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Infrared Cyclotron Resonance in *n*-Type InAs and InP

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Cyclotron resonance of conduction electrons in InAs and InP has been measured in the far infrared spectral region. The effective masses obtained for InAs show a variation with magnetic field indicative of the nonparabolic nature of the conduction band of this material.

I. INTRODUCTION

INFRARED cyclotron resonance of conduction electrons in InSb has been measured by Burstein *et al.*[1] and by Palik *et al.*[2] using steady magnetic fields and by Keyes *et al.*[3] using pulsed fields. With pulsed fields between 150 and 250 kgauss, Keyes *et al.*[3] have observed cyclotron resonance at room temperature in InAs and obtained a conduction-electron effective-mass ratio of about 0.030. Lax and Mavroides[4] have recently reviewed cyclotron resonance work to date. They discuss additional data by Keyes giving an effective mass ratio for InAs of about 0.015 at the bottom of the band.

We have observed cyclotron resonance absorption in InAs and InP in the spectral region between 50 and 160 μ using steady magnetic fields up to 75 kgauss. From these data for InAs and InP the effective mass variation with magnetic field has been obtained. Also, the mass at zero magnetic field (at the bottom of the conduction band) has been deduced. A preliminary report of some of the present work[5] was given at the International Conference on Semiconductor Physics, Prague, 1960.

II. EXPERIMENTAL TECHNIQUES AND RESULTS

The detailed experimental techniques are reported elsewhere.[2] The samples were polycrystalline made up of a few large single crystals and were made into thin sections about 20 μ thick mounted on silicon or crystal quartz backings. They had the following carrier concentrations N and mobilities μ at liquid nitrogen temperature. For InAs, $N=7\times10^{15}/\text{cm}^3$ and $\mu=70\,000$ cm^2/v sec. For InP, $N=6.1\times10^{15}/\text{cm}^3$ and $\mu=18\,600$ cm^2/v sec. The crystal orientations of the samples were not determined. It is probable that the conduction bands are spherical and the masses isotropic. In each case a magnetic field of 60 kgauss was enough to bring the cyclotron frequency into the accessible spectral region. Then, $\omega_c\tau=\mu H/10^8>1$, and a distinct resonance was observed. Measurements were made in the far infrared using a reststrahlen monochromator to obtain seven narrow bands of wavelengths. The wave numbers of the average wavelengths used are: KRS-5, 65 cm^{-1}; CsI, 70 cm^{-1}; CsBr, 85 cm^{-1}; KI, 111 cm^{-1}; KBr, 131 cm^{-1}; KCl, 164 cm^{-1}; and NaCl, 192 cm^{-1}. The magnetic field was varied from zero to 75 kgauss to move the cyclotron resonance band through the wavelength at which the observations were made. Typical results from InAs are shown in Fig. 1. Here the relative transmission of the sample at 111 cm^{-1} (90 μ) is plotted as a function of magnetic field. At room temperature the band is slightly asymmetrical, but becomes symmetrical at low temperature. As in the case of InSb,[2] the tail at high magnetic fields is probably due to transitions for $k_z\neq0$ between the same spin states of the nonparabolic Landau sub-bands $l=0$ and $l=1$ and transitions between higher Landau levels such as $l=1$ and $l=2$. Within experimental error, no difference was observed between the mass obtained from room temperature and low temperature data.

For InP, because of the large effective mass, the cyclotron resonance absorption band was observed only at the longest wavelengths and highest magnetic fields.

