Spectroscopy of Gamma Radiation from Nd^{144}, Sr^{88}, and Pb^{207}†

J. E. MONAHAN, S. RABOY, AND C. C. TRAIL
Argonne National Laboratory, Argonne, Illinois
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The energies of the cascade gamma rays in Nd^{144} are found to be 1487.0±1.1 kev and 696.7±0.6 kev and the measured energy of the crossover transition is 2186.0±2.2 kev. The agreement of these results is used to justify the claim of 0.1% accuracy for the scintillation spectrometer with anticoincidence annulus for the measurement of gamma-ray energies in the interval from 0.5 Mev to roughly 3.0 Mev. An energy of 570.8±0.5 kev is obtained for the low-energy radiation from Pb^{207} and energies of 1836.2±1.7 kev and 898.7±0.8 kev are reported for two Sr^{88} gamma rays. Also measurements are given for the relative intensities of the 2.18-Mev, 1.48-Mev, and 0.696-Mev gamma rays of Nd^{144}, the relative intensities of the 1.8-Mev and 0.898-Mev transitions in Sr^{88}, and the relative intensities of the 1.06-Mev and 0.57-Mev transitions in Pb^{207}.

I. INTRODUCTION

IN the analysis[1] of the spectra of gamma rays obtained with the scintillation spectrometer with an anticoincidence annulus,[2] the mean pulse height of the full-energy peaks are determined to within an uncertainty of from 0.4% to 1.0% of the width of a peak. In order to utilize this precision for the evaluation of the energies of gamma rays, it is necessary that the relation between energy and pulse height be determined with comparable accuracy. For any limited energy interval, the required calibration can be made by measuring the response of the scintillation spectrometer for several gamma rays of known energy within this interval. Between 0.3 Mev and 1.5 Mev the energies of a number of gamma rays have been determined with sufficient accuracy for this purpose. However, for transitions in the interval from 1.5 Mev to 2.7 Mev there seem to be no convenient sources of gamma rays with precisely measured energy. This interval, which was of particular interest in connection with another experiment,[3] is too large to allow an *a priori* estimate of the validity of any calibration function, particularly for energies in the neighborhood of 2.2 Mev.

The nucleus Nd^{144} is the source of three gamma rays with energies of about 2.18, 1.48, and 0.696 Mev. The latter two are in cascade from the level at 2.18 Mev, as shown in Fig. 1. Thus the energies of the two cascade gamma rays fall within an interval in which the spectrometer can be calibrated accurately. Since the sum of the energies in this cascade gives an independent determination of the energy of the crossover gamma ray, this source provides a means by which a given calibration relation may be tested in the critical neighborhood of 2.2 Mev.

The results of the present measurement of the energies of the gamma rays from Nd^{144} are 2186.0±2.2 kev, 1487.0±1.1 kev, and 696.7±0.6 kev. The directly measured energy of the crossover gamma ray and the sum of the independently measured energies of the cascade gamma rays are seen to agree within the experimental errors. It is believed that this agreement provides a confirmation of the validity of the present method of measurement and analysis.

More generally these results indicate that a representation of gamma-ray energies by a quadratic function of the observed pulse height is accurate to about 0.1% over the entire energy interval from 0.6 to 2.7 Mev. Previous studies[4,5] on the linearity of the response of scintillation counters over this energy interval were not accurate enough to detect the slight deviations represented by the quadratic term in the response function. In these earlier studies, no effort was made to suppress those events in which degraded quanta escape from the crystal. These events caused asymmetries in the full-energy peaks so that it was difficult to assess the accuracy with which their mean positions were determined.

In the present experiment it proved convenient to use the two gamma rays of Sr^{88} as auxiliary standards in the measurement of the energy of the 1.48-Mev gamma ray from Nd^{144}. The energies of these two gamma rays, as measured in this experiment, are 1836.2±1.7 kev and 898.7±0.8 kev. Also, as a by-product, we obtain the energy of the 0.571-Mev gamma ray in Pb^{207}, the relative intensities of the gamma rays of 2.18, 1.48, and 0.696 Mev in Nd^{144},

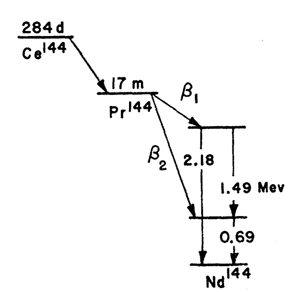

FIG. 1. Partial decay scheme of Ce^{144}.

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[1] J. E. Monahan, S. Raboy, and C. C. Trail (to be published).

[2] C. C. Trail and S. Raboy, Rev. Sci. Instr. **30**, 425 (1959).

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[4] Yu. A. Nemilov, I. I. Lomonsov, A. N. Pisarevskii, L. O. Soshin, and N. D. Teterim, Bull. Acad. Sci. U.S.S.R. Phys. Ser. (Columbia Tech. Translation) **23**, 246 (1959). A bibliography of such measurements can be obtained from this reference.

[5] R. W. Peele and T. A. Love, Oak Ridge National Laboratory Report ORNL-2801 (unpublished).

