cm^{-2} sec^{-1} above 25 kev. Their average energy is also somewhat lower than the subcascade photons discussed here thus making a small crystal a desirable choice. The experimental results concerning the cascade photons could be improved by making the size of the scintillation crystal larger. A reduction in the size and uncertainty of the photoconversion efficiency correction and improvement in the statistical accuracy would improve knowledge of the flux versus depth curve including a better determination of the intensity maximum. Knowledge of the angular distribution of the photons would provide more accurate absolute flux values than those reported here. That measurement, however, would require a much more refined experiment than the present one which involves only a single unshielded crystal.

(6) Omnidirectional particle fluxes as determined from a thin-wall Geiger tube and the vertical directional fluxes from a thin-wall counter telescope have been reported here. These were determined during August of 1959, a time when solar activity was still generally high and the primary cosmic-ray flux depressed. The last Forbush decrease before the measurements occurred on July 18, and for about 5 days before the observation the Deep River neutron monitor was quite level. The flux values determined from the two counters as a function of atmospheric depth appear in Fig. 1.

(7) Although the results here are in general agreement with the rocket observations of Northrop and Hostetler, they do not seem to bear out results also obtained in rockets by Kuperian and Friedman[4] during 1955 at a geomagnetic latitude of 41°. In particular these workers find a well defined maximum in the differential photon energy spectrum at 100 kev when the rockoon system is floating at a depth of 35 g cm^{-2}. At this depth the present results do not reflect a corresponding marked flattening of the integral spectrum.

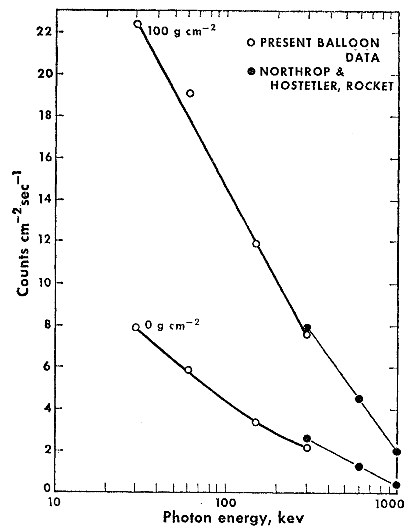

FIG. 7. Comparison of the present balloon results with rocket observations obtained by other workers.

[4] J. E. Kuperian, Jr., and H. Friedman, IGY Rocket Report Series, No. 1, 201 (1959).

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Conversion of Muonium into Antimuonium*

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A detailed analysis is made of the possible conversion of muonium into antimuonium in various environments. An assumed $\bar{\mu}e\bar{\mu}e$ weak interaction of the usual form and strength gives a probability of 2.5×10^{-5} in vacuum, even in the presence of reasonable external electric fields. In a solid the probability is less by at least 10, and probably 20, orders of magnitude. In an inert gas the probability is roughly to be divided by the numbers of collisions during a muon lifetime, and hence is quite small unless the pressure at room temperature is less than about 10^{-4} atm. Lowering the temperature does not help. A possible experiment is suggested.

I. INTRODUCTION

IT has been suggested[1,2] that muonium may be able to turn into antimuonium spontaneously. A specific example[3] of an interaction with this effect is

$$H=C\bar{\Psi}_\mu\gamma_\lambda(1+\gamma_5)\Psi_e\bar{\Psi}_\mu\gamma^\lambda(1+\gamma_5)\Psi_e, \tag{1}$$

which would yield a matrix element for conversion of $M(\equiv\mu^+e^-)$ into $\bar{M}(\equiv\mu^-e^+)$ equal to $\langle\bar{M}|H|M\rangle=\delta/2$,

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† Alfred P. Sloan Foundation Fellow.

[1] B. Pontecorvo, Zhur. Eksp. i Teoret. Fiz. 33, 549 (1957).

[2] G. Feinberg and S. Weinberg, Phys. Rev. Letters 6, 381 (1961).