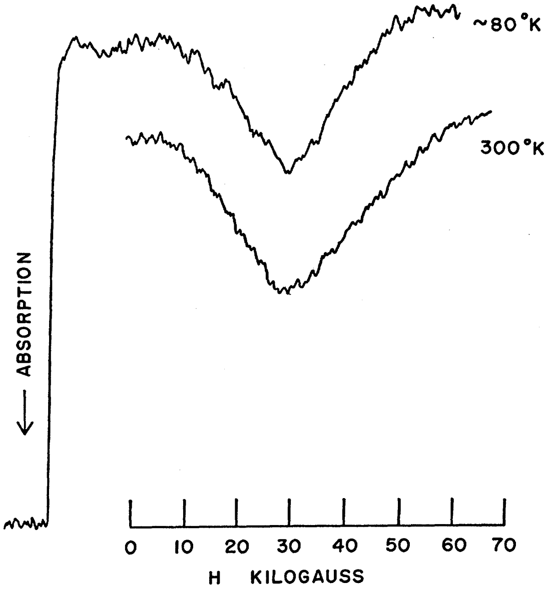

FIG. 1. Cyclotron resonance absorption in InAs at 111 cm^{-1} (90 μ).

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[1] E. Burstein, G. S. Picus and H. A. Gebbie, Phys. Rev. **103**, 825 (1956).

[2] E. D. Palik, G. S. Picus, S. Teitler and R. F. Wallis, Phys. Rev. **122**, 475 (1961).

[3] R. J. Keyes, S. Zwerdling, S. Foner, H. H. Kolm, and B. Lax, Phys. Rev. **104**, 1804 (1956).

[4] B. Lax and J. B. Mavroides, *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1960), Vol. 11.

[5] E. D. Palik, G. S. Picus, S. Teitler, and R. F. Wallis, Czechoslov. J. Phys. (to be published).

At room temperature the band was quite broad, but it sharpened appreciably at low temperature. For InP there appeared to be no difference between the respective room temperature and low temperature effective mass ratios.

The effective masses were computed using the formula $m^*=eH/\omega c$, where H is the magnetic field at which resonance occurs. In Fig. 2 are shown the results for InAs and InP. Here, the effective mass ratio m^*/m is plotted against magnetic field. The data show that the mass increases with increasing magnetic field. This is indicative of the nonparabolic nature of the conduction band in which the mass increases with increasing energy. The variation of mass with magnetic field has been discussed by Palik *et al.*[2] in connection with InSb. The theory outlined there yields that the energy difference ΔE between the two lowest Landau levels is given by

$$\Delta E=\hbar\omega_c[1-(\hbar\omega_c/E_G)(A-B)].$$

Equating $\Delta E=\hbar eH/m^*c$, an effective mass m^* is obtained which has a magnetic field dependence. Then

$$\frac{1}{m^*}=\frac{1}{m_0^*}[1-(\hbar\omega_c/E_G)(A-B)], \tag{1}$$

where

$$A=2\left(1-\frac{m_0^*}{m}\right)^2\frac{(1+\frac{1}{2}x^2)}{(1+\frac{1}{2}x)},$$

$$B=\frac{1}{2}\left(1-\frac{m_0^*}{m}\right)(1-x)$$

$$\times\left[\frac{(1+\frac{3}{4}x+\frac{1}{2}x^2)\left(1-\dfrac{m_0^*}{m}\right)}{(1+\frac{1}{2}x)^2}-\frac{4}{3}\frac{m_0^*}{m}\right],$$

$$x=[1+(\Delta/E_G)]^{-1},$$

$$\omega_c=eH/m_0^*c,$$

E_G is the band gap, Δ is the spin-orbit splitting, m_0^* is the effective mass at the bottom of the band in zero magnetic field, and m is the free-electron mass.

For the materials and magnetic fields considered in this paper, the quantity $(\hbar\omega_c/E_G)(A-B)$ is always small compared to unity. The expression for m^* can then be written in the approximate form,

