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# Annihilation Process of Neutrino Production in Stars

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The rate at which the energy of a blackbody radiation is converted into neutrinos by the pair annihilation process $e^-+e^+\rightarrow\nu+\bar{\nu}$ has been calculated. At $T\sim6\times10^9$ °K the relaxation time for such conversion process is around 100 sec for pure radiation. Since neutrinos have a very long mean free path ($\gg$stellar dimensions) they will escape, thus carrying away the energy. This process therefore will be of astrophysical importance. The rate of energy loss $dU/dt$ is tabulated, as a function of temperature and density, together with the chemical potential, the pressure, and the electron-positron energy. This table should be useful for numerical integrations of stellar structure equations in the temperature range $(0.5\text{–}10)\times10^9$ °K, and the density range 0–$10^9$ g/cm$^3$.

## I. INTRODUCTION

THE importance of neutrino production processes in stellar evolution has been discussed by a number of authors.[1–6] Here we shall only consider the annihilation process of neutrino production[5] whose energy loss rate has been shown to be larger than other hitherto known processes in the range of temperature in which one is usually interested ($5\times10^8$ °K$-10^{10}$ °K). We shall derive the exact formula for the rate of energy loss due to the annihilation process.

## II. THEORY

When $kT\sim mc^2$, the rest energy of electrons, then ($T\sim5\times10^9$ °K) electron-positron pairs will be created during collisions. Such pairs will annihilate later to form photons. Therefore pairs will exist in equilibrium with radiation. However, during the annihilation process there is a small possibility that a pair of neutrinos may result:

$$e^-+e^+\rightarrow\nu+\bar{\nu};\tag{1}$$

this process is the annihilation process of neutrino production, assuming the basic coupling for this reaction to be the same for $\mu$ decay. Because the mean free path of neutrinos is very long ($\gg$stellar dimensions) they will escape, thus carrying away the energy. The cross section $\sigma$ for process (1) is around $10^{-45}$ cm$^2$. The number of pairs per cm$^3$ in equilibrium with radiation at $kT\sim mc^2$ is around $n\approx m^3c^3/h^3=10^{30}$. Therefore the rate of energy loss is around

$$n^2\sigma cmc^2\approx10^{19}\ \text{ergs/cm}^3\text{-sec}.$$

The thermal energy content per gram of matter at $kT\sim mc^2$ is around $10^{19}$ ergs/g. Therefore the relaxation time for cooling is around $\rho$ sec, where $\rho$ is the density in g/cm$^3$. $\rho$ is around $10^7$. The relaxation time for cooling due to annihilation process is around or less than one year and is short compared with the usual time scale for stellar evolution ($\gtrsim10^6$ years). The consequence of the cooling effect has been briefly discussed.[5,6]

### (a) The Cross Section

The cross section for (1) has been calculated.[5] It is

$$\begin{aligned}\sigma&=\sigma_0[(E_T^2-1)/|\mathbf{v}|/c],\\ \sigma_0&=G^2m^2/24\pi=1.5\times10^{-45}\ \text{cm}^2,\end{aligned}\tag{2}$$

where $\mathbf{v}$ is the difference of velocities of the electron and the positron in the center-of-mass system (the system in which the sum of momenta of the electron and the positron is zero). $E_T$ is the total energy of the electron and the positron including their rest energy in the c.m. system, measured in units of $mc^2$. $G$ is the beta-coupling constant and numerically $GM_p^2=(1.01\pm0.01)\times10^{-5}$, where $M_p$ is the mass of the proton.

### (b) The Electron and Positron Densities

The electron and positron densities have been treated in a number of literatures.[7,8] Here we shall follow the approach of Landau and Lifshitz[8] and treat the equilibrium among photons and pairs as a chemical equilibrium:

$$e^-+e^+\rightleftharpoons\gamma,\tag{3}$$

where $\gamma$ stands for one or more photons. In order to have equilibrium, the chemical potential for photon $\mu_\gamma$ must be equal to the sum of those for electron and positron ($\mu_-+\mu_+$). Since $\mu_\gamma=0$,[9] we can write

$$\mu_-=-\mu_+=\mu.\tag{4}$$

In the limit $T\rightarrow0$, $\mu$ is the Fermi energy. The distribution functions $n_-$ and $n_+$ for electrons and

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[7] G. Wataghin, Phys. Rev. **66**, 149 (1944).
[8] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, translation by R. F. Peierls and E. Peierls (Pergamon Press, Ltd., London, 1958), p. 325.
[9] The free energy $F$ of a system of particles whose number is not fixed must be a minimum with respect to the number of particles $N$. Thus $\partial F/\partial N=0$. But $\partial F/\partial N=\mu$, the chemical potential. Therefore $\mu=0$. For details, see for example, L. D. Landau and E. M. Lifshitz, reference 8, p. 172.

positrons are then

$$n_{\pm}d^3p_{\pm}=\frac{2}{h^3}\frac{d^3p_{\pm}}{\exp[(E_{\pm}\pm\mu)/kT]+1}, \qquad (5)$$

where $E_{\pm}$ is the electron or positron total energy (including the rest energy $mc^2$), and $\mathbf{p}_{\pm}$ is the corresponding momentum: $E_{\pm}^2-|\mathbf{p}_{\pm}|^2=(mc^2)^2$. The total number of electrons and positrons per unit volume is

$$N_{\pm}=\frac{2}{h^3}\int_0^{\infty}\frac{d^3p_{\pm}}{\exp[(E_{\pm}\pm\mu)/kT]+1}=8\pi\frac{m^3c^3}{h^3}\int_0^{\infty}\frac{x^2dx}{\exp[\beta(y\pm\mu')]+1}, \qquad (6)$$

where $x=p/mc$, $y=E/mc^2$, $\beta=mc^2/kT$, and $\mu'=\mu/mc^2$. If there are $N_0$ electrons per unit volume present originally (i.e., $N_0=m\langle z\rangle$, where $m$ is the number density of heavy ions, $\langle z\rangle$ is their average charge in electronic charge units), then

$$N_--N_+=N_0, \qquad (7)$$

Equation (7) yields a relation among $N_0$, $\mu'$, and $T$.

### (c) Rate of Energy Loss

The rate of energy loss $dU/dt$ per unit volume per unit time is given as

$$-\frac{dU}{dt}=\int\int(E_-+E_+)(\sigma v)n_+n_-d^3p_+d^3p_-, \qquad (8)$$

where $(\sigma v)$ must be transformed to a dynamical system in which the electron and the positron have energies $E_-$ and $E_+$, momenta $\mathbf{p}_-$ and $\mathbf{p}_+$. This transformation will be done in the following subsection.

It is convenient to use dimensionless variables. We shall put $m=c=h=1$. The energy $E$ is then measured in units $mc^2$, momentum $p$ in units of $mc$, $v$ in units of $c$, $n_{\pm}$ in units of $(m^3c^3/h^3)$, and $U$ will be measured in units of $U_0$, where $U_0=(\sigma_0c)mc^2(mc/h)^6$. Thus

$$-\frac{1}{U_0}\frac{dU}{dt}=\int\int(E_++E_-)(\sigma v)n_+n_-d^3p_+d^3p_-. \qquad (9)$$

On the right-hand side dimensionless variables are used. $\sigma v$ is measured in units of $\sigma_0c$. Henceforth we also drop the prime on $\mu$ which is measured in units of $mc^2$.

### (d) Transformation of Dynamical Variables

Since at $kT\sim mc^2$ electrons or positrons will attain velocities comparable to $c$, relativistic transformations have to be used.

The following is a list of dynamical variables and their relations to each other:

- $E_T$: the total energy of the positron and electron in the center-of-mass system.
- $p_T$: the total momenta in the c.m. system ($p_T=0$, by the definition of the c.m. system).
- $E_{\pm}$: the energy of the positron (or electron) in the laboratory system. ($E_{\pm}$ includes the rest energy.)
- $\mathbf{p}_{\pm}$: the momentum of the positron (or electron) in the laboratory system.
- $\mathbf{v}_{\pm}$: the velocity of the positron (the electron) in the the laboratory system.
- $\mathbf{v}$: the difference of velocities of the positron and the electron in the c.m. system.
- $\mathbf{v}_c$: the velocity of the c.m. system with respect to the laboratory system.

