev meson laboratory kinetic energy. Since the time of the previous

[12] J. Hamilton and W. S. Woolcock, Phys. Rev. **118**, 291 (1960).

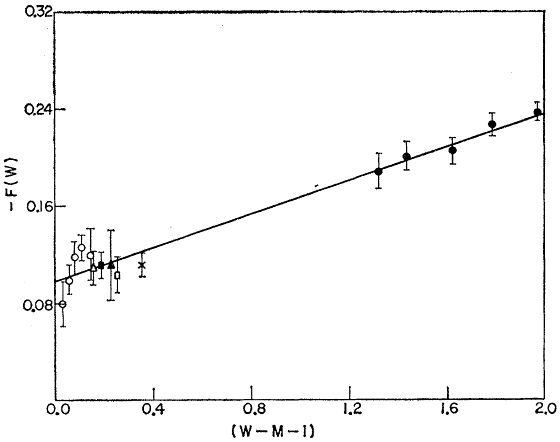

FIG. 1. S-wave pion nucleon scattering in the $T=\frac{3}{2}$ state. The quantities plotted are defined in Appendix II, and the solid line is a best least-mean-squares fit to the data. The sources of the plotted data are: ○—[G. E. Fischer and J. W. Jenkins, Phys. Rev. **116**, 749 (1959)]; Δ—[D. Miller and J. Ring, Phys. Rev. **117**, 582 (1960)]; ■—[W. B. Johnson and M. Camac, quoted in reference 17]; □—[S. W. Barnes, B. Rose, G. Giacomelli, J. Ring, K. Miyahe, and K. Kinsey, Phys. Rev. **117**, 226 (1960)]; ▲—[A. M. Sachs, H. Winick, and B. A. Wooten, Phys. Rev. **109**, 1733 (1958)]; ●—[Reported by B. Pontecorvo, Ninth Annual International Conference on High-energy Physics, Kiev, 1959 (unpublished)].

analyses additional data in the 300-Mev region has become available. If these points are considered along with the low-energy data and the points at 98, 150, and 170 Mev are ignored,[13] an excellent linear fit is obtained (see Fig. 1). The χ^2 criterion for this fit is approximately unity and the result is

$$-F_{\frac{3}{2}}(W) = (0.098\pm0.004)+(0.068\pm0.003)(W-M-1),$$

in units of the reciprocal π-meson mass. The threshold value of 0.098 is not very different from the previously accepted value[12] of 0.110 ± 0.004.

The s-wave scattering data for the isotopic spin one-half state present a much less pleasing appearance. The experiments that were ignored in the isotopic spin $\frac{3}{2}$ analysis were kept in this case on the theory that the existing scatter of data was so bad that nothing could make it worse. The best fit was the quadratic (a linear fit would have been obtained if the 98, 150, and 170 Mev points had been ignored)

$$F_{\frac{1}{2}}(W) = (0.139\pm0.031)-(0.084\pm0.069)(W-M-1) + (0.064\pm0.026)(W-M-1)^2.$$

The large errors and the χ^2 criterion of about 7 for this fit suggest that a rather careful weeding of the experiments is in order.

[13] A phase-shift analysis of the 98-Mev scattering quoted in reference 12 does not appear to have been published. There seems to be some question concerning the phase shift solutions at 150 and 170 Mev (see reference 12) although the phase shifts used by Hamilton at these energies seem consistent with the energy dependence of Appendix II.