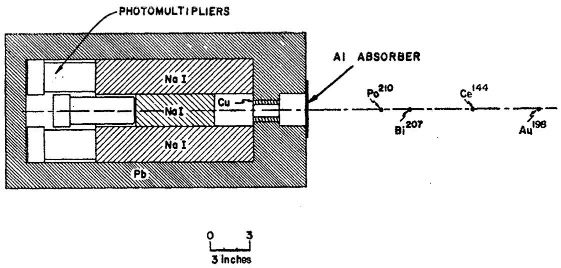

FIG. 2. Experimental arrangement for the measurement of the 696-kev gamma ray of Nd^{144}.

the relative intensities of the gamma rays of 1.8 and 0.898 Mev in Sr^{88}, and the relative intensities of the gamma rays of 1.06 and 0.571 Mev in Pb^{207}. These results are summarized in Tables III and VIII in Sec. III.

II. APPARATUS AND PROCEDURE

All spectra were obtained by use of the anti-coincidence spectrometer. These spectra are characterized by the suppression of events within the crystal that do not contribute to the full-energy peaks so that the full-energy peaks are apparently enhanced relative to the remainder of the spectrum. This eliminates much of the ambiguity in the identification of the mean position of each gamma-ray peak.

The energy of each of the three gamma rays from Nd^{144} was measured separately. In order to measure the energy of the gamma ray of about 696 kev, a spectrum of the gamma rays from Bi^{207} (1063.9±0.3 kev),[6] Po^{210} (803.3±0.4 kev),[7] Au^{198} (411.78±0.04 kev),[8] and Ce^{144} (unknown) was obtained by exposing the detection system simultaneously to these sources. The experimental arrangement for this measurement is shown in Fig. 2 and the measured spectrum is presented in Fig. 3(a). A sheet of aluminum 0.125 in. thick is placed between the sources and the detector to stop the energetic beta rays from the sources. The simultaneous collection of data from all sources tends to minimize the systematic errors caused by the drift of the photomultiplier, fluctuations in the power supply, and changes in the gain of the amplifiers. Also, the counting rate was restricted to moderate values in order to reduce the possible influence of counting rate on the relative positions of the full-energy peaks.

The spectra are recorded on a 256-channel analyzer and stored on punched paper tape. This tape is compatible with a program developed for the digital computer GEORGE at the Argonne National Laboratory. This program analyzes the spectra in a manner which we believe realizes the full potential of the counting statistics. The details of this program are given in Sec. III.

In the analysis of the data, provisions are made for the removal of background contributions to the full-energy peaks as well as contributions from gamma rays with an energy higher than that of the particular gamma ray being analyzed. In preparation for this analysis, separate measurements are made of the spectra of the following isolated sources: (i) Bi^{207} [Fig. 3(b)]; (ii) Zn^{65} [Fig. 3(c)]; (iii) Po^{210} [Fig. 3(d)]; (iv) Ce^{144} [Fig. 3(e)]; (v) Au^{198} [Fig. 3(f)]; and (vi) room background [Fig. 3(g)]. We shall refer to these six auxiliary spectra as background spectra for the measurement of the energy of the gamma ray of 0.696 Mev.

The full spectrum, which we shall call the data run, in the measurement of the energy of the 696-kev gamma ray consisted of five full-energy peaks. In addition to the peaks of the three standard energies and the lower energy cascade gamma ray of Nd^{144}, the full-energy peak of the 571-kev gamma ray of Pb^{207} was present. It proved convenient to carry out the background subtractions for this data run in two steps. First the gamma rays of 1.0639 Mev, 0.8033 Mev, and 0.571 Mev are considered as one group. These spectra are corrected for room background and the Ce^{144} source by subtracting spectrum (iv) with the normalization appropriate for the total live times for the accumulation of the data run and the background run. The full-energy peak corresponding to the 1.0639-Mev gamma ray is then analyzed[9] to obtain the parameters of the line shape of this transition for the data run and for the background spectrum (ii). In the latter case, the background (vi) is subtracted after normalization by the ratio of the live times for the accumulation of the respective data. To prepare for the analysis of the full-energy peak of the 0.8033-Mev gamma ray, the contribution of the 1.0639-Mev gamma ray in the region of the 0.8033-Mev gamma ray is removed by subtracting spectrum (ii) with normalization determined by the ratio of the areas observed for the 1.0639-Mev "line" in the data run and in the background run (ii). The analysis of the 0.8033-Mev peak in the data run is then carried out. The full-energy peak in the background run (iii) is analyzed next. The background of spectrum (vi) is subtracted first in this case. Finally the 0.571-Mev peak in the data run is analyzed after first using the background (iii) to subtract out the influence of the higher energy gamma rays in the vicinity of this peak. The normalization for this subtraction made use of the ratio of the area of the full-energy peak of the 0.8033-Mev gamma ray of the data run to the area of the corresponding peak of spectrum (iii). We note that each subtraction

[6] D. E. Alburger, Phys. Rev. 92, 1257 (1953).

[7] D. E. Alburger and M. H. L. Pryce, Phys. Rev. 95, 1482 (1954).

[8] D. E. Muller, H. C. Hoyt, D. J. Klein, and J. W. DuMond, Phys. Rev. 88, 775 (1945).