The above quantities are related by the following relations:

$$E_{\pm}^2-\mathbf{p}_{\pm}^2=1,\quad E_T^2-p_T^2=(E_++E_-)^2-(\mathbf{p}_++\mathbf{p}_-)^2,$$

Since $p_T=0$, we have

$$E_T^2=2+2E_+E_--2\mathbf{p}_+\cdot\mathbf{p}_-,$$
$$\tfrac{1}{2}|\mathbf{v}|=[(\tfrac{1}{2}E_T^2)-1]^{\frac{1}{2}}/(\tfrac{1}{2}E_T), \qquad (10)$$
$$\mathbf{v}_c=(\mathbf{p}_++\mathbf{p}_-)/(E_++E_-).$$

### (e) Calculations

The quantity $\sigma v$ has been given in the c.m. system and $n_+$, $n_-$ have been given in the lab system. In order to calculate $-dU/dt$ it is necessary to relate the two systems.

Consider in the center-of-mass system two groups of particles of density $n_+$ and $n_-$ passing against each other with velocities $\mathbf{v}_+$ and $\mathbf{v}_-$. The number of reactions per unit volume per unit time is given as

$$\sigma v n_+n_-,$$

where $\sigma$ is the reaction cross-section and $v=|\mathbf{v}_+-\mathbf{v}_-|$. The total number of reactions in the time interval $dT$ and the volume $dV$ is

$$\sigma v n_+n_-dVdT. \qquad (11)$$

This expression is an invariant. This can be seen as follows: We put the system in a box of volume $dV$ and attach to the box a clock. The number of reactions that occur inside the box in a time interval $dt$ registered by this clock should be an invariant. Since $dVdt$ is an invariant, so is $\sigma v n_+n_-$.

Therefore, we need to express $n_{\pm}$ in the c.m. system in terms of those in the lab system. We note that $n_+$ may be considered as the time component of the 4-vector $(n_+\mathbf{v}_+,n_+)$. Thus $n_{\pm}$ in the c.m. system (denoted by the superscript c.m.) is related to that in the lab system (denoted by the superscript $L$) by Lorentz transformations:

$$n_{\pm}^{\text{c.m.}}=n_{\pm}^{L}\frac{(1-\mathbf{v}_c\cdot\mathbf{v}_{\pm})}{(1-v_c^2)^{\frac{1}{2}}}. \qquad (12)$$

Equation (9) thus reduces to

$$-\frac{1}{U_0}\frac{dU}{dt}=\int\int(E_++E_-)(E_+^2-1)\times\frac{(1-\mathbf{v}_c\cdot\mathbf{v}_+)(1-\mathbf{v}_c\cdot\mathbf{v}_-)}{(1-v_c^2)}\times\frac{d^3p_+}{\exp[\beta(E_++\mu)]+1}\frac{d^3p_-}{\exp[\beta(E_--\mu)]+1}. \quad (13)$$

We have dropped the prime on $\mu$.

From Eqs. (10), we have

$$(1-v_c^2)=1-[(\mathbf{P}_++\mathbf{P}_-)^2/(E_++E_-)^2]=[E_T^2/(E_++E_-)^2],$$
$$1-\mathbf{v}_c\cdot\mathbf{v}_\pm=1-\frac{\mathbf{P}_++\mathbf{P}_-}{E_++E_-}\cdot\frac{\mathbf{P}_\pm}{E_\pm}=\tfrac{1}{2}E_T^2/[E_\pm(E_++E_-)]. \quad (14)$$

After substituting Eqs. (10) and (14) into Eq. (13), we find the integrand contains $|\mathbf{p}_+|$, $|\mathbf{p}_-|$, and $\theta\equiv\angle(\mathbf{p}_+,\mathbf{p}_-)$ as variables only. One can then integrate the rest of angular variables and obtain

$$-\frac{1}{U_0}\frac{dU}{dt}=32\pi^2\int\int\int(E_++E_-)[1+2E_+E_--2P_+P_-\cos\theta]\times[1+E_+E_--P_+P_-\cos\theta]\frac{d\cos\theta}{2E_+E_-}\times\frac{P_+^2dP_+}{\exp[\beta(E_++\mu)]+1}\frac{P_-^2dP_-}{\exp[\beta(E_--\mu)]+1}$$
$$=(32\pi^2/3)\{8\langle E_+\rangle\langle E_-^2\rangle+7N_+\langle E_-\rangle-2\langle E_-^2\rangle\langle 1/E_+\rangle+5N_-\langle 1/E_+\rangle+8\langle E_-\rangle\langle E_+^2\rangle+7N_-\langle E_+\rangle-2\langle E_+^2\rangle\langle 1/E_-\rangle+5N_+\langle 1/E_-\rangle\}, \quad (15)$$

where

$$\langle E_\pm{}^n\rangle=\int_0^\infty\frac{(1+P^2)^{n/2}P^2dP}{\exp\{\beta[(1+P^2)^{\frac{1}{2}}\pm\mu]\}+1},$$

and

$$N_\pm=\langle E_\pm{}^0\rangle. \quad (16)$$

Thus $dU/dt$ may be expressed in terms of the expectation values of $E$, $E^2$, $1/E$ of a Fermi gas.

### (f) Tabulation of $-dU/dt$

We have expressed $-dU/dt$ as a polynomial in the expectation values of $E$, $E^2$, $1/E$, and $N$. However, no known series expansions exist for them other than the simple case $\mu<1$. Even for the case $\mu<1$, the series expansion is sufficiently complicated.[10] It is therefore convenient to have $-dU/dt$ tabulated. This is done in Table I.

$-dU/dt$ is an explicit function of $\mu$ and $T$. $\mu$ is an implicit function of $N_0$ and $T$. In astrophysical applications it is more convenient to use $N_0$ and $T$ as variables. We therefore tabulate $-dU/dt$ as a function of $N_0$ and $T$. In the same table we also tabulate the pressure $P$ due to all electrons and positrons, the total energy $E$ of all electrons and positrons, and $\mu$.

The tabulated quantities are dimensionless. In order to obtain their values in suitable units (e.g., the cgs units), suitable constant multipliers carrying all dimensional units have to be used. Let the superscript $\alpha$ denote these quantities in cgs units, and let the superscript $t$ denote these quantities from our table. Then the $\alpha$ quantities are related to the $t$ quantities by the following relations:

Number density

$$N_0{}^\alpha=(8\pi m^3c^3/h^3)N_0{}^t;$$
$$8\pi m^3c^3/h^3=1.76\times10^{30}\text{ particles/cm}^3.$$

Temperature $T^\alpha=T^t\times10^9$ °K.

Density

$$\rho^\alpha=(8\pi m^3c^3/h^3)\mu_eM_pN_0{}^t;$$
$$(8\pi m^3c^3/h^3)M_p=2.94\times10^6\text{ g/cm}^3.$$

Rate of energy loss

$$-(dU/dt)^\alpha=-[(32\pi^2/3)\sigma_0cmc^2(mc/h)^6](dU/dt)^t;$$
$$(32\pi^2/3)\sigma_0cmc^2(mc/h)^6=1.88\times10^{19}\text{ ergs/cm}^3\text{-sec.}$$

Pressure

$$P^\alpha=(8\pi m^4c^5/3h^3)P^t;$$
$$8\pi m^4c^5/3h^3=4.808\times10^{23}\text{ d/cm}^2. \quad (17)$$

Energy density

$$E^\alpha\equiv\langle E_-\rangle^\alpha+\langle E_+\rangle^\alpha=(8\pi m^4c^5/h^3)E^t;$$
$$8\pi m^4c^5/h^3=1.44\times10^{24}\text{ ergs/cm}^3.$$

Chemical potential

$$\mu^\alpha=\mu^tmc^2;\quad mc^2=8.12\times10^{-7}\text{ ergs.}$$

On the other hand, $P^t$ is defined as

$$P^t=\int_0^\infty\frac{p^4}{E}dp\left[\frac{1}{\exp[\beta(E-\mu)]+1}+\frac{1}{\exp[\beta(E+\mu)]+1}\right]. \quad (18)$$

In Fig. 1 we plot $-dU/dt$ as a function of temperature for different values of $N_0$. In Fig. 2 we plot $-dU/dt$ as a function of $N_0$ for different values of $T$.

