Velocity-Dependent Correlations in the Statistical Distribution of the Electric Microfield in a Plasma

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The polarization of a plasma in the neighborhood of a moving ion depends on the ion velocity. This affects the distribution of the stochastic field acting upon the ion. The correction to the Holtsmark distribution due to the complete test particle—field particle correlation including this dynamic effect is calculated up to the order e^2. The result is: (1) a shift towards smaller fields, (2) anisotropy, and (3) velocity dependence, which is not necessarily equal to the zero velocity effect even on the average.

I. INTRODUCTION

NEW methods for calculating the probability $W(\mathbf{E})$ that a test particle traveling through a plasma experiences a given electric field $\mathbf{E}$, have been suggested recently. The original work on this problem is due to Holtsmark[1] who determined the probability $W(\mathbf{E})$ for the case when the test particle is a neutral atom. This calculation finds its application in problems related to the broadening of spectral lines.[2] Chandrasekhar[3] used the Holtsmark results to find the probability $W(\mathbf{F})$ for a force $\mathbf{F}$ exerted on a star, due to the gravitational attraction of the neighboring stars. The Holtsmark distribution is obtained by the complete neglect of the correlations between the particles, and by treating the stochastic field as a superposition of independent random events. In fact, of course, correlations do exist in the system and they cause deviations of various types from the Holtsmark distribution. One may conveniently classify them as (1) correlations between the plasma particles themselves, and (2) correlations between the test particles and the plasma particles.

Diverse approaches have been employed to include the correlations in the calculations of the probability distribution. A group of workers have concentrated on the effect of the collective correlations. Mayer[4] treats the system of field particles as a system of simple harmonic oscillators for small fields [small $\mathbf{E}$ in $W(\mathbf{E})$], and for large fields he takes into account only a single nearest neighbor. By using the Bohm-Pines[5] method of *collective coordinates* in separating the electric field into short- and long-range components, Broyles[6,7] has been able to consider these correlation effects rather accurately. Another school has used the *effective potential* of Debye-Hückel type[8] (which is again a result of collective correlations) to describe the field of the individual particles. Calculations have been made by Edmonds[9] and Hoffman and Theimer.[10] Ecker and Müller[11,12] have refined these methods and have been able to show by careful machine calculation[12] that one can approximate the collective correlations by using a cutoff at the field corresponding to the Debye length. Some further aspects of the effective potential method have been discussed by Theimer *et al.* in several articles.[10] A novel approach has been given recently by Baranger and Mozer.[13] It is based on a systematic cluster type expansion of the many-particle distribution and takes into account correlations of increasing order in the perturbation parameter e^2.

The correlations between the test particle and the plasma particles, if considered, are taken generally into account through the Boltzmann factor, eventually containing the Debye-Hückel potential. However, the concept of local equilibrium, which is the underlying physical picture, is hardly applicable to plasmas.[14] Instead, the distribution of field particles around a moving test particle results as a solution of the corresponding nonequilibrium problem.[14,15] Such a treatment reveals the essential dependence of the particle distribution on the test particle velocity. One can easily convince oneself that such a *polarization effect* results

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partly in an angular dependence of the field distribution, partly in a change in the distribution of the directionally averaged field. The present note is devoted to the explicit calculation of this effect. Our starting point is $f^{(1)}(\mathbf{r},\mathbf{v})$, the field particle distribution calculated by us in reference 14. In this treatment $f^{(1)}$ is correct up to e^2 and yields the distribution of the field particles within the Debye sphere. Outside the Debye sphere ($r>h$, $h^2=kT/4\pi e^2n$, or for wave numbers $k<h^{-1}$), $f^{(1)}$ has been taken to be zero. This approximation has been justified at length in reference 14. Thus our procedure consists of the following. The unshielded Coulomb field of the uncorrelated field particles is considered. The field particles are distributed according to the polarized perturbed density in the neighborhood of the test ion. The integration of $f^{(1)}$ around the ion is extended to finite region only. As a lower limit we take $b=e^2/kT$, the collision parameter: Within this sphere the linearization certainly breaks down,[14] but the contribution of the corresponding large field is not significant. For the upper limit, h is employed as explained in the foregoing. We may point out that in this way both the Boltzmann factor (up to e^2) and the screening (through the cutoff) are automatically included. Apart from these customary corrections a distinct velocity-dependent dynamical effect shows up, which in our approximation is additive.