[9] By the "analysis" of a line we shall mean the fitting of the observed line shape to the Gaussian function given in Eq. (1), Sec. III.

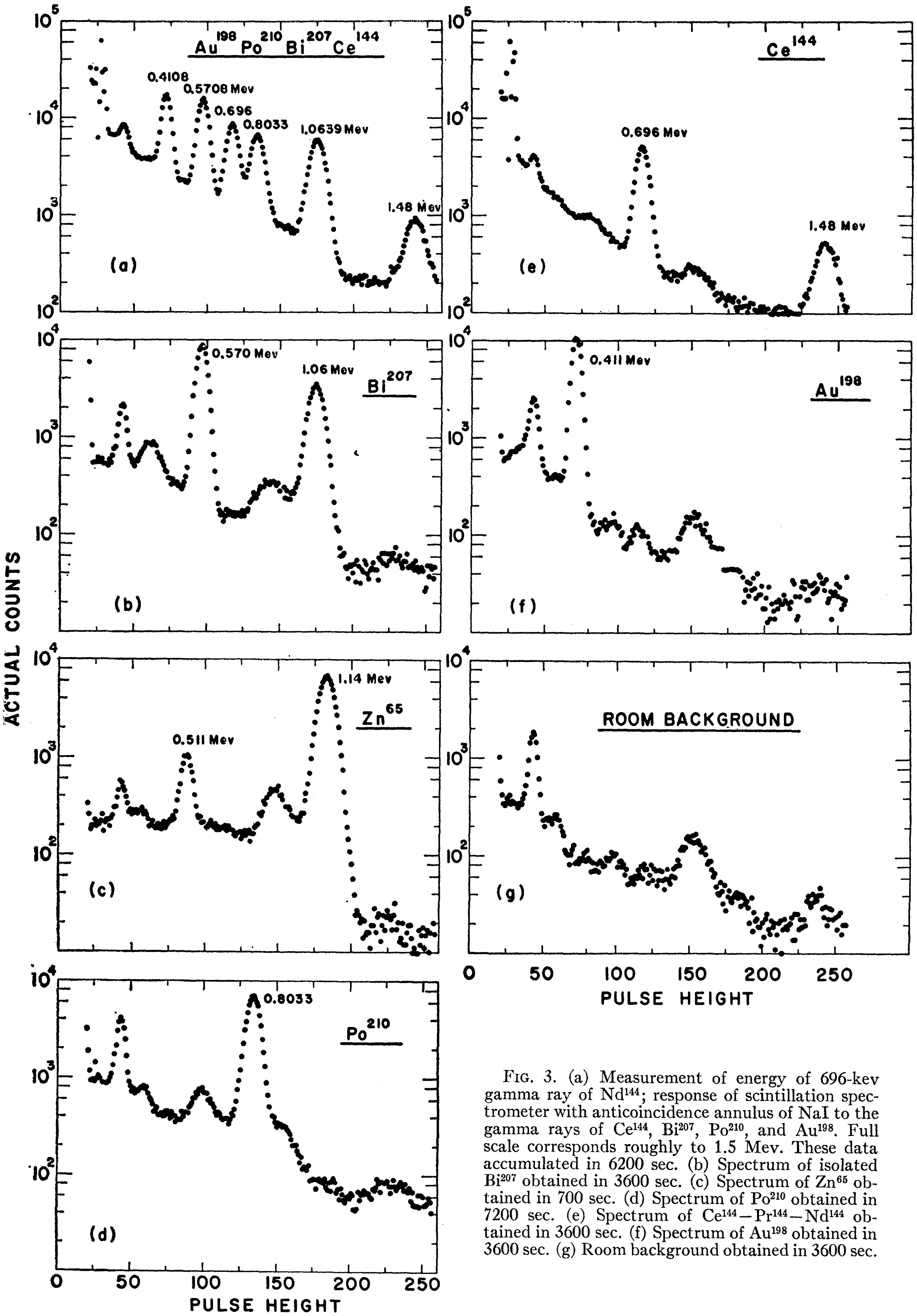

FIG. 3. (a) Measurement of energy of 696-kev gamma ray of Nd^{144}; response of scintillation spectrometer with anticoincidence annulus of NaI to the gamma rays of Ce^{144}, Bi^{207}, Po^{210}, and Au^{198}. Full scale corresponds roughly to 1.5 Mev. These data accumulated in 6200 sec. (b) Spectrum of isolated Bi^{207} obtained in 3600 sec. (c) Spectrum of Zn^{65} obtained in 700 sec. (d) Spectrum of Po^{210} obtained in 7200 sec. (e) Spectrum of $Ce^{144}-Pr^{144}-Nd^{144}$ obtained in 3600 sec. (f) Spectrum of Au^{198} obtained in 3600 sec. (g) Room background obtained in 3600 sec.