[3] S. L. Glashow [Phys. Rev. Letters 6, 196 (1961)] has remarked that only the charge-conjugation-invariant part of H can contribute to $\langle\bar{M}|H|M\rangle$, so that interactions may be constructed which forbid $M\to\bar{M}$ while allowing clashing beam reactions like $e^-+e^-\to\mu^-+\mu^-$. Such interactions have the property of conserving parity, if the relative $e-\mu$ parity is chosen imaginary; they allow $M\to\bar{M}$ if M has odd l, and $\bar{M}$ even l, or vice versa. The possibility of essentially imaginary relative parities can only arise when some quantum number (here, muon number) is multiplicatively conserved, as emphasized by G. Feinberg and S. Weinberg, Nuovo cimento 14, 571 (1959).

where

$$\delta = 16C/\pi a^3 = 2.1\times 10^{-12} \text{ ev}. \tag{2}$$

[This applies in both the $F=1$ and $F=0$ 1S hyperfine states. Here a is the Bohr radius $(m_e e^2)^{-1}$, and C is taken equal to the usual β-decay coupling constant C_V.] We shall discuss the $M\rightarrow\bar{M}$ charge-exchange process in detail, with emphasis on effects due to the environment of the muonium; these effects are important because δ is so small.

II. MUONIUM IN VACUUM

First we consider the case of muonium in a vacuum, in the presence of static external electromagnetic fields which break the original $M-\bar{M}$ degeneracy by an amount Δ. The true energy eigenstates are then not $M,\bar{M}$ but instead:

$$\begin{aligned} |M_1\rangle &= [2W(W-\Delta)]^{-\frac{1}{2}}(\delta|M\rangle+(W-\Delta)|\bar{M}\rangle),\\ |M_2\rangle &= [2W(W+\Delta)]^{-\frac{1}{2}}(-\delta|M\rangle+(W+\Delta)|\bar{M}\rangle),\end{aligned} \tag{3}$$

with energies differing by an amount

$$E_1-E_2=W\equiv(|\delta|^2+\Delta^2)^{\frac{1}{2}}. \tag{4}$$

If at $t=0$ the system is pure muonium, then at time t it will have developed an antimuonium component equal to

$$\begin{aligned}\langle\bar{M}|\Psi(t)\rangle &= \langle\bar{M}|M_1\rangle\langle M_1|M\rangle e^{-iE_1t} + \langle\bar{M}|M_2\rangle\langle M_2|M\rangle e^{-iE_2t}\\ &= ie^{-\frac{1}{2}(E_1+E_2)t}(\delta^*/2W)\sin Wt\end{aligned} \tag{5}$$

and the probability that the muon decays as μ^- rather than μ^+ is

$$P(\bar{M})=\int_0^\infty \lambda e^{-\lambda t}|\langle\bar{M}|\Psi(t)\rangle|^2=\frac{|\delta|^2}{2(|\delta|^2+\Delta^2+\lambda^2)}, \tag{6}$$

where $\lambda=3\times10^{-10}$ ev is the muon decay rate.

In the absence of external fields,

$$P(\bar{M})\simeq(|\delta|^2/2\lambda^2)=2.5\times10^{-5}, \tag{7}$$

a probability that does not seem too low for experimental observation. The signature of an $M\rightarrow\bar{M}$ charge exchange is the emission of a fast e^- with an e^+ left behind, instead of the reverse. The probability that the e^+ emitted when the μ^+ decays in ordinary muonium should give up ≥10 Mev of its energy to the bound e^- may be estimated semiclassically by folding the Bhabha cross section into the Michel spectrum shape, and comes out $\simeq10^{-10}$; hence a fast e^- is a sure sign that $\bar{M}$ was formed.

Clearly external fields will quench the $M\rightarrow\bar{M}$ conversion unless

$$\Delta\ll W\simeq\lambda=3\times10^{-10} \text{ ev}. \tag{8}$$

It may be surprising that charge exchange is unquenched for $\Delta\geq|\delta|$ as long as $\Delta\ll\lambda$; the reason is that as Δ increases beyond δ, M_1 approaches M and M_2 approaches $\bar{M}$, but also the interference "beat" between M_1 and M_2 speeds up.