[10] See Appendix.

TABLE I. Table of $\log_{10}N_0$, $\log_{10}(-dU/dt)$, $\log_{10}P$, $\log_{10}E$, and $\mu$. The units of these quantities have been given in Eqs. (17) and (18).

| $\log_{10}(N_0)$ | $\log_{10}(-dU/dt)$ | $\log_{10}(P)$ | $\log_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=0.5$ | | | | |
| -5.00 | -11.59223 | -5.59805 | -4.94329 | 0.31308 |
| -4.50 | -11.59228 | -5.09837 | -4.44372 | 0.40980 |
| -4.00 | -11.59253 | -4.59824 | -3.94374 | 0.50659 |
| -3.50 | -11.59335 | -4.09780 | -3.44369 | 0.60351 |
| -3.00 | -11.59594 | -3.59643 | -2.94351 | 0.70079 |
| -2.50 | -11.60412 | -3.09215 | -2.44297 | 0.79921 |
| -2.00 | -11.62974 | -2.57907 | -1.94126 | 0.90119 |
| -1.80 | -11.65137 | -2.36837 | -1.73983 | 0.94431 |
| -1.60 | -11.68514 | -2.15226 | -1.53761 | 0.98991 |
| -1.40 | -11.73743 | -1.92862 | -1.33420 | 1.03930 |
| -1.20 | -11.81746 | -1.69515 | -1.12904 | 1.09435 |
| -1.00 | -11.93797 | -1.44992 | -0.92143 | 1.15766 |
| -0.90 | -12.01844 | -1.32259 | -0.81644 | 1.19344 |
| -0.80 | -12.11557 | -1.19216 | -0.71052 | 1.23261 |
| -0.70 | -12.23200 | -1.05882 | -0.60356 | 1.27572 |
| -0.60 | -12.37057 | -0.92286 | -0.49546 | 1.32332 |
| -0.50 | -12.53425 | -0.78466 | -0.38613 | 1.37603 |
| -0.40 | -12.72614 | -0.64465 | -0.27550 | 1.43446 |
| -0.30 | -12.94945 | -0.50328 | -0.16351 | 1.49926 |
| -0.20 | -13.20745 | -0.36098 | -0.05015 | 1.57109 |
| -0.10 | -13.50354 | -0.21816 | 0.06462 | 1.65061 |
| 0.00 | -13.84120 | -0.07513 | 0.18078 | 1.73852 |
| 0.05 | -14.02675 | -0.00362 | 0.23937 | 1.78585 |
| 0.10 | -14.22408 | 0.06784 | 0.29830 | 1.83554 |
| 0.15 | -14.43365 | 0.13923 | 0.35757 | 1.88770 |
| 0.20 | -14.65597 | 0.21053 | 0.41716 | 1.94242 |
| 0.25 | -14.89153 | 0.28172 | 0.47708 | 1.99979 |
| $T=0.6$ | | | | |
| -5.00 | -9.58660 | -5.47539 | -4.88837 | 0.15040 |
| -4.50 | -9.58661 | -5.01445 | -4.42754 | 0.26183 |
| -4.00 | -9.58678 | -4.51864 | -3.93186 | 0.37746 |
| -3.50 | -9.58736 | -4.01874 | -3.43225 | 0.49366 |
| -3.00 | -9.58922 | -3.51778 | -2.93214 | 0.61021 |
| -2.50 | -9.59511 | -3.01466 | -2.43167 | 0.72777 |
| -2.00 | -9.61358 | -2.50501 | -1.93018 | 0.84846 |
| -1.80 | -9.62921 | -2.29704 | -1.72892 | 0.89879 |
| -1.60 | -9.65367 | -2.08491 | -1.52696 | 0.95131 |
| -1.40 | -9.69170 | -1.86681 | -1.32395 | 1.00721 |
| -1.20 | -9.75024 | -1.64061 | -1.11936 | 1.06820 |
| -1.00 | -9.83908 | -1.40417 | -0.91251 | 1.13671 |
| -0.90 | -9.89883 | -1.28160 | -0.80798 | 1.17477 |
| -0.80 | -9.97134 | -1.15601 | -0.70257 | 1.21599 |
| -0.70 | -10.05880 | -1.02743 | -0.59615 | 1.26091 |
| -0.60 | -10.16355 | -0.89602 | -0.48863 | 1.31010 |
| -0.50 | -10.28811 | -0.76204 | -0.37988 | 1.36419 |
| -0.40 | -10.43512 | -0.62582 | -0.26984 | 1.42381 |
| -0.30 | -10.60733 | -0.48777 | -0.15844 | 1.48963 |
| -0.20 | -10.80755 | -0.34831 | -0.04562 | 1.56234 |
| -0.10 | -11.03869 | -0.20786 | 0.06863 | 1.64263 |
| 0.00 | -11.30375 | -0.06678 | 0.18430 | 1.73121 |
| 0.10 | -11.60581 | 0.07459 | 0.30140 | 1.82883 |
| 0.20 | -11.94808 | 0.21600 | 0.41987 | 1.93623 |
| 0.30 | -12.33395 | 0.35724 | 0.53967 | 2.05422 |
| 0.40 | -12.76696 | 0.49817 | 0.66075 | 2.18363 |
| 0.50 | -13.25089 | 0.63867 | 0.78302 | 2.32534 |
| 0.60 | -13.78977 | 0.77870 | 0.90642 | 2.48028 |
| 0.70 | -14.38790 | 0.91820 | 1.03087 | 2.64945 |
| $T=0.7$ | | | | |
| -4.50 | -8.10942 | -4.84463 | -4.31327 | 0.12310 |
| -4.00 | -8.10952 | -4.43871 | -3.90745 | 0.24494 |
| -3.50 | -8.10995 | -3.95057 | -3.41952 | 0.37886 |
| -3.00 | -8.11134 | -3.45105 | -2.92064 | 0.51454 |
| -2.50 | -8.11576 | -2.94879 | -2.42034 | 0.65127 |
| $T=0.7$ | | | | |
| -2.00 | -8.12964 | -2.44140 | -1.91903 | 0.79081 |
| -1.80 | -8.14140 | -2.23525 | -1.71792 | 0.84845 |
| -1.60 | -8.15985 | -2.02581 | -1.51618 | 0.90806 |
| -1.50 | -8.17266 | -1.91940 | -1.41498 | 0.93892 |
| -1.40 | -8.18862 | -1.81156 | -1.31349 | 0.97071 |
| -1.30 | -8.20847 | -1.70206 | -1.21164 | 1.00364 |
| -1.20 | -8.23308 | -1.59061 | -1.10937 | 1.03797 |
| -1.10 | -8.26351 | -1.47695 | -1.00658 | 1.07401 |
| -1.00 | -8.30097 | -1.36083 | -0.90317 | 1.11211 |
| -0.90 | -8.34688 | -1.24203 | -0.79905 | 1.15269 |
| -0.80 | -8.40284 | -1.12039 | -0.69409 | 1.19622 |
| -0.70 | -8.47067 | -0.99585 | -0.58817 | 1.24323 |
| -0.60 | -8.55236 | -0.86845 | -0.48119 | 1.29428 |
| -0.50 | -8.65004 | -0.73831 | -0.37301 | 1.35000 |
| -0.40 | -8.76600 | -0.60568 | -0.26356 | 1.41105 |
| -0.30 | -8.90264 | -0.47088 | -0.15274 | 1.47812 |
| -0.20 | -9.06243 | -0.33430 | -0.04050 | 1.55190 |
| -0.10 | -9.24794 | -0.19632 | 0.07320 | 1.63312 |
| 0.00 | -9.46179 | -0.05733 | 0.18836 | 1.72252 |
| 0.10 | -9.70669 | 0.08231 | 0.30498 | 1.82086 |
| 0.20 | -9.98547 | 0.22230 | 0.42302 | 1.92891 |
| 0.30 | -10.30104 | 0.36238 | 0.54243 | 2.04748 |
| 0.40 | -10.65651 | 0.50237 | 0.66315 | 2.17741 |
| 0.50 | -11.05512 | 0.64212 | 0.78511 | 2.31959 |
| 0.60 | -11.50037 | 0.78153 | 0.90824 | 2.47497 |
| 0.70 | -11.99596 | 0.92054 | 1.03245 | 2.64454 |
| 0.80 | -12.54588 | 1.05912 | 1.15766 | 2.82937 |
| 0.90 | -13.15441 | 1.19725 | 1.28378 | 3.03062 |
| 1.00 | -13.82617 | 1.33493 | 1.41074 | 3.24951 |
| 1.10 | -14.56612 | 1.47219 | 1.53845 | 3.48739 |
| 1.20 | -15.37963 | 1.60904 | 1.66684 | 3.74568 |
| $T=0.8$ | | | | |
| -4.00 | -6.96642 | -4.26086 | -3.77620 | 0.12651 |
| -3.50 | -6.96672 | -3.87638 | -3.39188 | 0.26195 |
| -3.00 | -6.96778 | -3.39161 | -2.90760 | 0.41463 |
| -2.50 | -6.97119 | -2.89137 | -2.40887 | 0.57034 |
| -2.00 | -6.98194 | -2.38569 | -1.90786 | 0.72879 |
| -1.80 | -6.99107 | -2.18082 | -1.70687 | 0.79382 |
| -1.60 | -7.00541 | -1.97329 | -1.50532 | 0.86064 |
| -1.50 | -7.01537 | -1.86814 | -1.40424 | 0.89500 |
| -1.40 | -7.02781 | -1.76182 | -1.30290 | 0.93021 |
| -1.30 | -7.04330 | -1.65411 | -1.20123 | 0.96646 |
| -1.20 | -7.06255 | -1.54475 | -1.09917 | 1.00399 |
| -1.10 | -7.08639 | -1.43350 | -0.99664 | 1.04308 |
| -1.00 | -7.11583 | -1.32009 | -0.89355 | 1.08408 |
| -0.90 | -7.15203 | -1.20428 | -0.78978 | 1.12738 |
| -0.80 | -7.19632 | -1.08587 | -0.68522 | 1.17343 |
| -0.70 | -7.25022 | -0.96473 | -0.57976 | 1.22275 |
| -0.60 | -7.31541 | -0.84078 | -0.47328 | 1.27589 |
| -0.50 | -7.39376 | -0.71407 | -0.36564 | 1.33348 |
| -0.40 | -7.48725 | -0.58475 | -0.25675 | 1.39618 |
| -0.30 | -7.59800 | -0.45304 | -0.14651 | 1.46470 |
| -0.20 | -7.72820 | -0.31926 | -0.03485 | 1.53974 |
| -0.10 | -7.88014 | -0.18376 | 0.07828 | 1.62207 |
| 0.00 | -8.05619 | -0.04693 | 0.19290 | 1.71244 |
| 0.10 | -8.25876 | 0.09089 | 0.30900 | 1.81162 |
| 0.20 | -8.49039 | 0.22935 | 0.42657 | 1.92043 |
| 0.30 | -8.75367 | 0.36817 | 0.54555 | 2.03967 |
| 0.40 | -9.05135 | 0.50712 | 0.66588 | 2.17021 |
| 0.50 | -9.38631 | 0.64603 | 0.78750 | 2.31295 |
| 0.60 | -9.76164 | 0.78475 | 0.91032 | 2.46883 |
| 0.70 | -10.18060 | 0.92320 | 1.03426 | 2.63886 |
| 0.80 | -10.64671 | 1.06133 | 1.15923 | 2.82412 |
| 0.90 | -11.16372 | 1.19908 | 1.28514 | 3.02576 |