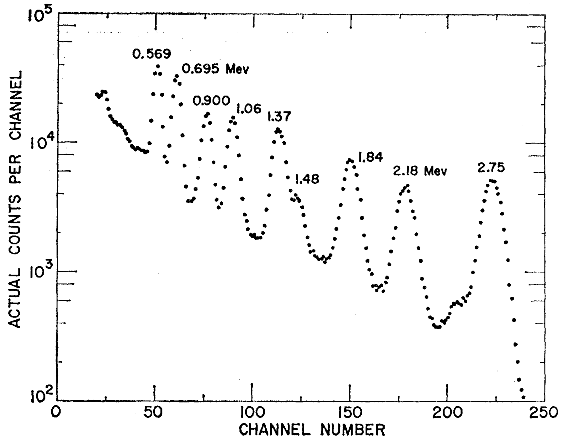

FIG. 4. Measurement of the energy of the 2.18-Mev gamma ray of Nd^{144}; response of scintillation spectrometer to the gamma rays of Ce^{144}, Na^{24}, Bi^{207}, and Y^{88}.

is made for the entire range of the multichannel analyzer so that the background of a particular source is removed only once.

In the analysis described above, the spectrum of the 1.12-Mev gamma ray of Zn^{65} was used to simulate the spectrum of the 1.0639-Mev gamma ray of Bi^{207} in the region of the full-energy peak of the 0.571-Mev gamma ray. We have also carried out the analysis of the full-energy peak of the 0.571-Mev transition without any attempt to remove the influence of the 1.06-Mev gamma ray. Its contribution is then contained in the value of the parameter α_4, defined below, which describes a constant level of background in the response of the counter system. These two methods proved to be equivalent for the determination of the energy of the 0.571-Mev gamma ray. That is, the two methods yield mean positions which differ by less than 0.1 of the uncertainty associated with them.

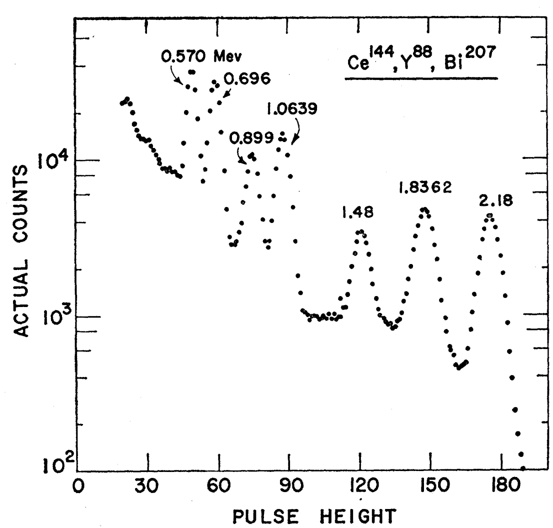

FIG. 5. Measurement of the energy of the 1.48-Mev gamma ray of Nd^{144}; response of scintillation spectrometer to the gamma rays of Ce^{144}, Bi^{207}, and Y^{88}.

The second step in the background subtraction procedure for the determination of the energy of the 696-kev gamma ray involves the analysis of the full-energy peaks (in the data run) associated with the gamma rays of energy 0.696 Mev (Ce^{144}) and 0.412 Mev (Au^{198}). We begin by subtracting spectrum (i), normalized to the respective live times of the counter, from the data run. The peak of the 0.8033-Mev gamma ray in the data run, as well as the corresponding peak of spectrum (iii), is analyzed. In the latter case spectrum (vi), normalized to a corresponding live time of the counter, is subtracted first. The procedures outlined above are repeated to obtain the analysis of the peaks of the 0.696- and 0.411-Mev gamma rays.

A similar procedure is used to determine the parameters for the crossover transition, the 2.18-Mev line of Nd^{144}. The spectrum of this gamma ray is obtained simultaneously with the spectra of "standard" gamma rays in the same manner as described for the measurement of the energy of the 0.696-Mev gamma ray. The "standards" used in this case are the (2753.3 ± 1.0)- and the (1368.6 ± 0.3)-kev gamma rays[10] of Mg^{24}, the (1063.9 ± 0.3)- and the (570.8 ± 0.5)-kev gamma rays of Pb^{207}, and the (696.7 ± 0.6)-kev gamma ray of Nd^{144}. The energies of the latter two gamma rays were determined with different standards in the measurement described above. The spectrum from a typical run is presented in Fig. 4. The gamma rays of Y^{88} (1.8 and 0.90 Mev) are included in these data in anticipation of their use in the measurement of the 1.48-Mev cascade gamma ray.

The analysis of these full-energy peaks is carried out as outlined above with a few minor changes. In the analysis of the 2.75-Mev gamma ray of Mg^{24}, the contribution of the crossover transition in Y^{88} (with an energy of about 2.74 Mev) is removed by subtraction of the appropriate isolated spectrum. Because of a weak 1.74-Mev transition in Pb^{207} a separate subtraction, using the spectrum of a Bi^{207} source, is carried out in the neighborhood of the 1.84-Mev peak of Y^{88}.

In the measurement of the 1.48-Mev cascade gamma ray of Nd^{144}, the interference of the 1.368-Mev transition prevents the use of Na^{24} as a source of "standard" gamma rays. We therefore use the gamma rays of Y^{88} and the 2.18-Mev gamma ray of Nd^{144} as "standards" in this case. A typical spectrum is presented in Fig. 5.