In estimating Δ it is important to note that a constant electric field gives no contribution. This is because the only scalars that the energy may depend upon in a state with angular momentum $\mathbf{J}$ are $\mathbf{E}^2$ and $(\mathbf{E}\cdot\mathbf{J})^2$ (since $\mathbf{E}\cdot\mathbf{J}$ is pseudoscalar); both scalars are even in $\mathbf{E}$ so that they are the same for M and $\bar{M}$. In the presence of an additional constant magnetic field $\mathbf{H}$, Δ will still be zero for states with $\mathbf{H}\cdot\mathbf{J}=0$, since the energy can at most depend upon $\mathbf{E}^2$, $(\mathbf{E}\cdot\mathbf{J})^2$, $(\mathbf{E}\cdot\mathbf{H})^2$, $\mathbf{H}^2$, and $[\mathbf{J}\cdot(\mathbf{E}\times\mathbf{H})]^2$, all of which are even in $\mathbf{E}$ and $\mathbf{H}$. The 1S M and $\bar{M}$ states with $F=1$, $F_z=\pm1$ are split by a constant field in the z direction by an amount $|\Delta|\simeq eH/2m_e$; thus, in order to avoid quenching in these states, H_z must be kept below about 0.01 gauss.

In an inhomogeneous electric field, Δ will contain only odd powers of $\mathbf{E}$, and since M and $\bar{M}$ are neutral the lowest term will be of order E^3; to lowest order in gradients this term is[4]

$$\Delta=(213/8)e^{-4}a^6\mathbf{E}\cdot\nabla(\mathbf{E}^2). \tag{9}$$

For fields varying in distances of order 1 mm, this gives $\Delta\leq\lambda$ for $E\leq10^8$ v/cm, and so we may safely ignore Δ for any reasonable external field.

III. MUONIUM IN CRYSTALS

If muonium is trapped in an ionic crystal, Δ [as given by (9)] will be of the order of several ev, lowering $P(\bar{M})$ by a factor $\sim10^{-20}$. However, in substances composed of neutral atoms, such as molecular crystals of the inert gases, this term in Δ will be many orders of magnitude smaller. In such substances the main contribution to Δ will arise from the overlap of the muonium and crystal atom wave functions, in the same way that the repulsive force between crystal atoms arises from the overlap of their wave functions. If we write the energy of M in a crystal as $+E_M=-E_V-E_E+E_X$, where the three terms represent energies due to Van der Waals forces, ordinary electrostatic forces (due to overlap), and exchange forces, then the energy of $\bar{M}$ is $-E_V+E_E$, so Δ is $-2E_E+E_X$. If for illustration we assume that these energies have the same ratios as for the neon-neon interaction,[5] then $E_X\simeq4E_E$ and $E_X-E_E\simeq\frac{1}{2}E_V$ so $\Delta\simeq\frac{2}{3}|E_M|$. In any case it seems reasonable to estimate Δ as being not less than the crystal binding energy *per electron*, which for neon (where it is particularly low) is 0.40 kcal/mole $=1.7\times10^{-3}$ ev/electron. Even here the $M\rightarrow\bar{M}$ conversion is quenched by a factor $<10^{-14}$.

Several new features enter for muonium trapped in other materials. For example, in semiconductors[6] the

[4] The numerical factor here was computed together with C. Schwartz, using a method developed by him in Ann. Phys. **2**, 156 (1959).

[5] V. Deitz, J. Franklin Inst. **219**, 459 (1935). W. E. Bleick and J. E. Mayer, J. Chem. Phys. **2**, 252 (1934).

[6] G. Feher, R. Prepost, and A. M. Sachs, Phys. Rev. Letters **5**, 515 (1960).