The integration of the unperturbed part of the distribution $f^{(0)}(v)$ is extended over the whole space and results in the usual Holtzmark type $C(\mathbf{p})$ (Chandrasekhar's[3] notation is used). The probability distribution $W(\mathbf{E})$, however, is not a linear functional of $C(\mathbf{p})$ and the additivity does not prevail in the final result.

The reader should be warned here that our procedure is definitely not consistent. The correlation between the plasma particles themselves has a contribution of the order e^2, and if there is no reason to the contrary this gives a correction to the Holtsmark distribution of the same order of magnitude as that considered here. The justification of the omission of this factor is that we believe that (this effect being physically distinct) its influence should be considered separately. In fact, this has been the chief concern of many previous investigations. In principle we might improve upon our calculations in order to include these field particle–field particle correlations by including results from other works. We may use the $r_{\max}=h$ cutoff for the undisturbed part of the distribution (Ecker[11]) or we may add to $C(\mathbf{p})$ the second-order correction $h_2(p)$ as calculated by Baranger and Mozer.[13] The latter procedure is in our opinion the most consistent, corresponding to the spirit of the perturbation analysis employed here. In the first approximation the two effects (ours, and the field particle–field particle correlations) are additive, and therefore the superposition of distinct corrections to $C(\mathbf{p})$ is admissible. The nonlinear dependence of $W(\mathbf{E})$ on $C(\mathbf{p})$, of course, mixes the various corrections finally.

Recently Baranger and Mozer[13] have extended their cluster-expansion method to the case of a charged test particle. Thus they succeeded in carrying out a systematic analysis consistent with e^2, and their work in this respect is superior to ours. On the other hand, they go beyond the customary Debye scheme in the definition of the correlations, using constant-density and Debye-type distributions for large and small relative velocities, respectively. These in fact constitute the two limiting cases of our Eq. (7) for $v\gg v_T$ (thermal velocity) and $v\ll v_T$.

II. FORMULATION OF THE PROBLEM

We consider the probability $W(\mathbf{E})d\mathbf{E}$ that a *moving* ion experiences an electric field in the range $\mathbf{E}$ to $\mathbf{E}+d\mathbf{E}$ in a plasma. The probability distribution $W(\mathbf{E})$ can be obtained by applying the usual Markov method as obtained, e.g., by Chandrasekhar.[3] Chandrasekhar's basic assumptions ($V\to\infty$, $N\to\infty$, $n_0=N/V$ $=$constant, no correlations between the sources of the field) and notations will be adopted in this paper. The field of an individual particle is taken as

$$\mathbf{E}_i=e_2\mathbf{r}/r^3, \tag{1}$$

and no explicit shielding effect is considered. (Here and in the following, subscript 2 and i refer to the field particles and subscript 1 refers to the test particle.) However, in the correction we calculate, the particles outside the Debye sphere do not contribute. Therefore, to be able to apply the Markov procedure, we need the additional stricter condition:

$$N_h\gg1, \quad N_h\approx h^3n_0; \tag{2}$$

that is, the number of particles within the Debye sphere of radius h should be large. This requirement is well satisfied under the usual circumstances in a high-temperature plasma, the ratio h/d ($d=$interparticle distance; $n_0=d^{-3}$) being large.