TABLE I (*continued*)

| $\log_{10}(N_0)$ | $\log_{10}(-dU/dt)$ | $\log_{10}(P)$ | $\log_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=0.8$ | | | | |
| 1.00 | −11.73570 | 1.33647 | 1.41192 | 3.24502 |
| 1.10 | −12.36702 | 1.47347 | 1.53947 | 3.48323 |
| 1.20 | −13.06239 | 1.61012 | 1.66772 | 3.74182 |
| $T=0.9$ | | | | |
| −3.50 | −6.04887 | −3.73214 | −3.28744 | 0.15622 |
| −3.00 | −6.04965 | −3.32849 | −2.88416 | 0.31261 |
| −2.50 | −6.05233 | −2.83951 | −2.39636 | 0.48561 |
| −2.00 | −6.06085 | −2.33607 | −1.89659 | 0.66286 |
| −1.80 | −6.06810 | −2.13220 | −1.69578 | 0.73533 |
| −1.60 | −6.07949 | −1.92609 | −1.49440 | 0.80944 |
| −1.50 | −6.08742 | −1.82189 | −1.39344 | 0.84736 |
| −1.40 | −6.09733 | −1.71671 | −1.29223 | 0.88606 |
| −1.30 | −6.10969 | −1.61034 | −1.19072 | 0.92571 |
| −1.20 | −6.12507 | −1.50259 | −1.08885 | 0.96654 |
| −1.10 | −6.14416 | −1.39319 | −0.98655 | 1.00881 |
| −1.00 | −6.16779 | −1.28191 | −0.88372 | 1.05285 |
| −0.90 | −6.19691 | −1.16850 | −0.78027 | 1.09902 |
| −0.80 | −6.23264 | −1.05273 | −0.67608 | 1.14776 |
| −0.70 | −6.27629 | −0.93442 | −0.57103 | 1.19957 |
| −0.60 | −6.32929 | −0.81344 | −0.46500 | 1.25500 |
| −0.50 | −6.39325 | −0.68976 | −0.35787 | 1.31466 |
| −0.40 | −6.46992 | −0.56342 | −0.24952 | 1.37921 |
| −0.30 | −6.56117 | −0.43458 | −0.13984 | 1.44937 |
| −0.20 | −6.66898 | −0.30348 | −0.02876 | 1.52586 |
| −0.10 | −6.79541 | −0.17042 | 0.08380 | 1.60946 |
| 0.00 | −6.94259 | −0.03574 | 0.19786 | 1.70094 |
| 0.10 | −7.11274 | 0.10020 | 0.31343 | 1.80111 |
| 0.20 | −7.30813 | 0.23707 | 0.43050 | 1.91078 |
| 0.30 | −7.53113 | 0.37455 | 0.54902 | 2.03080 |
| 0.40 | −7.78422 | 0.51239 | 0.66893 | 2.16204 |
| 0.50 | −8.07001 | 0.65038 | 0.79017 | 2.30541 |
| 0.60 | −8.39124 | 0.78835 | 0.91265 | 2.46187 |
| 0.70 | −8.75086 | 0.92619 | 1.03629 | 2.63243 |
| 0.80 | −9.15201 | 1.06381 | 1.16099 | 2.81817 |
| 0.90 | −9.59807 | 1.20115 | 1.28667 | 3.02025 |
| 1.00 | −10.09265 | 1.33819 | 1.41324 | 3.23992 |
| 1.10 | −10.63966 | 1.47492 | 1.54062 | 3.47851 |
| 1.20 | −11.24331 | 1.61133 | 1.66871 | 3.73746 |
| 1.30 | −11.90814 | 1.74743 | 1.79746 | 4.01832 |
| 1.40 | −12.63904 | 1.88325 | 1.92678 | 4.32277 |
| 1.50 | −13.44131 | 2.01879 | 2.05661 | 4.65263 |
| $T=1.0$ | | | | |
| −3.40 | −5.29073 | −3.43919 | −3.02902 | 0.10454 |
| −3.00 | −5.29118 | −3.22990 | −2.81997 | 0.21580 |
| −2.50 | −5.29325 | −2.78792 | −2.37893 | 0.39827 |
| −2.00 | −5.30011 | −2.29092 | −1.88486 | 0.59346 |
| −1.80 | −5.30596 | −2.08812 | −1.68451 | 0.67335 |
| −1.60 | −5.31518 | −1.88321 | −1.48342 | 0.75478 |
| −1.40 | −5.32965 | −1.67546 | −1.28150 | 0.83854 |
| −1.20 | −5.35219 | −1.46365 | −1.07845 | 0.92587 |
| −1.00 | −5.38701 | −1.24614 | −0.87376 | 1.01859 |
| −0.80 | −5.44012 | −1.02107 | −0.66672 | 1.11934 |
| −0.60 | −5.51973 | −0.78669 | −0.45645 | 1.23168 |
| −0.50 | −5.57270 | −0.66565 | −0.34979 | 1.29358 |
| −0.40 | −5.63645 | −0.54199 | −0.24194 | 1.36017 |
| −0.30 | −5.71264 | −0.41578 | −0.13281 | 1.43214 |
| −0.20 | −5.80307 | −0.28720 | −0.02228 | 1.51026 |
| −0.10 | −5.90959 | −0.15648 | 0.08970 | 1.59528 |
| 0.00 | −6.03417 | −0.02392 | 0.20320 | 1.68803 |
| 0.10 | −6.17882 | 0.11014 | 0.31823 | 1.78931 |
| 0.20 | −6.34563 | 0.24537 | 0.43477 | 1.89997 |
| 0.30 | −6.53678 | 0.38146 | 0.55281 | 2.02087 |
| 0.40 | −6.75454 | 0.51814 | 0.67227 | 2.15290 |