$M \to \bar{M}$ conversion is stopped more often by $\mu - e$ system being broken up than by muon decay. A generalization of the argument leading to (6) gives in this case

$$P(\bar{M}) \leq |\delta|/4\lambda\Delta = 2.5\times 10^{-5}(\lambda/2\Delta), \quad (10)$$

which is larger than before but still much too small. (This upper bound is attained only if every $\bar{M}$ breakup leads to an observed μ^- decay or absorption, and if every M breakup leads to M being formed again, and if the average breakup rate of M and $\bar{M}$ is equal to Δ.) Another effect which slows down the $M \to \bar{M}$ process arises in materials with a large dielectric constant ϵ; a is increased[7] by a factor ϵ, decreasing δ by ϵ^{-3} and also increasing Δ. An effect which could, in principle, increase the chance of seeing antimuonium is the direct *decay* of M into $\bar{M}$ or free μ^-, e^+. Since the energy, Δ, available is so tiny, this process is extremely slow compared to ordinary muon decay, for any coupling C comparable to the weak interactions. In fact, it will also be slow compared to $M \to \nu_e + \bar{\nu}_\mu$ whose rate is $\sim 10^{-9}$ that of μ decay. The reason we are so much better off looking for a "Gell-Mann–Pais" effect is that δ^2 goes as $C_V{}^2$ while λ^2 goes as $C_V{}^4$, as pointed out by Pontecorvo.[1]

One may also ask what happens if we try to *drive* muonium into antimuonium by adding a time-varying external contribution to Δ. It can easily be shown that we can never do better in this way than with $\Delta = 0$. If Δ has a constant part Δ_0 which we try to counteract with a varying part $\Delta_1 \sin\omega t$, and if $|\Delta_0 - n\omega| \ll \omega$ for some integer n, then

$$P(\bar{M}) \simeq \frac{|\delta|^2}{2[\lambda^2 + (\Delta_0 - n\omega)^2]} J_n{}^2(\Delta_1/\omega). \quad (11)$$

It is probably impossible to find any practicable values of n, ω, and Δ_1 with which we could eliminate the quenching due to a Δ_0 of order $10^{-3}-10$ ev.

Evidently it is hopeless to look for $M \to \bar{M}$ conversion in a solid or liquid, if δ is as small as estimated in (1). Since δ could conceivably be much larger, it might be worthwhile to check whether $M \to \bar{M}$ is actually absent in a molecular crystal.

IV. MUONIUM IN GASES

In treating the $M \to \bar{M}$ process in a gas we shall assume that the muonium system is scattered incoherently by the gas molecules, except of course for the coherent forward scattering responsible for the index of refraction. However, we do not want to assume that in general the muonium simply moves classically from molecule to molecule. In this situation it seems essential to use a statistical matrix treatment.

Suppose that we refer to the sequence of elastic scatterings up to time t as a "history" H, with probability $P(H)$. [The sum of the $P(H)$ is the probability that a decay or inelastic collision has not yet occurred, and hence vanishes as $t \to \infty$.] Each history gives rise to a 2-dimensional state vector $u(H)$ with components $u_1 = \langle M|\Psi\rangle$ and $u_2 = \langle \bar{M}|\Psi\rangle$, normalized in the sense that

$$\|u\|^2 = |u_1|^2 + |u_2|^2 = 1. \quad (12)$$

The statistical matrix $\rho(t)$ is defined as

$$\rho(t) = \textstyle\sum_H P(H) u(H) u^\dagger(H). \quad (13)$$

At time $t' = t + dt$, the history H' might consist of either:

(i) A history H followed by elastic scattering through an angle θ, giving

$$u(H') = F(\theta)u(H)/\|F(\theta)u(H)\|, \quad (14)$$

$$P(H') = \|F(\theta)u(H)\|^2 P(H) nv dt, \quad (15)$$

where $F(\theta)$ is the matrix

$$F(\theta) = \begin{pmatrix} f(\theta) & 0 \\ 0 & \bar{f}(\theta) \end{pmatrix}, \quad (16)$$

and n is the number density of gas molecules, v is the muonium velocity (assumed fixed), and f and $\bar{f}$ are the M and $\bar{M}$ elastic scattering amplitudes.