To take care of the correlations between the test particle and the field particles, one considers the density of the field particles around the test ion, $n(\mathbf{r})$. It is customary to regard this as the static pair correlation function (pertaining to a test particle at rest), given by the Boltzmann factor containing the effective Debye potential[3]

$$n(r)=n\exp[-(b/r)e^{-r/h}], \tag{3}$$

where $b=e_1e_2/kT$ is the collision diameter and n_0 is the average density. To be consistent with our cutoff approximation and in virtue of the perturbation approach we apply (and in the spirit of the Debye approximation, too), we make the following simplification:

$$\begin{aligned} n(r)&=n_0\exp(-b/r), & r&<h\\ &=n_0, & r&>h. \end{aligned} \tag{4}$$

We make use of the fact that

$$b/d\ll1, \tag{5}$$

and get

$$n(r)=n_0(1-b/r), \quad b<r<h$$
$$=n_0, \quad r>h. \tag{6}$$

In fact, instead of (6) we wish to use the more exact dynamical correlations, which result from the first-order solution of the Boltzmann-Vlasov equation. This has been given in reference 14,

$$n(\mathbf{r})=n_0\{1-(b/r)[1-\Phi(\alpha^{\frac{1}{2}}\mathbf{r}\cdot\mathbf{v}/r)]$$
$$\times\exp(-\alpha v^2[1-(\mathbf{r}\cdot\mathbf{v}/rv)^2])\}, \quad b<r<h \tag{7}$$
$$=n_0, \quad r<b, \quad r>h,$$

where $\mathbf{v}$ is the velocity of the test particle and where $\alpha=m_2/2kT$ is characteristic for the thermal velocity of the plasma. Φ is defined by

$$\Phi(x)\equiv\frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt.$$

The essential difference between (6) and (7) emerges (i) through the velocity dependence and (ii) through the anisotropic distribution around the test particle (compare with Fig. 2 in reference 14). These two effects represent the essential departure in the present paper from previous considerations. In contrast to the customary isotropic $W(\mathbf{E})$ it results in a probability distribution $W(\mathbf{E})$ depending on the direction of $\mathbf{E}$.

To proceed, we follow Chandrasekhar's considerations.[3] The probability distribution is given through the characteristic function $A(\mathbf{p})$ as

$$W(\mathbf{E})=(2\pi)^{-3}\int d\mathbf{p}\, A(\mathbf{p})\exp(-i\mathbf{p}\cdot\mathbf{E}), \tag{8}$$

$$A(\mathbf{p})=\prod_{i=1}^{N}\int d\mathbf{r}\,\tau_i(\mathbf{r}_i)\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r}_i)]. \tag{9}$$

In (9), $\mathbf{E}_i(\mathbf{r}_i)$ is the field of the ith particle situated at the point $\mathbf{r}_i$ with respect to the test particle. $\tau_i(\mathbf{r}_i)$ governs the probability of occurrence of the ith particle at the point $\mathbf{r}_i$. Supposing that only statistical fluctuations compatible with the average density $n(\mathbf{r})$ given by (7) occur,

$$\tau_i(\mathbf{r}_i)=n(\mathbf{r})/N_h, \tag{10}$$

and assuming N and N_h to be very large, one gets

$$A(\mathbf{p})=e^{-C(\mathbf{p})}, \tag{11}$$

where

$$C(\mathbf{p})=\int d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\}n(\mathbf{r}). \tag{12}$$

III. THE CHARACTERISTIC FUNCTION

The exact computation of $C(\mathbf{p})$ unfortunately cannot be performed. To overcome the complexities of the integration we simplify matters by restricting ourselves to the low-velocity approximation. Nevertheless, no significant part of the problem will be lost in this way. We approximate $n(\mathbf{r})$ by expanding it with respect to $\mathbf{v}$, retaining first-order terms only:

$$n(\mathbf{r})=n_0\{1-(b/r)[1-(2/\sqrt{\pi})\mathbf{w}\cdot(\mathbf{r}/r)]\}, \quad b<r<h \tag{13}$$

$$=n_0, \quad r<b, \; r>h \tag{13}$$

where

$$\mathbf{w}=\alpha^{\frac{1}{2}}\mathbf{v}.$$

Then (12) is replaced by

$$C(\mathbf{p})=\int d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\}$$
$$\times n_0[1-(b/r)+(2/\sqrt{\pi})b\mathbf{w}\cdot(\mathbf{r}/r^2)]. \tag{14}$$

To carry out the integration, we consider the three parts of the bracket separately.