$$m^*=m_0^*[1+(\hbar\omega_c/E_G)(A-B)], \tag{2}$$

which illustrates the linear dependence of m^* on H. The results given by Eqs. (1) and (2) take into account only interactions between valence and conduction bands. In principle, interactions with higher bands could be included as discussed by Bowers and Yafet.[6] A rough estimate indicates that inclusion of higher bands might increase the slope of the m^*/m vs H plot for InAs by a few percent and for InP by possibly 20%. Assuming for InAs at liquid nitrogen temperature that $E_G=0.41$ ev,[7] $\Delta=0.43$ ev,[7] and $m_0^*/m=0.023$, the dependence of the effective mass on magnetic field was calculated using Eq. (1) and the results are given by the solid line for InAs in Fig. 2. The pulsed field work of Keyes *et al.*[3] giving an effective mass of ~0.03 at fields between 150 and 250 kgauss at room temperature, is consistent with the present results. Lax and Mavroides[4] present other room temperature cyclotron resonance data of Keyes which suggests a zero-magnetic-field mass ratio of ~0.015. The present data indicate at most a very slight temperature dependence of the effective mass in InAs. We have estimated the temperature change of effective mass due to dilation of the lattice and the subsequent change in band gap[2] and the temperature change in mass due to the distribution of carriers in the nonparabolic band.[8] The first effect increases the effective mass by about 6% as the temperature decreases from room temperature to liquid nitrogen temperature while the second effect decreases the effective mass as the temperature decreases. For the InAs sample used, these two effects roughly cancel indicating little or no change in effective mass with temperature in agreement with our observations. In view of these considerations, it seems unlikely that the discrepancy between the value of 0.015 discussed by Lax and Mavroides and the value 0.023 reported here can be attributed solely to a temperature dependence of the mass.

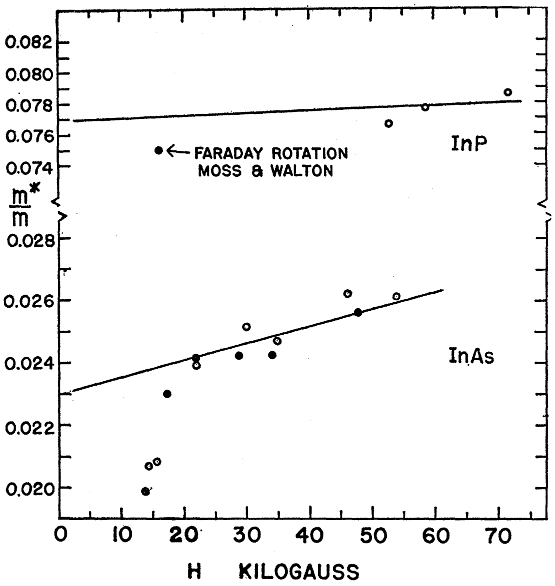

FIG. 2. Effective-mass ratio vs magnetic field for InAs and InP from cyclotron resonance data obtained at liquid nitrogen temperature. Open circles indicate data obtained with orientation I and solid circles data obtained with orientation II.

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[6] R. Bowers and Y. Yafet, Phys. Rev. **115**, 1165 (1959).

[7] F. Matossi and F. Stern, Phys. Rev. **111**, 472 (1958).

[8] R. F. Wallis, J. Phys. Chem. Solids **4**, 101 (1958).

Matossi and Stern[7] use a value of 0.021 for the conduction electron effective mass ratio in their discussion of optical absorption of p-type InAs. Also, electrical measurements by Sledek[9,10] and Frederikse and Hosler[11] give values of about 0.020.

For InP, assuming at liquid nitrogen temperature that $E_G = 1.31$ ev,[12] $\Delta = 0.24$ ev,[12] and $m_0^*/m = 0.077$, a theoretical curve based on Eq. (1) has been calculated and is shown as a solid line in Fig. 2. A comparison between cyclotron resonance and Faraday rotation masses for InP can be made in analogy to the comparison for InSb discussed by Palik *et al.*[2] The work of Moss and Walton[12,13] yields an effective-mass ratio of 0.075 ± 0.008 for a sample with $N = 1.07 \times 10^{16}/\mathrm{cm}^3$. This point is shown on the plot of Fig. 2. The Faraday rotation and cyclotron resonance data agree to within experimental error.

In the present work the uncertainty in wavelength is about ±2% and the uncertainty in the resonant magnetic field is about ±5%. We estimate the effective mass ratios for InAs and InP to be 0.023 ± 0.002 and 0.077 ± 0.005, respectively.