(ii) A history H followed by no decay or collisions, except for the unavoidable coherent forward scattering. This gives

$$u(H') = (1 + A dt) u(H) / \|(1 + A dt) u(H)\|, \quad (17)$$

$$P(H') = 1 - u(H)^\dagger B u(H) dt, \quad (18)$$

where

$$A = \begin{bmatrix} \frac{2\pi i n v}{k} f(0) - iE_0 - \frac{\lambda}{2} & -i\frac{\delta}{2} \\ -i\frac{\delta^*}{2} & \frac{2\pi i n v}{k}\bar{f}(0) - i\bar{E}_0 - \frac{\lambda}{2} \end{bmatrix}, \quad (19)$$

$$B = \begin{bmatrix} \omega_c + \lambda & 0 \\ 0 & \bar{\omega}_c + \lambda \end{bmatrix}. \quad (20)$$

Here E_0 and $\bar{E}_0$ are the M and $\bar{M}$ energies between collisions, $k \cong m_\mu v$, and ω_c and $\bar{\omega}_c$ are the total (elastic plus inelastic) M and $\bar{M}$ collision rates. It follows from the optical theorem that $A + A^\dagger = -B$, so that

$$P(H') = \|(1 + A dt) u(H)\|^2. \quad (21)$$

Now at time t' the statistical matrix is

$$\begin{aligned} \rho(t') &= \textstyle\sum_{H'} P(H') u(H') u(H')^\dagger \\ &= \textstyle\sum_H P_H \Big\{ nv dt \int F(\theta) u(H) u(H)^\dagger F^\dagger(\theta) d\Omega \\ &\qquad + (1 + A dt) u(H) u(H)^\dagger (1 + A dt)^\dagger \Big\} \\ &= hv dt \int F(\theta) \rho(t) F^\dagger(\theta) d\Omega \\ &\qquad + (1 + A dt) \rho(t) (1 + A dt)^\dagger, \end{aligned} \quad (22)$$

[7] G. Feher, Phys. Rev. **114**, 1219 (1959).

and hence, finally,

$$d\rho/dt=A\rho+\rho A^\dagger+nv\int F(\theta)\rho F^\dagger(\theta)d\Omega. \qquad (23)$$

We have derived here four coupled linear differential equations for the four components of $\rho(t)$. It might be necessary to go through the straightforward but tedious task of solving them if (as in an experiment with gated counters) it were necessary to know the time dependence of $\rho(t)$. We shall only solve for $P(\bar{M})$.

It will be assumed that an $M\rightarrow\bar{M}$ conversion is registered if either the muon decays as a μ^- or an inelastic collision breaks up the μe system into a μ^- and e^+, the μ^- subsequently decaying or being absorbed. The rate for either occurrence is $\bar{\omega}=\bar{\omega}_I+\lambda$, where $\bar{\omega}_I$ is the rate for inelastic $\bar{M}$ collisions. (The corresponding rates for M are ω_I and $\omega=\omega_I+\lambda$.) Hence

$$P(\bar{M})=\bar{\omega}\langle\bar{M}|I|\bar{M}\rangle, \qquad (24)$$

where

$$I=\int_0^\infty \rho(t)dt. \qquad (25)$$

If we integrate (23) from 0 to ∞, and use the fact that $\rho(\infty)=0$, we obtain

$$\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}=\rho(0)=AI+IA^\dagger+nv\int F(\theta)IF^\dagger(\theta)d\Omega, \quad (26)$$

and solving these four linear algebraic equations we have

$$P(\bar{M})=\frac{|\delta|^2\Lambda}{2\omega[\Lambda^2+\Delta^2+(\Lambda^2|\delta|^2/\omega\bar{\omega})]} \qquad (27)$$

$$\simeq\frac{|\delta|^2\Lambda}{2\omega(\Lambda^2+\Delta^2)} \qquad (28)$$

where now

$$\Lambda=\Lambda_0-\mathrm{Re}\mathcal{E}, \qquad (29)$$

$$\Delta=\Delta_0-(2\pi nv/k)\ \mathrm{Re}[f(0)-\bar{f}(0)]-\mathrm{Im}\mathcal{E}. \qquad (30)$$