(a) The first term,

$$C_1(\mathbf{p})=n_0\int d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\}, \tag{15}$$

is identical with the $C(\mathbf{p})$ of the Holtsmark distribution and yields[3]

$$C_1(\mathbf{p})=E_n^{\frac{3}{2}}p^{\frac{3}{2}},$$
$$E_n=(4/15)^{\frac{2}{3}}2\pi e_2 n_0^{\frac{2}{3}}=2.61\,E_d, \tag{16}$$
$$E_d=e_2/d^2.$$

E_d is the field corresponding to the interparticle distance.

(b) In the second term,

$$C_2(\mathbf{p})=-n_0 b\int_{r=b}^{r=h} d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\}1/r, \tag{17}$$

we introduce the new variables

$$\mathbf{u}=\mathbf{E}_i(\mathbf{r})=e_2\mathbf{r}/r^3, \quad d\mathbf{r}=-\tfrac{1}{2}e_2^{\frac{3}{2}}u^{-9/2}d\mathbf{u}, \tag{18}$$

and we obtain with $z=\cos(\mathbf{p},\mathbf{u})$

$$C_2(p)=-\pi b n_0 e_2\int_{e_2/h^2}^{e_2/b^2}\frac{du}{u^2}\int_{-1}^{1}dz[1-\exp(ipuz)]. \tag{19}$$

In the above integral we replace the upper limit by infinity and the lower one by zero. The justification of this procedure is as follows. The corrections to the Holtsmark distribution that we take into account, are of the first order in e^2, or in ϵ_0 [in terms of the dimensionless parameter ϵ_0 which is defined by (34)]. Any term of higher order in it can be omitted or added according to convenience. The change in the upper limit amounts to the neglect of the integral

$$\pi b n_0 e_2 \int_{e_2/b^2}^{\infty} \frac{du}{u^2} \int_{-1}^{1} dz[1-\exp(ipuz)] \approx b n_0 e \int_{e_2/b^2}^{\infty} \frac{du}{u^2} = b^3 n_0 \approx \epsilon_0^2. \quad (20)$$

To see the value of the term added through the alteration of the lower limit, we expand the integrand for small values of u:

$$(1/u^2)[1-\exp(ipuz)] = (1/u^2)(-ipuz+p^2u^2z^2+\cdots). \quad (21)$$

The first term vanishes in the z integration. Then we are left with the integral

$$I=\pi b n_0 e_2 \int_0^{e_2/h^2} p^2 du \int_{-1}^{1} dz \approx b n_0 e_2^2 h^{-2} p^2. \quad (22)$$

For the sake of an order-of-magnitude estimate, we set

$$p \approx E_n^{-1} \approx \epsilon_0 h^2/e, \quad (23)$$

and obtain that I is indeed proportional to ϵ_0^2.

Thus, we put

$$C_2(\mathbf{p}) = -2\pi b n_0 e_2 \int_0^{\infty} [1-(\sin pu/pu)] du/u^2, \quad (24)$$

and obtain

$$C_2(\mathbf{p}) = -(\pi^2/2) b e_2 n_0 p = -(\pi/8) E_1 p, \quad (25)$$

where

$$E_1 = e_1/h^2 \quad (26)$$

is the field produced by the test particle on the surface of the Debye sphere.

(c) The third term describing the dynamical correlation can be treated along similar lines:

$$C_3(\mathbf{p}) = (2/\sqrt{\pi}) b n_0 \int_{r=b}^{r=h} d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\} \times \mathbf{w}\cdot(\mathbf{r}/r^2). \quad (27)$$

Substituting (18) in (27), we have

$$C_3(p) = -(1/\sqrt{\pi}) b n_0 e_2 \int_{u=e_2/b^2}^{u=e_2/h^2} d\mathbf{u}[1-\exp(i\mathbf{p}\cdot\mathbf{u})] \times \mathbf{w}\cdot(\mathbf{u}/u^5). \quad (28)$$