In calculating the effective mass from the position of the cyclotron resonance line as observed in transmission, corrections arising from magneto-plasma effects may be necessary if measurements are made near the plasma frequency ω_p. A particularly important correction[4] involving the so-called depolarization factor L occurs if the sample surface is parallel to the magnetic field H_z, and the radiation electric field E_x is plane-polarized perpendicular to H_z, hereafter referred to as orientation I. In this case for the rectangular thin sections used, the pertinent depolarizing factor is unity. For $\omega\tau \gg 1$, the resonance condition leading to a maximum in the real part of the conduction is

$$\omega_L^2 + \omega_c^2 - \omega^2 = 0$$

or

$$\omega_c = \omega \left(1 - \frac{\omega_L^2}{\omega^2}\right)^{\frac{1}{2}}, \tag{3}$$

where $\omega_L^2 = L 4\pi N e^2/\epsilon_0 m^*$. If $L = 1$, $\omega_L = \omega_p$, the usual plasma frequency.

For the other orientation with the plane of the sample perpendicular to H_z and the radiation field E_x perpendicular to H_z, hereafter referred to as orientation II, the pertinent depolarization factor is zero and no correction of this kind is necessary. For InAs with $\omega_p = 40\ \mathrm{cm}^{-1}$ both orientations were used. For a specific orientation, the room temperature and low temperature data were essentially the same. At high magnetic fields the data for both orientations were the same giving the same effective masses. However, at low magnetic fields where the resonance occurred at $\omega < 2\omega_p$, there were differences in the data for the two orientations. The low temperature data for both orientations are shown in Fig. 2. These data indicate that the correction for orientation I discussed above is not important when $\omega > 2\omega_p$. For the low field data where $\omega < 2\omega_p$, the correction for orientation I specified by Eq. (3) gives qualitatively reasonable results. Quantitatively the corrections appear to be somewhat too large compared to the data for orientation II.

The data of Fig. 2 for orientation II fall on a straight line except for the lowest field point observed with KRS-5. Ignoring the low points, a straight line given by Eq. (2) fits the other points for both orientations.

There is another allied reason why the line observed in transmission deviates from its expected position when ω is only slightly larger than ω_p. Whereas the real part of the conductivity and the imaginary part of the dielectric constant peak at ω_c for orientation II, the absorption constant and transmission do not peak at ω_c. The transmission can be altered by the rapidly varying reflectivity in the region near ω_p when the magnetic field is varied for both orientations I and II.[4] The transmission of a sample taking into account the varying reflectivity can be estimated by extensive calculations. Such calculations qualitatively suggest that for ω near ω_p the transmission as a function of magnetic field is altered so that the minimum transmission occurs at a lower field than that corresponding to the maximum in the real part of the conductivity. A correction of this kind appears to be necessary for the KRS-5 data in Fig. 2 for orientation II. The raw data provide further evidence that the line is distorted due to large changes in reflectivity, for as the field is increased beyond the apparent resonance line, the transmission increases appreciably above the zero field value.

The quantitative determination of the corrections to be applied when ω is near ω_p may be complicated by the presence of inhomogeneities in the particular InAs sample used. If the carrier concentration is non-uniform, the proper plasma frequency for making the corrections may not be the same as that calculated from the mean carrier density.

For InP at liquid nitrogen temperature $\omega_p = 24\ \mathrm{cm}^{-1}$ ($417\ \mu$). The measurements made at high magnetic fields using KRS-5, CsI, and CsBr reststrahlen plates were sufficiently far removed from ω_p that no significant correction was necessary.

We have made Faraday rotation measurements on a sample of InAs taken from the same larger piece of material from which the cyclotron resonance sample was made. The measurements were made at liquid nitrogen temperature in the spectral region from 15 to 20 μ using low magnetic fields. The usual straight line variation of rotation angle with magnetic field was observed at several wavelengths. For the InAs sample at liquid nitrogen temperature with a thickness $d = 0.138$ cm and carrier concentration $N = 7 \times 10^{15}/\mathrm{cm}^3$, an effec-

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[9] R. J. Sledek, Phys. Rev. **105**, 460 (1957).