Also

$$\Delta_0=E_0-\bar{E}_0 \qquad (31)$$

is the $M-\bar{M}$ energy split between collisions,

$$\Lambda_0=\lambda+\tfrac{1}{2}(\omega_c+\bar{\omega}_c) \qquad (32)$$

is the average rate of collisions or decay for M and $\bar{M}$, and

$$\mathcal{E}=nv\int f(\theta)\bar{f}^*(\theta)d\Omega. \qquad (33)$$

The parameter $\mathcal{E}$ represents the degree of coherence between the phase of the elastically scattered M and $\bar{M}$. We always have

$$|\mathcal{E}|\leq\tfrac{1}{2}(\omega_E+\bar{\omega}_E)\leq\Lambda_0, \qquad (34)$$

where ω_E and $\bar{\omega}_E$ are the elastic collision rates:

$$\omega_E=nv\int|f(\theta)|^2d\Omega=\omega_c+\lambda-\omega,$$
$$\bar{\omega}_E=nv\int|\bar{f}(\theta)|^2d\Omega=\bar{\omega}_c+\lambda-\bar{\omega}. \qquad (35)$$

The terms in Δ involving $f(0)$ and $\bar{f}(0)$ represent the fact that a difference in index of refraction can act like a mass splitting.

We will examine the behavior of $P(\bar{M})$ in both the high- and low-temperature regions, defined by the conditions that $kR\gg1$ and $kR\ll1$, respectively, where R is an interaction range of the order of 10^{-8} cm. At room temperature $kR\simeq1$ for thermalized muonium.

For $kR\gg1$, many partial waves contribute to $f(\theta)$ and $\bar{f}(\theta)$. Since these may partially cancel in $\mathcal{E}$ but not in ω_E or $\bar{\omega}_E$, we expect that inequality (34) becomes $|\mathcal{E}|\ll\Lambda_0$, so that $\Lambda\simeq\Lambda_0$. Furthermore,

$$\frac{2\pi nv}{k}\ \mathrm{Re}f(0)=\sum_l\frac{\pi nv}{k^2}(2l+1)\ \mathrm{Im}[e^{2i\delta_l}]. \qquad (36)$$

If every term in the sum had its maximum value (up to $l\simeq kR$), this sum would be of order $\pi nvR^2\sim\omega_c$. Since there will be many cancellations, we expect that

$$(2\pi nv/k)|\mathrm{Re}f(0)|\ll\omega_c,$$

and likewise for $\bar{f}(0)$. Assuming also that $|\Delta_0|\ll\Lambda_0$ (see below) we have now $|\Delta|\ll\Lambda_0$, and (28) becomes

$$P(\bar{M})\simeq|\delta|^2/2\omega\Lambda_0. \qquad (37)$$

(This result may also be obtained by assuming that after each elastic collision the μe system is an incoherent mixture of M and $\bar{M}$.)

At all reasonable temperatures in a dilute inert gas, ω_I will probably be small compared to λ. It is known experimentally[8] that a good fraction of the muonium formed in argon at room temperature and 50 atm lasts long enough for the μ^+ to decay. (The same is probably not true for $\bar{M}$.) Hence $\omega\simeq\lambda$, so that

$$P(\bar{M})\simeq|\delta|^2/2\lambda\Lambda_0=2.5\times10^{-5}/N, \qquad (38)$$

where $N=\Lambda_0/\lambda$ is (for large N) the mean of the number of collisions suffered during a muon lifetime for M and $\bar{M}$. It makes no difference whether the $\bar{M}$ collisions are elastic or inelastic, providing that an $\bar{M}$ breakup is detectable through μ^- decay or absorption. We may estimate N roughly (for $N>1$) as

$$N\simeq\pi nvR^2/\lambda\simeq\pi(kR)Rn/m_\mu\lambda$$
$$\simeq n(kR)/(3\times10^{15}\ \mathrm{cm}^{-3}), \qquad (39)$$

so at room temperature we need $n\ll3\times10^{15}$ cm^{-3} to avoid quenching of the $M\rightarrow\bar{M}$ process. (In the

[8] V. W. Hughes, D. W. McColm, K. Ziock, and R. Prepost, Phys. Rev. Letters 5, 63 (1960).

experiment quoted,[8] N was roughly 10^6 and hence C would have to be 10^3 times larger than C_V to have produced one $\bar{M}$ per 10^5 M.) Raising the temperature improves our approximations but worsens the situation.