Choosing in $\mathbf{u}$ space the coordinate system so that

$$\mathbf{p} = \mathbf{1}_z p, \quad \mathbf{w} = w(\mathbf{1}_z \cos\eta + \mathbf{1}_x \sin\eta), \quad (29)$$

where η is the angle between $\mathbf{w}$ and $\mathbf{p}$, we get with $E_2 = e_2/h^2$

$$C_3(\mathbf{p}) = -2\sqrt{\pi} n_0 b e_2 \mathbf{w}\cdot(\mathbf{p}/p) \times \int_{E_2}^{e_2/b^2} \frac{du}{u^2} \int_{-1}^{1} dz[1-\exp(ipuz)]z. \quad (30)$$

Now, it is easy to see that the foregoing considerations used in changing the limits of the integral allow here as well of the replacement of the upper limit by infinity, but do not apply to the lower limit where the integral exhibits a logarithmic divergence. Thus we integrate with finite limits (corresponding to the Debye sphere). E_2 is carried through as a parameter, and we get

$$\begin{aligned} C_3(\mathbf{p}) &= i(4/3)(\sqrt{\pi}) n_0 b e_2 \mathbf{w}\cdot\mathbf{p}[4/3-\mathrm{Ci}(pE_2)] \\ &= i(E_1/3\sqrt{\pi})[4/3-\mathrm{Ci}(pE_2)]\mathbf{w}\cdot\mathbf{p}., \end{aligned} \quad (31)$$

where[16]

$$\mathrm{Ci}(x) \equiv -\int_x^{\infty} dt \cos t/t$$

is the cosine integral, which diverges logarithmically for small values of x (large distance). This is due to the polarization effect and the accumulation of charges in the wake of the moving test particle. In fact, the Debye screening makes this contribution finite. Actually it remains finite even if $h \to \infty$, if this limit is taken properly considering that in this case both E_1 and E_2 vanish, and a weakening of the correlation accompanies the increase of the Debye length.

To conclude this section, we write the characteristic function combining the three terms,

$$A(\mathbf{p}) = \exp\{-E_n^{\frac{3}{2}} p^{\frac{3}{2}} + (\pi/8) E_1 p - i(1/3\sqrt{\pi}) E_1 [4/3-\mathrm{Ci}(pE_2)]\mathbf{w}\cdot\mathbf{p}\}. \quad (32)$$

IV. THE DISTRIBUTION FUNCTION

To evaluate $W(\mathbf{E})$, we substitute (32) into (8) and carry out the integration. It is convenient to employ dimensionless quantities. We define

$$\boldsymbol{\varepsilon} \equiv \mathbf{E}/E_n, \quad (33)$$

and similarly instead of E_1 and E_2 we write

$$\begin{aligned} \epsilon_1 &= E_1/E_n = (e_1/e_2)\epsilon_0, \\ \epsilon_0 &= E_2/E_n = (15/4)^{\frac{2}{3}}(1/2\pi)(d/h)^2 \\ &= 2(15/4)^{\frac{2}{3}}(n^{\frac{1}{3}}/kT)e^2. \end{aligned} \quad (34)$$

Changing the variables of the integration:

$$\mathbf{x} = E_n \mathbf{p}, \quad d\mathbf{x} = E_n^3 d\mathbf{p}, \quad (35)$$

we obtain

$$W(\boldsymbol{\varepsilon}) = (2\pi E_n)^{-3} \int d\mathbf{x}\{\exp[-x^{\frac{3}{2}} + (8/\pi)(e_1/e_2)\epsilon_0 x] - i(1/3\sqrt{\pi})(e_1/e_2)\epsilon_0[\tfrac{4}{3}-\mathrm{Ci}(x\epsilon_0)]\mathbf{w}\cdot\mathbf{x} - i\mathbf{x}\cdot\boldsymbol{\varepsilon})\}. \quad (36)$$

We use

$$\mathbf{D}(x) = \boldsymbol{\varepsilon} + \mathbf{w} g(x,\epsilon_0), \quad (37)$$

where

$$g(x,\epsilon_0) = (1/3\sqrt{\pi})(e_1/e_2)\epsilon_0[\tfrac{4}{3}-\mathrm{Ci}(\epsilon_0 x)] \quad (38)$$

is a known function of x with ϵ_0 as a parameter. Thus.