III. RESULTS

Table I contains the results of the analysis of a typical run in the measurement of the energy of the 0.696-Mev gamma ray of Nd^{144}. The parameters listed in the table correspond to the parameters in a function

[10] A. Hedgran and D. Lind, Arkiv Fysik **5**, 177 (1952) and private communication.

TABLE I. The parameters of Eq. (1) determined from the observed response of the scintillation detector by the modified χ^2 minimum method. The results are for a typical set of data, run number 825.

Energy of gamma ray (kev)	Peak height α_1 (counts)	Mean of Gaussian α_2 (channels)	Standard deviation α_3 (channels)	Residual background α_4 (counts)	χ^2 This calculation	χ^2 Expected value
1063.9	5574±52	174.785±0.038	7.04 ±0.09	62±55	21	17
803.3	5766±44	133.574±0.06	5.97 ±0.09	152±38	9	12
696. (unknown)	7700±170	116.736±0.048	5.51 ±0.14	310±200	11	9
571. (unknown)	13752±72	96.795±0.023	4.79 ±0.05	170±70	5	8
411.8	14986±48	71.595±0.009	4.058±0.020	76±45	2	8

of the form

$$C(x)=\alpha_1 G(x;\alpha_2,\alpha_3)+\alpha_4+\alpha_5 G(x;\alpha_2-d,\alpha_3), \quad (1)$$

where $C(x)$ is the measured response at pulse height x, and $\alpha_1, \alpha_2, \cdots, \alpha_5$ are parameters whose values are determined from the corrected data spectrum by means of the modified χ^2 minimum procedure, and

$$G(y;\alpha_2,\alpha_3)=\exp[-(y-\alpha_2)^2/\alpha_3^2]. \quad (2)$$

It is seen from Eqs. (1) and (2) that α_1 and α_2 represent the amplitude and mean pulse height, respectively, of the full-energy peak, and α_3 is a measure of its width. The α_4 is a measure of a constant background that is not removed by the subtraction procedures. In sources with two or more gamma rays, it is sometimes not possible to remove the influence of the higher energy gamma ray in the region of the full-energy peak of the lower energy gamma rays. The parameter α_4 is used to represent these contributions. We also introduce a fifth parameter α_5 in Eq. (1) to describe the response to back-scattered events within the crystal. These are caused by large-angle Compton scatterings in which the degraded gamma rays escape the detector through the entrance aperture and, therefore, fail to trigger an anticoincidence pulse. This type of event leads to a small peak not more than 250 kev lower than the mean of the full-energy peak. Since the shape and mean position, (α_2-d) in Eq. (1), of this secondary peak can be calculated with reasonable accuracy, it is necessary only to determine its amplitude α_5 in order to include these events in the response function. For all full-energy peaks, with the exception of that at 2.7533-Mev, the contribution to the full-energy peak from α_5 is essentially zero; and even in the case of the 2.7533-Mev peak, the correlation between α_2 and α_5 is sufficiently small that any reasonable value of α_5 produces a negligible shift in α_2.

The calculated value of χ^2 provides a measure of the "goodness of fit" of the assumed response function, Eq. (1), to the observed spectrum. The calculated and expected values of this quantity are listed for each full-energy peak in the last two columns of Table I.

The mean positions of the full-energy peaks are determined within an uncertainty which varies from 1.0% to 0.4% of the width of the peak. This accuracy is limited only by counting statistics and the stability of the counting system, provided that the shape of the peak is known. The values calculated for χ^2 give a measure of the validity of the assumptions about the shape of the full-energy peak and the errors quoted in the table are adjusted in the usual way for external consistency.

In order to determine the energy of the 696-kev gamma ray, the relation between energy and pulse height of the spectrometer system must be determined from the energies of the three standard gamma rays. The least-squares values of the coefficients of an assumed linear response in the energy interval from 411 kev to 1.06 Mev are given in Table II for five separate experimental "runs." Listed in the last column of Table II are the calculated values of χ^2 for each "run." Theoretically these values should be distributed as χ^2 with one degree of freedom. Actually the observed distribution is somewhat narrower than the theoretical distribution so we conclude that the assumption of a linear relationship between energy and pulse height within this energy interval is consistent with the data to within the accuracy of the "standard" energies.

We used this opportunity to make an independent measurement of the energy of the 571-kev gamma ray in Pb^{207}. There are two values reported in the literature[11]; 569.7±0.1 kev by Bäckström and 568.9 ±0.3 kev by Yavin and Schmidt. These results do not agree within the stated errors and our preference, therefore, is not to use the average of these results. Both papers are very tersely written and we are prevented from making an evaluation of the work for systematic errors. Our own result turned out to be 570.8±0.5 kev which, we note, does not agree with either of the two earlier results. In the subsequent measurement on gamma rays of higher energies we use our value of the energy for this gamma ray. The results of our measurements on these transitions are summarized in Table III. In the calculation of the standard deviations associated with these results, the error asssociated with each individual determination is treated as entirely systematic so that the average result of a number of runs has, in general, about the same error associated with it as do the individual runs. The justification for this procedure is that we believe the main sources of error in our energy determination

TABLE II. Least-squares values of the energy-calibration equation $E=a+b\alpha_2$ for the energy interval from 0.41 to 1.06 Mev.