| $\log_{10}(N_0)$ | $\log_{10}(-dU/dt)$ | $\log_{10}(P)$ | $\log_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=1.0$ | | | | |
| 0.50 | −7.00128 | 0.65515 | 0.79311 | 2.29698 |
| 0.60 | −7.27951 | 0.79231 | 0.91522 | 2.45408 |
| 0.70 | −7.59191 | 0.92948 | 1.03853 | 2.62523 |
| 0.80 | −7.94133 | 1.06655 | 1.16295 | 2.81152 |
| 0.90 | −8.33083 | 1.20344 | 1.28837 | 3.01410 |
| 1.00 | −8.76368 | 1.34010 | 1.41472 | 3.23423 |
| 1.10 | −9.24342 | 1.47652 | 1.54189 | 3.47324 |
| 1.20 | −9.77385 | 1.61268 | 1.66982 | 3.73258 |
| 1.30 | −10.35908 | 1.74857 | 1.79841 | 4.01380 |
| 1.40 | −11.00353 | 1.88421 | 1.92760 | 4.31859 |
| 1.50 | −11.71198 | 2.01960 | 2.05732 | 4.64876 |
| 1.60 | −12.48957 | 2.15477 | 2.18750 | 5.00627 |
| 1.70 | −13.34188 | 2.28973 | 2.31810 | 5.39323 |
| $T=1.2$ | | | | |
| −2.90 | −4.09862 | −2.80738 | −2.45439 | 0.10824 |
| −2.50 | −4.09951 | −2.62864 | −2.27612 | 0.23151 |
| −2.00 | −4.10389 | −2.20368 | −1.85309 | 0.44711 |
| −1.80 | −4.10783 | −2.00766 | −1.65872 | 0.54068 |
| −1.60 | −4.11410 | −1.80657 | −1.46020 | 0.63641 |
| −1.40 | −4.12399 | −1.60196 | −1.25955 | 0.73447 |
| −1.20 | −4.13948 | −1.39375 | −1.05736 | 0.83573 |
| −1.00 | −4.16354 | −1.18098 | −0.85354 | 0.94171 |
| −0.90 | −4.18008 | −1.07243 | −0.75088 | 0.99717 |
| −0.80 | −4.20051 | −0.96214 | −0.64759 | 1.05474 |
| −0.70 | −4.22566 | −0.84989 | −0.54359 | 1.11487 |
| −0.60 | −4.25648 | −0.73549 | −0.43874 | 1.17807 |
| −0.50 | −4.29404 | −0.61877 | −0.33292 | 1.24490 |
| −0.40 | −4.33955 | −0.49960 | −0.22601 | 1.31601 |
| −0.30 | −4.39437 | −0.37795 | −0.11788 | 1.39209 |
| −0.20 | −4.45994 | −0.25386 | −0.00842 | 1.47391 |
| −0.10 | −4.53781 | −0.12745 | 0.10247 | 1.56224 |
| 0.00 | −4.62964 | 0.00108 | 0.21485 | 1.65793 |
| 0.10 | −4.73715 | 0.13147 | 0.32877 | 1.76182 |
| 0.20 | −4.86213 | 0.26343 | 0.44425 | 1.87480 |
| 0.30 | −5.00646 | 0.39667 | 0.56126 | 1.99777 |
| 0.40 | −5.17209 | 0.53088 | 0.67977 | 2.13166 |
| 0.50 | −5.36107 | 0.66581 | 0.79973 | 2.27741 |
| 0.60 | −5.57554 | 0.80122 | 0.92104 | 2.43602 |
| 0.70 | −5.81778 | 0.93692 | 1.04363 | 2.60855 |
| 0.80 | −6.09022 | 1.07277 | 1.16740 | 2.79610 |
| 0.90 | −6.39544 | 1.20865 | 1.29225 | 2.99984 |
| 1.00 | −6.73621 | 1.34448 | 1.41809 | 3.22103 |
| 1.10 | −7.11552 | 1.48020 | 1.54482 | 3.46103 |
| 1.20 | −7.53657 | 1.61577 | 1.67235 | 3.72128 |
| 1.30 | −8.00281 | 1.75118 | 1.80060 | 4.00334 |
| 1.40 | −8.51796 | 1.88641 | 1.92950 | 4.30890 |
| 1.50 | −9.08602 | 2.02147 | 2.05895 | 4.63979 |
| 1.60 | −9.71131 | 2.15635 | 2.18891 | 4.99796 |
| 1.70 | −10.39850 | 2.29107 | 2.31932 | 5.38554 |
| 1.80 | −11.15261 | 2.42564 | 2.45011 | 5.80483 |
| 1.90 | −11.97906 | 2.56006 | 2.58123 | 6.25830 |
| 2.00 | −12.88370 | 2.69436 | 2.71266 | 6.74863 |
| 2.10 | −13.87284 | 2.82854 | 2.84434 | 7.27871 |
| 2.20 | −14.95330 | 2.96261 | 2.97625 | 7.85168 |
| $T=1.4$ | | | | |
| −2.50 | −3.18950 | −2.31725 | −2.00970 | 0.11895 |
| −2.00 | −3.19188 | −2.08140 | −1.77504 | 0.30353 |
| −1.80 | −3.19445 | −1.91763 | −1.61240 | 0.40285 |
| −1.60 | −3.19876 | −1.73202 | −1.42859 | 0.50938 |
| −1.40 | −3.20572 | −1.53497 | −1.23433 | 0.62042 |
| −1.20 | −3.21675 | −1.33137 | −1.03500 | 0.73531 |
| −1.00 | −3.23402 | −1.12267 | −0.83274 | 0.85476 |

TABLE I (*continued*)