[16] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945).

with $y=\cos(\mathbf{x},\mathbf{D})$, (36) yields

$$W(\boldsymbol{\varepsilon})=(1/4\pi^2E_n{}^3)\int_0^\infty dx\,\exp[-x^{\frac32}+(\pi/8)(e_1/e_2)\epsilon_0 x]\times\int_{-1}^{1}dy\,\exp[-ixD(x)y]$$

$$=(1/2\pi^2E_n{}^3)\int_0^\infty dx\,x\,\exp[-x^{\frac32}+(\pi/8)(e_1/e_2)\epsilon_0 x]\times[\sin xD(x)/D(x)].\quad(39)$$

It is convenient to write $W(\boldsymbol{\varepsilon})$ as

$$W(\boldsymbol{\varepsilon})=(1/4\pi E_n{}^3\epsilon^2)G(\boldsymbol{\varepsilon}),\quad(40)$$

where

$$G(\boldsymbol{\varepsilon})=(2\epsilon/\pi)\int_0^\infty dx\,x\,\exp[-x^{\frac32}+(\pi/8)(e_1/e_2)\epsilon_0 x]\times\{\sin[\epsilon x|(\boldsymbol{\varepsilon}/\epsilon)+(\mathbf{w}g(x,\epsilon_0)/\epsilon)|]/|(\boldsymbol{\varepsilon}/\epsilon)+(\mathbf{w}g(x,\epsilon_0)/\epsilon)|\}\quad(41)$$

is now the modified Holtsmark distribution.

It is interesting to compare qualitatively this result with the well-known Holtsmark function [see reference 3, Eq. (553)]

$$H(\epsilon)=(2\epsilon/\pi)\int_0^\infty dx\,x\,\exp(-x^{\frac32})\sin\epsilon x.\quad(42)$$

$G(\boldsymbol{\varepsilon})$ and $H(\epsilon)$ are identical either in the trivial case $e_1=0$ (no charge on the test particle) or when $\epsilon_0=0$, which is equivalent to no screening or infinite temperature. The departure of $G(\boldsymbol{\varepsilon})$ from $H(\epsilon)$ originates from three distinct reasons:

(i) The field particle–field particle correlation which brings about the screening and the finite range of the electric field. The main effect of this correlation is absent in our treatment, but it appears in the cutoff we applied in the calculations of the velocity-dependent term.

(ii) The static correlation between the charged test particle and the field particles (Boltzmann factor). This correlation is borne out by the exponential term, $\exp[(\pi/8)(e_1/e_2)\epsilon_0 x]$, in the integrand of $G(\boldsymbol{\varepsilon})$. This point has been discussed by a number of authors.[2,6,7,10,12] We remark only that in the case of ion-ion correlations, a case that mostly pertains to experimental interest, it results in the increase at smaller ϵ values.

(iii) The dynamic velocity-dependent correlation between the test particle and the field particles, as reflected by the expression $|(\boldsymbol{\varepsilon}/\epsilon)+(\mathbf{w}g(x)/\epsilon)|$. This represents the main effect from the point of view of the present work. It gives rise to the anisotropy of $G(\boldsymbol{\varepsilon})$, a direction distinguished by $\mathbf{w}$ being given in the distribution function. Again for ion-ion correlations small fields parallel to the motion of the test ion have an increased probability, while for antiparallel fields the probability for large fields becomes greater.