Run	Energy E of standard gamma ray (kev)	Mean position α_2 (channels)	a (kev)	b (kev/channel)	χ^2
822	1063.9 ±0.3	177.011±0.037	−52.15 ±0.38	6.3040±0.0033	0.47
	803.3 ±0.4	135.811±0.075			
	411.77±0.04	73.587±0.017			
823	1063.9 ±0.3	174.830±0.038	−39.672±0.055	6.3119±0.0005	0.01
	803.3 ±0.4	133.566±0.062			
	411.77±0.04	71.523±0.017			
825	1063.9 ±0.3	174.785±0.038	−40.636±0.078	6.3190±0.0007	0.02
	803.3 ±0.4	133.574±0.06			
	411.77±0.04	71.595±0.009			
828	1063.9 ±0.3	173.953±0.034	−37.272±0.069	6.3299±0.0006	0.02
	803.3 ±0.4	132.804±0.027			
	411.77±0.04	70.940±0.010			
831	1063.9 ±0.3	174.540±0.029	−40.196±0.23	6.326 ±0.002	0.13
	803.3 ±0.4	133.306±0.042			
	411.77±0.04	71.456±0.027			

are possible errors in the value of the energy assigned to the standard gamma rays and systematic deviations of the true relation between energy and pulse height from that which we have assumed. Both of these sources result in a systematic error that cannot be reduced by simple repetitions of measurement.

The 570.8- and the 696.7-kev gamma rays were used as standards, together with the 1368.6- and the 2753.3-kev gamma rays of Na^{24} and the 1.06-Mev line of Bi^{207}, to measure the energies of the prominent gamma rays from Y^{88} and the 2.18-Mev line of Nd^{144}. A typical spectrum in this series of data runs is shown in Fig. 4. These data were analyzed in a manner similar to that described for the spectrum of Fig. 3. Tables IV and V contain the channel positions of the mean of the full-energy peaks for the standard and unknown energies, respectively. Also given in Table IV are the parameters of a calibration curve for the spectrometer over an energy interval from 0.571 to 2.75 Mev. This calibration curve includes a quadratic term in the mean pulse height in addition to the constant and linear terms. The number of degrees of freedom for the theoretical distribution of the values of χ^2 is 2, except for the runs numbered 157 and 158, where a malfunction of the printer prevented the analysis of one full-energy peak. The computed values of χ^2 vary from about 3 to 20 for the quadratic calibration function. For a linear calibration within this energy interval, the computed values of χ^2 vary from 30 to 80 and the average ratio of the values of χ^2 for these two calibration functions is 5.9. If equal validity is assumed for both calibrations, the expected value of this ratio is 1.5. This indicates that the quadratic term in the interpolation function improves the agreement to an extent greater than would normally be expected by the introduction of one additional free parameter. However, even for the quadratic formula the distribution of observed values of χ^2 does not agree with the theoretical distribution as well as might be expected. We shall return to this point below.

TABLE III. Energy of "696"-kev and "571"-kev gamma rays of Nd^{144} and Pb^{207}, respectively.

Run	Channel position of unknown	Energy from linear fit (kev)
822	118.67 ±0.03	696.0±0.5
	98.845±0.034	571.0±0.5
823	116.49 ±0.06	695.6±0.7
	96.707±0.034	570.7±0.5
825	116.736±0.048	697.0±0.6
	96.795±0.023	571.0±0.5
828	116.021±0.039	697.1±0.5
	96.063±0.021	570.8±0.4
831	116.599±0.033	697.4±0.5
	96.568±0.014	570.7±0.5
Averages	696.7±0.6[a]	
	570.8±0.5[a]	

[a] Errors of individual runs are treated as systematic rather than random.

The weighted average of the energies of the two gamma rays in Sr^{88} and the highest energy line from Nd^{144} are 898.7±0.8 kev, 1836.2±1.7 kev, and 2186.0 ±2.2 kev, respectively. A summary of these results for each of eight separate determinations is included in Table V. For the same reasons as given before, the errors associated with the individual runs include a normalization to the expected value of χ^2 and are treated as entirely systematic.

These three lines are now used as secondary standards in the measurement of the energy of the 1.5-Mev gamma ray of Nd^{144}. This part of the experiment is similar to the previous data runs, except that the Na^{24}

TABLE IV. The parameters of the calibration relation $E=a+bx_2+cx_2^2$ in the energy interval from 0.57 Mev to 2.75 Mev.