| $\text{Log}_{10}(N_0)$ | $\text{Log}_{10}(-dU/dt)$ | $\text{Log}_{10}(P)$ | $\text{Log}_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=1.4$ | | | | |
| −0.80 | −3.26074 | −0.90850 | −0.62802 | 0.98060 |
| −0.60 | −3.30153 | −0.68763 | −0.42048 | 1.11570 |
| −0.50 | −3.32910 | −0.57421 | −0.31542 | 1.18793 |
| −0.40 | −3.36270 | −0.45859 | −0.20934 | 1.26406 |
| −0.30 | −3.40342 | −0.34066 | −0.10212 | 1.34478 |
| −0.20 | −3.45246 | −0.22035 | 0.00636 | 1.43083 |
| −0.10 | −3.51111 | −0.09771 | 0.11621 | 1.52301 |
| 0.00 | −3.58079 | 0.02718 | 0.22752 | 1.62215 |
| 0.10 | −3.66298 | 0.15414 | 0.34036 | 1.72914 |
| 0.20 | −3.75925 | 0.28295 | 0.45476 | 1.84490 |
| 0.30 | −3.87124 | 0.41334 | 0.57072 | 1.97035 |
| 0.40 | −4.00067 | 0.54503 | 0.68823 | 2.10645 |
| 0.50 | −4.14934 | 0.67777 | 0.80725 | 2.25421 |
| 0.60 | −4.31915 | 0.81130 | 0.92769 | 2.41464 |
| 0.70 | −4.51210 | 0.94541 | 1.04948 | 2.58881 |
| 0.80 | −4.73029 | 1.07991 | 1.17253 | 2.77786 |
| 0.90 | −4.97601 | 1.21466 | 1.29674 | 2.98298 |
| 1.00 | −5.25165 | 1.34954 | 1.42200 | 3.20544 |
| 1.10 | −5.55982 | 1.48447 | 1.54822 | 3.44660 |
| 1.20 | −5.90328 | 1.61938 | 1.67531 | 3.70792 |
| 1.30 | −6.28503 | 1.75423 | 1.80317 | 3.99097 |
| 1.40 | −6.70828 | 1.88899 | 1.93171 | 4.29745 |
| 1.50 | −7.17650 | 2.02366 | 2.06087 | 4.62919 |
| 1.60 | −7.69340 | 2.15821 | 2.19057 | 4.98814 |
| 1.70 | −8.26302 | 2.29265 | 2.32075 | 5.37645 |
| 1.80 | −8.88968 | 2.42698 | 2.45134 | 5.79641 |
| 1.90 | −9.57805 | 2.56121 | 2.58230 | 6.25050 |
| 2.00 | −10.33316 | 2.69533 | 2.71357 | 6.74140 |
| 2.10 | −11.16045 | 2.82937 | 2.84513 | 7.27202 |
| 2.20 | −12.06577 | 2.96332 | 2.97693 | 7.84549 |
| 2.30 | −13.05543 | 3.09720 | 3.10894 | 8.46518 |
| 2.40 | −14.13624 | 3.23101 | 3.24112 | 9.13474 |
| 2.50 | −15.31556 | 3.36476 | 3.37347 | 9.85815 |
| $T=1.6$ | | | | |
| −2.30 | −2.46133 | −1.94389 | −1.67321 | 0.10107 |
| −2.00 | −2.46221 | −1.87362 | −1.60342 | 0.19060 |
| −1.50 | −2.46854 | −1.55232 | −1.28499 | 0.44085 |
| −1.20 | −2.47921 | −1.27064 | −1.00754 | 0.62716 |
| −1.00 | −2.49189 | −1.06832 | −0.80997 | 0.75929 |
| −0.80 | −2.51171 | −0.85891 | −0.60762 | 0.89795 |
| −0.60 | −2.54223 | −0.64287 | −0.40171 | 1.04526 |
| −0.50 | −2.56300 | −0.53212 | −0.29742 | 1.12321 |
| −0.40 | −2.58844 | −0.41936 | −0.19214 | 1.20475 |
| −0.30 | −2.61943 | −0.30446 | −0.08578 | 1.29052 |
| −0.20 | −2.65698 | −0.18733 | 0.02179 | 1.38125 |
| −0.10 | −2.70218 | −0.06792 | 0.13068 | 1.47772 |
| 0.00 | −2.75622 | 0.05375 | 0.24097 | 1.58078 |
| 0.10 | −2.82041 | 0.17758 | 0.35276 | 1.69132 |
| 0.20 | −2.89611 | 0.30342 | 0.46610 | 1.81026 |
| 0.30 | −2.98479 | 0.43107 | 0.58102 | 1.93859 |
| 0.40 | −3.08797 | 0.56028 | 0.69751 | 2.07728 |
| 0.50 | −3.20727 | 0.69080 | 0.81555 | 2.22737 |
| 0.60 | −3.34439 | 0.82240 | 0.93508 | 2.38990 |
| 0.70 | −3.50110 | 0.95482 | 1.05603 | 2.56599 |
| 0.80 | −3.67932 | 1.08788 | 1.17830 | 2.75679 |
| 0.90 | −3.88105 | 1.22140 | 1.30180 | 2.96350 |
| 1.00 | −4.10845 | 1.35525 | 1.42643 | 3.18743 |
| 1.10 | −4.36381 | 1.48929 | 1.55209 | 3.42993 |
| 1.20 | −4.64959 | 1.62347 | 1.67867 | 3.69250 |
| 1.30 | −4.96845 | 1.75769 | 1.80609 | 3.97670 |
| 1.40 | −5.32322 | 1.89193 | 1.93425 | 4.28424 |
| 1.50 | −5.71696 | 2.02615 | 2.06307 | 4.61695 |
| 1.60 | −6.15295 | 2.16033 | 2.19247 | 4.97682 |
| 1.70 | −6.63474 | 2.29446 | 2.32238 | 5.36596 |
| 1.80 | −7.16613 | 2.42852 | 2.45275 | 5.78670 |
| 1.90 | −7.75124 | 2.56252 | 2.58352 | 6.24151 |
| $T=1.6$ | | | | |
| 2.00 | −8.39449 | 2.69645 | 2.71462 | 6.73308 |
| 2.10 | −9.10064 | 2.83033 | 2.84603 | 7.26431 |
| 2.20 | −9.87484 | 2.96414 | 2.97771 | 7.83834 |
| 2.30 | −10.72263 | 3.09790 | 3.10961 | 8.45856 |
| 2.40 | −11.64998 | 3.23161 | 3.24170 | 9.12862 |
| 2.50 | −12.66335 | 3.36527 | 3.37397 | 9.85248 |
| 2.60 | −13.76969 | 3.49889 | 3.50638 | 10.63441 |
| 2.70 | −14.97651 | 3.63248 | 3.63893 | 11.47900 |
| $T=1.8$ | | | | |
| −2.00 | −1.85720 | −1.61303 | −1.37303 | 0.12037 |
| −1.50 | −1.86081 | −1.43354 | −1.19534 | 0.32733 |
| −1.00 | −1.87749 | −1.01291 | −0.78137 | 0.65782 |
| −0.80 | −1.89240 | −0.81113 | −0.58494 | 0.80816 |
| −0.60 | −1.91567 | −0.60038 | −0.38200 | 0.96756 |
| −0.50 | −1.93162 | −0.49208 | −0.27875 | 1.05138 |
| −0.40 | −1.95126 | −0.38183 | −0.17441 | 1.13856 |
| −0.30 | −1.97531 | −0.26953 | −0.06894 | 1.22969 |
| −0.20 | −2.00460 | −0.15510 | 0.03773 | 1.32544 |
| −0.10 | −2.04006 | −0.03848 | 0.14568 | 1.42658 |
| 0.00 | −2.08271 | 0.08035 | 0.25501 | 1.53395 |
| 0.10 | −2.13369 | 0.20137 | 0.36579 | 1.64842 |
| 0.20 | −2.19420 | 0.32449 | 0.47811 | 1.77095 |
| 0.30 | −2.26554 | 0.44954 | 0.59200 | 1.90252 |
| 0.40 | −2.34909 | 0.57635 | 0.70748 | 2.04414 |
| 0.50 | −2.44630 | 0.70469 | 0.82453 | 2.19688 |
| 0.60 | −2.55870 | 0.83432 | 0.94312 | 2.36182 |
| 0.70 | −2.68792 | 0.96503 | 1.06318 | 2.54010 |
| 0.80 | −2.83567 | 1.09659 | 1.18463 | 2.73288 |
| 0.90 | −3.00379 | 1.22881 | 1.30738 | 2.94141 |
| 1.00 | −3.19422 | 1.36154 | 1.43133 | 3.16700 |
| 1.10 | −3.40902 | 1.49465 | 1.55638 | 3.41104 |
| 1.20 | −3.65043 | 1.62801 | 1.68242 | 3.67502 |
| 1.30 | −3.92082 | 1.76156 | 1.80935 | 3.96052 |
| 1.40 | −4.22274 | 1.89522 | 1.93708 | 4.26926 |
| 1.50 | −4.55894 | 2.02895 | 2.06553 | 4.60309 |
| 1.60 | −4.93235 | 2.16272 | 2.19460 | 4.96398 |
| 1.70 | −5.34616 | 2.29649 | 2.32423 | 5.35408 |
| 1.80 | −5.80376 | 2.43025 | 2.45434 | 5.77569 |
| 1.90 | −6.30884 | 2.56400 | 2.58489 | 6.23131 |
| 2.00 | −6.86534 | 2.69772 | 2.71581 | 6.72363 |
| 2.10 | −7.47752 | 2.83140 | 2.84705 | 7.25557 |
| 2.20 | −8.14996 | 2.96506 | 2.97858 | 7.83025 |
| 2.30 | −8.88761 | 3.09869 | 3.11036 | 8.45106 |
| 2.40 | −9.69580 | 3.23228 | 3.24235 | 9.12168 |
| 2.50 | −10.58027 | 3.36585 | 3.37453 | 9.84605 |
| 2.60 | −11.54721 | 3.49939 | 3.50687 | 10.62845 |
| 2.70 | −12.60331 | 3.63290 | 3.63934 | 11.47349 |
| $T=2.0$ | | | | |
| −1.90 | −1.34162 | −1.34985 | −1.13546 | 0.10039 |
| −1.50 | −1.34341 | −1.27049 | −1.05703 | 0.23719 |
| −1.00 | −1.35484 | −0.94846 | −0.73985 | 0.55479 |
| −0.80 | −1.36603 | −0.76134 | −0.55684 | 0.71348 |
| −0.60 | −1.38393 | −0.55855 | −0.36016 | 0.88377 |
| −0.50 | −1.39635 | −0.45315 | −0.25873 | 0.97329 |
| −0.40 | −1.41172 | −0.34548 | −0.15577 | 1.06612 |
| −0.30 | −1.43066 | −0.23566 | −0.05144 | 1.16274 |
| −0.20 | −1.45384 | −0.12369 | 0.05420 | 1.26375 |
| −0.10 | −1.48207 | −0.00957 | 0.16118 | 1.36984 |
| 0.00 | −1.51621 | 0.10674 | 0.26953 | 1.48183 |
| 0.10 | −1.55726 | 0.22523 | 0.37933 | 1.60057 |
| 0.20 | −1.60628 | 0.34584 | 0.49065 | 1.72703 |
| 0.30 | −1.66442 | 0.46849 | 0.60353 | 1.86218 |
| 0.40 | −1.73292 | 0.59301 | 0.71801 | 2.00707 |
| 0.50 | −1.81311 | 0.71923 | 0.83408 | 2.16277 |