[10] R. J. Sledek, Phys. Rev. **110**, 817 (1958).

[11] H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **110**, 880 (1958).

[12] T. S. Moss and A. K. Walton, Physica **25**, 1142 (1959).

[13] A. K. Walton, (private communication).

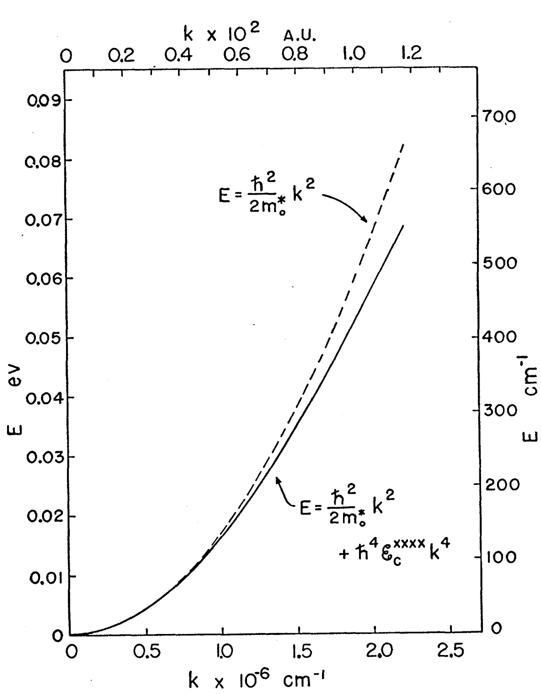

FIG. 3. Parabolic conduction band of InAs with $m_0^*=0.023$ and nonparabolic band as deduced from cyclotron resonance.

tive mass ratio of 0.026±0.003 was obtained. For the pure sample used, this represents the mass ratio near the bottom of the band. The uncertainty here is large because of the fact that six quantities must be measured to determine the effective-mass ratio, carrier concentration N, thickness d, magnetic field H, wavelength λ, index of refraction n, and rotation angle θ. Uncertainty in N contributes the largest uncertainty in m^*. The effective-mass ratio obtained from low-field Faraday rotation is in reasonable agreement with the one obtained from cyclotron resonance. We feel that cyclotron resonance gives the better value.

Also, Faraday rotation measurements were made at liquid nitrogen temperature on a sample of InAs with 9.6×10^{16} carriers/cm^3. An effective-mass ratio of 0.030 was obtained. The Faraday rotation and cyclotron resonance masses for InAs were compared in a manner analogous to the comparison carried out for InP above. The effective-mass ratio point 0.030 falls at an equivalent magnetic field of about 80 kgauss, in reasonable agreement with the cyclotron data considering experimental uncertainties.

Faraday rotation measurements were also made at room temperature for the InAs sample with 9.6×10^{16}/cm^3. A room temperature effective-mass ratio of 0.032 was obtained. The change of effective mass with temperature is discussed by Cardona[14] who finds that dilational effects of the lattice and carrier distribution effects in the nonparabolic band can account for such changes in masses measured by Faraday rotation. For a degenerate sample with half the carrier concentration used in the present experiment, Cardona calculates that the two temperature effects will produce a net decrease in mass of about 8% in going from room temperature to liquid nitrogen temperature and measures a 5% decrease. This is in agreement with our observations.

The results for InAs have been used to calculate the shape of the conduction band. In Fig. 3 the parabolic band given by $E=\hbar^2k^2/2m_0^*$ for a fixed effective-mass ratio of 0.023 is shown along with the nonparabolic band given by $E=(\hbar^2k^2/2m_0^*)+\hbar^4\mathcal{E}_c^{xxxx}k^4$.[2] The energy in ev and cm^{-1} is plotted against the wave vector in cm^{-1} and a.u. (atomic units).

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