Standard γ-ray energy E (kev)	Run 804	Run 806	Run 807	Run 810	Run 811	Run 157	Run 158	Run 159
	Mean position of full-energy peak, x_2							
2753.3±1.0	223.40 ±0.03	222.72 ±0.03	222.67 ±0.03	221.07 ±0.04	223.09 ±0.05	225.974±0.026	225.55 ±0.03	226.38 ±0.05
1368.6±0.3	113.449±0.028	112.890±0.022	112.896±0.010	112.015±0.015	113.815±0.029	114.485±0.017	114.230±0.020	114.876±0.020
1063.9±0.3	89.762±0.016	89.201±0.013	89.182±0.013	88.305±0.018	90.139±0.010	90.073±0.013	89.712±0.016	90.333±0.014
696.7±0.5	60.785±0.007	60.229±0.018	60.240±0.015	59.620±0.015	61.390±0.014	61.168±0.012	Not in this program	61.378±0.008
570.8±0.5	50.970±0.015	50.455±0.009	50.450±0.006	49.909±0.004	51.632±0.017	Not used in this run	50.537±0.012	51.110±0.023
	Parameters of quadratic fit							
a	−83.0 ±3.2	−77.3 ±2.9	−78.4 ±2.6	−74.3 ±1.1	−95.0 ±2.1	−81.3 ±3.8	−65.6 ±2.3	−75.8 ±3.5
b	12.86±0.06	12.88±0.05	12.90±0.05	12.97±0.02	12.93±0.04	12.81±0.06	12.93 ±0.04	12.67±0.06
1000c	− 0.72±0.22	− 0.75±0.21	− 0.82±0.19	− 0.79±0.08	− 0.71±0.14	− 1.18±0.21	− 0.612±0.15	− 0.78±0.23
χ^2	19	20	19	3.1	7.6	9.3	7.4	20

TABLE V. Energies of unknown gamma rays of Nd^{144} and Sr^{88} from the calibration of the response of the scintillation spectrometer given in Table IV.

Run	Mean position of full-energy peak of unknown (channels)	Energy determined from quadratic fit[a] (kev)
804	178.46 ±0.04	2188.9±3.1
	150.657±0.029	1837.9±2.5
	76.712±0.015	899.2±1.2
806	177.67 ±0.04	2187.1±2.9
	149.996±0.023	1837.5±2.2
	76.129±0.012	898.8±1.1
807	177.699±0.025	2188.5±2.6
	150.03 ±0.03	1838.9±2.0
	76.128±0.016	899.1±1.1
810	176.04 ±0.04	2183.8±1.5
	148.69 ±0.03	1836.2±1.1
	75.360±0.009	898.4±0.5
811	178.04 ±0.04	2184.2±2.2
	150.587±0.017	1835.7±1.5
	77.217±0.015	899.0±0.8
157	180.183±0.024	2188.3±2.4
	151.903±0.020	1837.2±2.0
	77.057±0.007	898.7±1.1
158	179.795±0.024	2186.2±2.0
	151.454±0.018	1833.9±1.6
	76.635±0.018	899.1±1.0
159	180.547±0.027	2186.7±3.1
	152.227±0.028	1835.2±2.5
	77.237±0.030	898.4±1.5
	Weighted averages	
		2186.0±2.2
		1836.2±1.7
		898.7±0.8

[a] These errors include systematic errors as suggested by the χ^2 test. The final error is the arithmetic average of the systematic errors.

source has been removed. A typical spectrum is shown in Fig. 5, and a summary of the data is given in Table VI. Again a nonlinear term is postulated for the calibration function and the theoretical distribution of χ^2 again has two degrees of freedom. In this case, for which the energy interval is from 0.57 to 2.18 Mev, the agreement between the distribution of observed values of χ^2 and the theoretical distribution is excellent.

The weighted average of four determinations of the energy of this cascade gamma ray in Nd^{144} is 1487.0±1.1 kev. Again the errors of the individual runs have been treated as systematic.

In Table VII is presented a summary of the results for the Nd^{144} gamma rays. Here the sum of the energies of the cascade is given as 2183.7±1.7 kev, and this value is to be compared with the energy of the crossover gamma ray which was measured to be 2186.0±2.2 kev. Treating the difference between these measured values as a sample drawn from a normal population with zero mean, it is easily shown that 4 of 10 similar experiments would result in a difference at least as large as that

TABLE VI. The parameters of the calibration equation $E=a+b\alpha_2+c\alpha_2^2$ in the energy interval from 0.57 Mev to 2.18 Mev. The calculated energy of the 1.48-Mev gamma ray of Nd^{44} is given for each run.

Energy E of "standard" γ ray (kev)	Mean position Run 815	Run 816	Run 819	Run 820
2186.0±2.2	176.460±0.027	176.298±0.022	175.564±0.015	177.994±0.032
1836.2±1.7	148.965±0.027	148.872±0.028	148.236±0.033	150.51 ±0.05
1063.9±0.3	88.490±0.011	88.463±0.009	87.859±0.014	89.987±0.021
898.7±0.8	75.493±0.025	75.505±0.012	74.960±0.021	77.230±0.020
696.7±0.6	59.710±0.012	59.730±0.010	59.190±0.021	61.330±0.008
570.8±0.5	49.984±0.007	50.010±0.008	49.490±0.011	51.632±0.031
1480-kev "unknown"	121.572±0.027	121.563±0.035	120.879±0.028	123.08 ±0.04
	Parameters of quadratic fit			
a	−70.2 ±1.6	−71.3 ±1.5	−66.7 ±1.3	−94.7 ±2.6
b	12.85±0.04	12.87±0.03	12.912±0.028	12.94±0.05
$1000c$	− 0.38±0.19	− 0.36±0.18	− 0.48 ±0.15	− 0.69±0.26
χ^2	2.5	2.3	1.4	3.2
Energy of 1480-kev	1486.6 ±1.0	1487.4 ±1.1	1487.1 ±1.0	1486.9 ±1.4
	Weighted average for 1480-kev gamma ray =1487.0±1.1 kev			

obtained here. We interpret this to indicate that the procedures of measurement and analysis described herein yield results that are internally consistent.