TABLE I (*continued*)

| $\mathrm{Log}_{10}(N_0)$ | $\mathrm{Log}_{10}(-dU/dt)$ | $\mathrm{Log}_{10}(P)$ | $\mathrm{Log}_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=2.0$ | | | | |
| 0.60 | −1.90637 | 0.84693 | 0.95172 | 2.33040 |
| 0·70 | −2.01420 | 0.97590 | 1.07088 | 2.51112 |
| 0.80 | −2.13817 | 1.10593 | 1.19148 | 2.70613 |
| 0.90 | −2.27994 | 1.23682 | 1.31345 | 2.91671 |
| 1.00 | −2.44131 | 1.36838 | 1.43668 | 3.14417 |
| 1.10 | −2.62415 | 1.50049 | 1.56108 | 3.38992 |
| 1.20 | −2.83051 | 1.63299 | 1.68653 | 3.65548 |
| 1.30 | −3.06254 | 1.76581 | 1.81294 | 3.94244 |
| 1.40 | −3.32258 | 1.89884 | 1.94021 | 4.25252 |
| 1.50 | −3.61310 | 2.03204 | 2.06824 | 4.58759 |
| 1.60 | −3.93679 | 2.16535 | 2.19696 | 4.94963 |
| 1.70 | −4.29653 | 2.29874 | 2.32627 | 5.34079 |
| 1.80 | −4.69539 | 2.43217 | 2.45611 | 5.76338 |
| 1.90 | −5.13671 | 2.56564 | 2.58641 | 6.21992 |
| 2.00 | −5.62406 | 2.69912 | 2.71713 | 6.71308 |
| 2.10 | −6.16129 | 2.83260 | 2.84819 | 7.24579 |
| 2.20 | −6.75254 | 2.96609 | 2.97956 | 7.82120 |
| 2.30 | −7.40228 | 3.09956 | 3.11120 | 8.44268 |
| 2.40 | −8.11531 | 3.23303 | 3.24308 | 9.11392 |
| 2.50 | −8.89682 | 3.36649 | 3.37515 | 9.83886 |
| 2.60 | −9.75241 | 3.49994 | 3.50740 | 10.62180 |
| 2.70 | −10.68808 | 3.63337 | 3.63980 | 11.46732 |
| 2.80 | −11.71034 | 3.76680 | 3.77233 | 12.38043 |
| 2.90 | −12.82620 | 3.90021 | 3.90497 | 13.36651 |
| 3.00 | −14.04322 | 4.03362 | 4.03771 | 14.43136 |
| 3.10 | −15.36955 | 4.16702 | 4.17054 | 15.58127 |
| 3.20 | −16.81398 | 4.30040 | 4.30343 | 16.82301 |
| $T=2.5$ | | | | |
| −1.50 | −0.31169 | −0.80644 | −0.64102 | 0.11380 |
| −1.00 | −0.31538 | −0.70203 | −0.53839 | 0.33288 |
| −0.80 | −0.32024 | −0.59218 | −0.43057 | 0.48157 |
| −0.60 | −0.32924 | −0.43775 | −0.27948 | 0.66138 |
| −0.50 | −0.33591 | −0.34775 | −0.19176 | 0.76045 |
| −0.40 | −0.34445 | −0.25163 | −0.09838 | 0.86463 |
| −0.30 | −0.35522 | −0.15079 | −0.00081 | 0.97357 |
| −0.20 | −0.36869 | −0.04622 | 0.09989 | 1.08726 |
| −0.10 | −0.38536 | 0.06142 | 0.20305 | 1.20595 |
| 0.00 | −0.40584 | 0.17173 | 0.30825 | 1.33015 |
| 0.10 | −0.43082 | 0.28450 | 0.41525 | 1.46053 |
| 0.20 | −0.46109 | 0.39956 | 0.52394 | 1.59791 |
| 0.30 | −0.49749 | 0.51682 | 0.63429 | 1.74323 |
| 0.40 | −0.54100 | 0.63616 | 0.74629 | 1.89749 |
| 0.50 | −0.59265 | 0.75746 | 0.85993 | 2.06181 |
| 0.60 | −0.65356 | 0.88057 | 0.97520 | 2.23733 |
| 0.70 | −0.72493 | 1.00532 | 1.09207 | 2.42527 |
| 0.80 | −0.80806 | 1.13151 | 1.21050 | 2.62689 |
| 0.90 | −0.90433 | 1.25897 | 1.33042 | 2.84350 |
| 1.00 | −1.01519 | 1.38750 | 1.45175 | 3.07651 |
| 1.10 | −1.14223 | 1.51694 | 1.57440 | 3.32736 |
| 1.20 | −1.28711 | 1.64713 | 1.69826 | 3.59760 |
| 1.30 | −1.45162 | 1.77794 | 1.82323 | 3.88888 |
| 1.40 | −1.63768 | 1.90924 | 1.94921 | 4.20295 |
| 1.50 | −1.84733 | 2.04095 | 2.07609 | 4.54170 |
| 1.60 | −2.08275 | 2.17298 | 2.20379 | 4.90714 |
| 1.70 | −2.34631 | 2.30527 | 2.33220 | 5.30145 |
| 1.80 | −2.64051 | 2.43777 | 2.46126 | 5.72695 |
| 1.90 | −2.96806 | 2.57043 | 2.59087 | 6.18618 |
| 2.00 | −3.33186 | 2.70322 | 2.72098 | 6.68184 |
| 2.10 | −3.73503 | 2.83612 | 2.85152 | 7.21687 |
| 2.20 | −4.18092 | 2.96910 | 2.98244 | 7.79441 |
| 2.30 | −4.67315 | 3.10214 | 3.11368 | 8.41787 |
| 2.40 | −5.21558 | 3.23524 | 3.24521 | 9.09094 |
| 2.50 | −5.81240 | 3.36838 | 3.37699 | 9.81758 |
| 2.60 | −6.46811 | 3.50156 | 3.50898 | 10.60209 |
| $T=2.5$ | | | | |
| 2.70 | −7.18755 | 3.63476 | 3.64116 | 11.44907 |
| 2.80 | −7.97594 | 3.76799 | 3.77350 | 12.36353 |
| 2.90 | −8.83892 | 3.90123 | 3.90598 | 13.35085 |
| 3.00 | −9.78255 | 4.03449 | 4.03858 | 14.41686 |
| 3.10 | −10.81339 | 4.16777 | 4.17128 | 15.56784 |
| 3.20 | −11.93848 | 4.30105 | 4.30407 | 16.81058 |
| $T=3.0$ | | | | |
| −1.20 | 0.48400 | −0.39505 | −0.26385 | 0.12820 |
| −1.00 | 0.48324 | −0.37637 | −0.24555 | 0.20061 |
| −0.80 | 0.48144 | −0.33517 | −0.20516 | 0.30884 |
| −0.60 | 0.47750 | −0.25556 | −0.12718 | 0.46107 |
| −0.50 | 0.47421 | −0.19774 | −0.07061 | 0.55416 |
| −0.40 | 0.46970 | −0.12796 | −0.00242 | 0.65761 |
| −0.30 | 0.46368 | −0.04753 | 0.07602 | 0.77027 |
| −0.20 | 0.45582 | 0.04168 | 0.16283 | 0.89100 |
| −0.10 | 0.44575 | 0.13780 | 0.25609 | 1.01899 |
| 0.00 | 0.43305 | 0.23929 | 0.35423 | 1.15378 |
| 0.10 | 0.41724 | 0.34497 | 0.45605 | 1.29534 |
| 0.20 | 0.39775 | 0.45405 | 0.56076 | 1.44397 |
| 0.30 | 0.37393 | 0.56601 | 0.66784 | 1.60028 |
| 0.40 | 0.34505 | 0.68049 | 0.77699 | 1.76506 |
| 0.50 | 0.31031 | 0.79725 | 0.88804 | 1.93930 |
| 0.60 | 0.26879 | 0.91610 | 1.00088 | 2.12408 |
| 0.70 | 0.21953 | 1.03686 | 1.11544 | 2.32061 |
| 0.80 | 0.16144 | 1.15937 | 1.23166 | 2.53016 |
| 0.90 | 0.09337 | 1.28344 | 1.34948 | 2.75410 |
| 1.00 | 0.01408 | 1.40890 | 1.46883 | 2.99384 |
| 1.10 | −0.07776 | 1.53558 | 1.58963 | 3.25090 |
| 1.20 | −0.18358 | 1.66331 | 1.71178 | 3.52686 |
| 1.30 | −0.30491 | 1.79195 | 1.83518 | 3.82342 |
| 1.40 | −0.44337 | 1.92135 | 1.95973 | 4.14236 |
| 1.50 | −0.60073 | 2.05139 | 2.08532 | 4.48561 |
| 1.60 | −0.77883 | 2.18198 | 2.21186 | 4.85521 |
| 1.70 | −0.97969 | 2.31302 | 2.33925 | 5.25336 |
| 1.80 | −1.20544 | 2.44443 | 2.46739 | 5.68243 |
| 1.90 | −1.45838 | 2.57615 | 2.59620 | 6.14495 |
| 2.00 | −1.74097 | 2.70814 | 2.72560 | 6.64366 |
| 2.10 | −2.05583 | 2.84034 | 2.85552 | 7.18151 |
| 2.20 | −2.40581 | 2.97272 | 2.98590 | 7.76166 |
| 2.30 | −2.79394 | 3.10526 | 3.11667 | 8.38755 |
| 2.40 | −3.22348 | 3.23791 | 3.24779 | 9.06286 |
| 2.50 | −3.69796 | 3.37068 | 3.37922 | 9.79159 |
| 2.60 | −4.22113 | 3.50353 | 3.51090 | 10.57800 |
| 2.70 | −4.79709 | 3.63645 | 3.64281 | 11.42676 |
| 2.80 | −5.43019 | 3.76944 | 3.77492 | 12.34287 |
| 2.90 | −6.12515 | 3.90248 | 3.90720 | 13.33172 |
| 3.00 | −6.88706 | 4.03556 | 4.03963 | 14.39914 |
| 3.10 | −7.72139 | 4.16868 | 4.17218 | 15.55143 |
| 3.20 | −8.63403 | 4.30183 | 4.30485 | 16.79538 |
| $T=3.5$ | | | | |
| −1.00 | 1.13473 | −0.06726 | 0.03914 | 0.13098 |
| −0.80 | 1.13405 | −0.05284 | 0.05326 | 0.20564 |
| −0.60 | 1.13243 | −0.02008 | 0.08533 | 0.31881 |
| −0.50 | 1.13095 | 0.00793 | 0.11274 | 0.39361 |
| −0.40 | 1.12878 | 0.04648 | 0.15046 | 0.48198 |
| −0.30 | 1.12567 | 0.09709 | 0.19995 | 0.58414 |
| −0.20 | 1.12132 | 0.16021 | 0.26162 | 0.69959 |
| −0.10 | 1.11543 | 0.23510 | 0.33470 | 0.82723 |
| 0.00 | 1.10765 | 0.32018 | 0.41758 | 0.96580 |
| 0.10 | 1.09759 | 0.41349 | 0.50830 | 1.11413 |
| 0.20 | 1.08483 | 0.51325 | 0.60502 | 1.27150 |
| 0.30 | 1.06890 | 0.61798 | 0.70631 | 1.43763 |
| 0.40 | 1.04921 | 0.72663 | 0.81109 | 1.61268 |
| 0.50 | 1.02516 | 0.83846 | 0.91868 | 1.79717 |