For $kR \ll 1$ the scattering amplitudes $f(\theta)$ and $\bar{f}(\theta)$ approach constants a, $\bar{a}$. In this limit

$$\Lambda \to \lambda+\tfrac{1}{2}(\omega_c+\bar{\omega}_c)+2\pi nv|a-\bar{a}|^2 \to \lambda+\tfrac{1}{2}\bar{\omega}_I, \tag{40}$$

$$\Delta \to \Delta_0-(2\pi nv/k)\,\mathrm{Re}(a-\bar{a})-4\pi nv\,\mathrm{Im}[a\bar{a}^*] \to \Delta_0-(2\pi n/m_\mu)\,\mathrm{Re}(a-\bar{a}). \tag{41}$$

In deriving (41) we are using the fact that there are exothermic inelastic channels open to $\bar{M}$ (transfer of the μ^- to a low Bohr orbit about a nucleus) so $\bar{\omega}_I$ becomes constant as $v\to 0$, while ω_E, ω_I, and $\bar{\omega}_E$ all tend to zero. Unless Rea and Re$\bar{a}$ happen to be equal (which seems highly unlikely), $\Delta-\Delta_0$ will be of order $\pi nR/m_\mu$, or roughly as large as ω_c or $\bar{\omega}_c$ would be for $kR=1$. We are assuming that Δ_0 is less than this, and it seems reasonable that $\lambda+\frac{1}{2}\bar{\omega}_I$ is not much greater than this, so

$$P(\bar{M})\simeq\frac{|\delta|^2(\lambda+\frac{1}{2}\bar{\omega}_I)}{(\pi nR/m_\mu)^2}\lesssim\frac{2.5\times10^{-5}}{N'}, \tag{42}$$

where

$$N'=\pi Rn/m_\mu\lambda\simeq n/(3\times10^{15}\ \mathrm{cm}^3). \tag{43}$$

Hence the index of refraction effect prevents the lowering of the temperature (with n fixed) from being of any help in preventing quenching.

We have been making the assumption that the $M-\bar{M}$ energy split between collisions Δ_0 is less than Λ_0, which in turn must, to avoid quenching, be not much greater than $\lambda=3\times10^{-10}$ ev. This condition applies not only to the part of Δ_0 arising from external fields, but also to the part due to the "tail" of the interactions between M or $\bar{M}$ and the gas molecules, and is needed to ensure that the scattering is actually incoherent.

In order to see whether $\Delta_0 \leq \lambda$ in an inert gas, we must take account of two types of interaction between μe and the gas atoms. One is a "dispersion" force, arising as a term of second order in the dipole-dipole interaction ($\sim r^{-3}$) and of first order in the quadrupole-quadrupole interaction ($\sim r^{-5}$), and hence varying as r^{-11}. (There are no terms of total order 1 since the systems are neutral and none of total order 2 since they would not contribute to Δ_0. There are no terms of third order in the dipole-dipole interaction because they would not conserve parity.) If we assume that in atomic units (a.u.) $\Delta_0 \simeq r^{-11}$ then $\Delta_0 \simeq \lambda$ for $r\simeq 10$ a.u., or about 10^{-7} cm.

A second kind of interaction arises from overlap of μe and gas atom wave functions. Such terms vary exponentially with r. If we assume that in atomic units $\Delta_0\simeq e^{-r}$, then $\Delta_0\simeq\lambda$ for $r\simeq 25$ a.u., or again about 10^{-7} cm. This sets an upper bound on n of about 10^{21} cm^{-3}, which is much less stringent than the limit set by collisional quenching.

V. CONCLUSION AND PROSPECTS

It is clear that we can only hope to see muonium become antimuonium in a very dilute inert gas, with $n \ll 3\times10^{15}$ cm^{-3}. (At room temperature this would mean $p \ll 10^{-4}$ atm.) Since there seems little hope of forming muonium in such a medium, we must look for some way of forming it in a solid and getting it out into the gas in a short time. The best chance seems to be to pass a μ^+ beam through a series of many thin plates with dilute argon or vacuum in between, and hope that a sufficient fraction of the muonium formed in the plates can diffuse out and not be adsorbed. The energy of the μ^+ beam, plate spacing, plate thickness, and plate material should be chosen to maximize this probability.

For a discussion of the implications of observing $M\to\bar{M}$, see reference 2.

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