We arbitrarily average the crossover value with the cascade sum to obtain the value 2185±2 kev for the energy of this crossover transition.

As mentioned before, the agreement obtained with a quadratic calibration function in the energy interval 0.57–2.75 Mev is somewhat disappointing. This can be attributed to the inadequacy of the calibration function, various approximations in the analysis, errors in the determination of the mean pulse heights, or errors in the assignment of energy values to the "standards." Undoubtedly all of these possibilities contribute to some extent. It should be noted, however, that the claim of 0.1% accuracy of calibration over this energy interval is not inconsistent with the agreement obtained by use of the quadratic calibration function. Furthermore, this degree of accuracy is supported by the agreement obtained for the crossover transition and the cascade sum in Nd^{144} as well as the results given in Table VI for calibration in the energy interval from 0.57 Mev to 2.18 Mev.

TABLE VII. Measurement of the energy of the Ce^{144} gamma rays. All energies are in kev.

	Energy	Standards	
Lowest energy gamma ray	696.7±0.6	Au^{198}	411.8±0.3
		Bi^{207}	570.8±0.5 1063.9±0.3
		Po^{210}	803.3±0.4
Intermediate energy gamma ray	1487.0±1.1	Bi^{207}	570.8±0.5 1063.9±0.3
		Y^{88}	1836.2±1.7 898.7±0.8
		Ce^{144}	2186.0±2.2 696.7±0.6
Highest energy gamma ray	2186.0±2.2	Na^{24}	2753.3±1.0 1368.6±0.3
		Bi^{207}	570.8±0.5 1063.9±0.3
		Ce^{144}	696.7±0.6
Sum of cascade	2183.7±1.7	(Errors treated as systematic.)	
Average for cross over transition	2185 ±2		

Finally, Table VIII presents the relative intensities of the gamma rays in Sr^{88}, Nd^{144}, and Pb^{207}. These numbers are obtained as a byproduct of the fit of the data in the form of Eq. (1) since the product $\alpha_1\alpha_3$ is a measure of the area of the photopeak. When this area is corrected for the photopeak efficiency of the spectrometer, obtained from a Monte Carlo calculation,[12] a measure of the relative intensities of the gamma rays from each source is obtained. In the ratio of the areas of the full-energy peaks of any pair of gamma rays, the errors associated with the amplitudes of the peaks and with the widths of the peaks are less than 2% each. Because of the uncertainties in the pertinent cross sections, we assign an error of 5% to the Monte Carlo calculation of the efficiency of the system for each gamma ray. The total error assigned to the ratios of intensities is therefore 8%. The ratios of the intensities of the gamma rays are corrected for the losses in the Al absorber (0.125 in. thick). This amounts to a 2% decrease in the ratio of the intensity of the 1836.2-kev gamma ray to that of the 898.7-kev gamma ray of Sr^{88}, and a 2% increase in the ratio of the intensity of the 696.7-kev gamma ray to that of the 2185-kev gamma ray of Nd^{144}.

The 898-kev gamma ray of Sr^{88} is part of a cascade from the level at 2.74 Mev to the level at 1.84 Mev. Since the direct transition from the 2.74-Mev level to the ground level is only about 0.5% of the intensity of the cascade decay, the relative intensities of these cascade transitions directly give the branching ratio

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TABLE VIII. Relative intensities of gamma rays from Sr^{88}, Nd^{144}, and Pb^{207}.

Emitting nucleus	Gamma-ray energy (kev)	Relative intensity
Sr^{88}	898.7	1
	1836.2	1.11±0.08
Nd^{144}	2185	1
	1487	0.40±0.03
	696.7	2.06±0.14
Pb^{207}	1063.9	0.76±0.06
	570.8	1

of the decay of Y^{88} to these levels. Our value for the ratio of the β decay to the 1.84-Mev level relative to that to the 2.73-Mev level is 0.11±0.08. Lazar[13] obtained a value of 0.09 for this ratio. It is difficult for us to assess the error assignment in Lazar's work because of the very brief description of the intensity measurements.

The relative intensities of the gamma rays in Nd^{144} imply a branching ratio of 1.19±0.10 to 1 for the β decay of Pr^{144} to the 0.696- and 2.18-Mev levels. Graham *et al.*[14] report a value of 1.3 for this ratio. Their measurement is also based upon the measurement of gamma-ray intensities. Kreger and Cook[15] report 1.14 for this ratio and Alburger and Kraushaar[16] report about 1:1. Burmistrov[17] reports that the ratios of the intensities of the 0.696-, 1.48-, and 2.18-Mev gamma rays are (2.1±0.6):(0.34±0.10):1; and Porter and Day[18] report (1.49±0.09):(0.29±0.03):(0.68±0.10) for the same gamma rays.

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