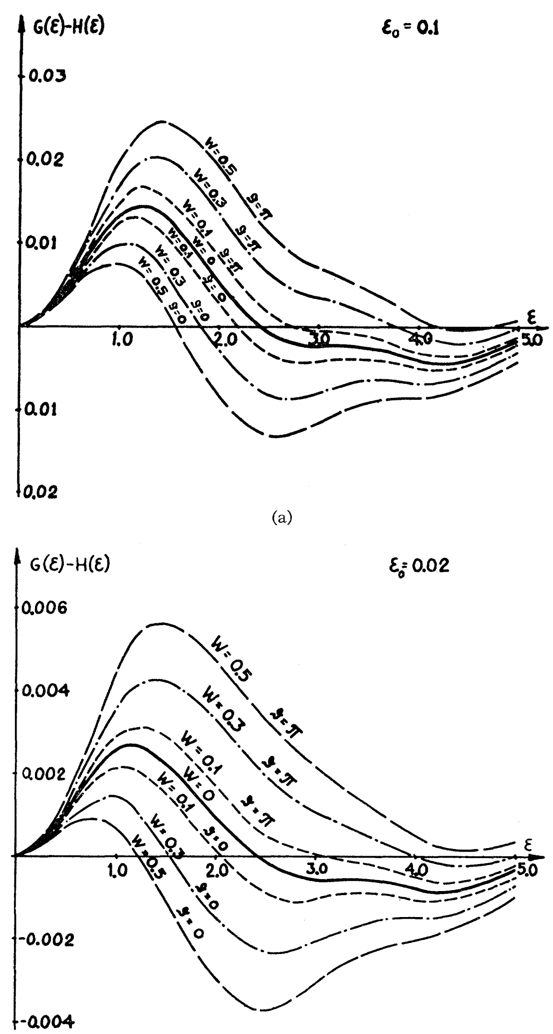
V. NUMERICAL CALCULATION OF $G(\boldsymbol{\varepsilon})$

The modified Holtsmark distribution $G(\boldsymbol{\varepsilon})$ has been calculated numerically on the WEGEMATIC of the Weizmann Institute of Science, Rehovoth, using (41) with the following choice of parameters:

(1) The test particle and the field particle charges are equal: $e_1=e_2$.

(2) The test particle velocity takes four values: $w=\alpha^{\frac12}v=0$, 0.1, 0.3, 0.5.

(3) The field points parallel, antiparallel, or perpendicular to the velocity $\mathbf{v}$; with ϑ (the angle between $\boldsymbol{\varepsilon}$ and $\mathbf{v}$) being, respectively, $\vartheta=0$, $(\pi/2)$, π.



(a)

(b)

FIG. 1(a) Deviations from the Holtsmark distribution $H(\epsilon)$ due to the static and dynamic correlations between the test particle and the plasma particle. The cutoff parameter ϵ_0, as defined be Eq. (34), is taken to be 0.1. Curves are given for $w=\alpha^{\frac12}v=0$, 0.1, 0.3, 0.5 and angles $\vartheta=0$, $(\pi/2)$, π. The curves corresponding to $\vartheta=\pi/2$ for the above velocity values coincide with the $w=0$ curve (pure static effect). (b) The same as (a), with $\epsilon_0=0.02$.

(4) The cutoff parameter [Eq. (34)] takes the values: $\epsilon_0 = 0.02$, 0.05, 0.1.

The results are given in Figs. 1(a), 1(b), 2, and 3.

VI. DISCUSSION OF RESULTS

In this paper the modification due to the velocity-dependent correlation between a test ion and the surrounding plasma particles in the distribution function of the field acting upon the ion has been considered. First-order corrections in e^2 [or in $\epsilon_0 \approx (n^{\frac{1}{3}}/kT)e^2$] have been retained, but the more or less familiar field particle-field particle correlations have not been included explicitly. The numerical calculations are correct for test-particle velocities smaller than the thermal velocity. An expected drift in the Holtsmark distribution results, but apart from that there is a novel effect of a *direction* and a *velocity* dependence in the probability distribution. One may speculate about the experimental ramification of these results. The quasi-static Stark broadening of spectral lines emitted by the ion moving in a plasma will follow the shape of the distribution, after a directional average is performed. If the emitting ions are in thermal equilibrium, the final broadening appears as the weighted sum over the velocity distribution of the lines calculated for a particular velocity. One can roughly estimate whether there is a difference between this effect and the broadening calculated through the static correlation. To do this we write

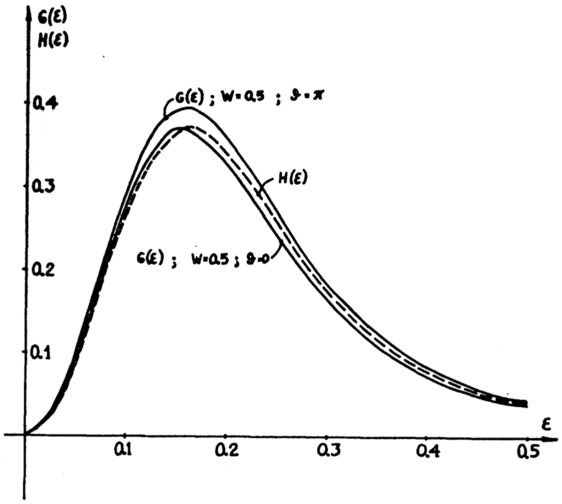


FIG. 3. The probability of obtaining a field ϵ for $\epsilon_0=0$ [$H(\epsilon)$] and for $\epsilon_0=0.1$. Test particle velocity $w=\alpha^{\frac{1}{2}}v=0.5$; $\vartheta=0$, π.

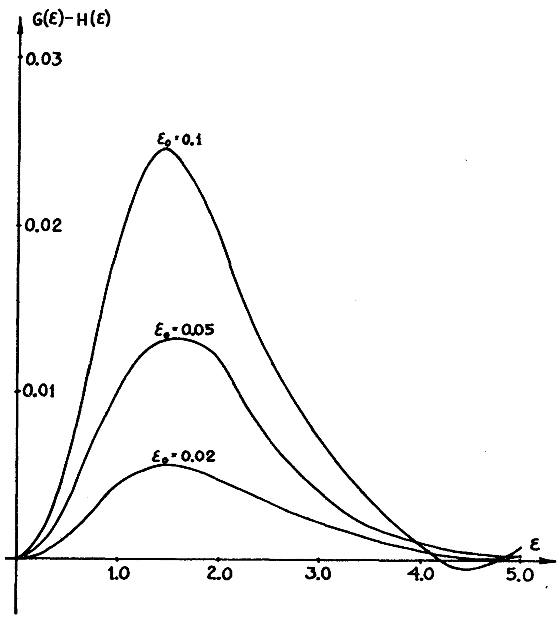


FIG. 2. Deviation from the Holtsmark distribution $H(\epsilon)$ due to the dynamic correlations, for $w=0.5$, $\vartheta=\pi$, and $\epsilon_0=0.02$, 0.05, 0.1.

$$\begin{aligned} A(\mathbf{p},\mathbf{v}) &= \exp[-C_0(p)-\Delta C(\mathbf{p},\mathbf{v})] \\ &= \exp\{-C_0(p)\}[1-\Delta C(\mathbf{p},\mathbf{v})] \\ &= A_0(p)-\Delta A(\mathbf{p},\mathbf{v}), \end{aligned} \tag{43}$$

where with the aid of (7)

$$\begin{aligned} \Delta C(\mathbf{p},\mathbf{v}) = n_0 \int d\mathbf{r}\{1-\exp[i\mathbf{p}\cdot\mathbf{E}_i(\mathbf{r})]\} \\ \times (b/r)[1-\Phi(\alpha^{\frac{1}{2}}\mathbf{v}\cdot(\mathbf{r}/r))] \\ \times \exp\{-\alpha v^2[1-(\mathbf{v}\cdot\mathbf{r}/vr)^2]\}. \end{aligned} \tag{44}$$

Then calculating both $\langle A(\mathbf{p},\mathbf{v})\rangle$ and $\Delta A(\mathbf{p},0)$ by averaging over a Maxwell distribution for $\mathbf{v}$, we obtain, independently of the temperature, that

$$\langle A(\mathbf{p},\mathbf{v})\rangle = (\pi/2)A(\mathbf{p},0). \tag{45}$$

Thus one should expect that this additional broadening would become detectable under suitable circumstances.

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