TABLE I (*continued*)

| $\text{Log}_{10}(N_0)$ | $\text{Log}_{10}(-dU/dt)$ | $\text{Log}_{10}(P)$ | $\text{Log}_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=3.5$ | | | | |
| 0.60 | 0.99601 | 0.95297 | 1.02862 | 1.99193 |
| 0.70 | 0.96098 | 1.06982 | 1.14062 | 2.19798 |
| 0.80 | 0.91917 | 1.18873 | 1.25452 | 2.41651 |
| 0.90 | 0.86961 | 1.30951 | 1.37018 | 2.64884 |
| 1.00 | 0.81126 | 1.43194 | 1.48750 | 2.89639 |
| 1.10 | 0.74295 | 1.55586 | 1.60639 | 3.16069 |
| 1.20 | 0.66347 | 1.68110 | 1.72676 | 3.44336 |
| 1.30 | 0.57148 | 1.80749 | 1.84852 | 3.74611 |
| 1.40 | 0.46556 | 1.93489 | 1.97155 | 4.07079 |
| 1.50 | 0.34418 | 2.06316 | 2.09576 | 4.41935 |
| 1.60 | 0.20570 | 2.19218 | 2.22104 | 4.79386 |
| 1.70 | 0.04839 | 2.32185 | 2.34730 | 5.19655 |
| 1.80 | −0.12964 | 2.45206 | 2.47443 | 5.62982 |
| 1.90 | −0.33037 | 2.58274 | 2.60234 | 6.09623 |
| 2.00 | −0.55596 | 2.71382 | 2.73094 | 6.59854 |
| 2.10 | −0.80869 | 2.84524 | 2.86016 | 7.13972 |
| 2.20 | −1.09104 | 2.97694 | 2.98992 | 7.72296 |
| 2.30 | −1.40563 | 3.10888 | 3.12015 | 8.35171 |
| 2.40 | −1.75530 | 3.24103 | 3.25080 | 9.02967 |
| 2.50 | −2.14308 | 3.37336 | 3.38182 | 9.76084 |
| 2.60 | −2.57225 | 3.50583 | 3.51315 | 10.54954 |
| 2.70 | −3.04631 | 3.63843 | 3.64475 | 11.40040 |
| 2.80 | −3.56905 | 3.77114 | 3.77659 | 12.31846 |
| 2.90 | −4.14451 | 3.90394 | 3.90864 | 13.30911 |
| 3.00 | −4.77709 | 4.03681 | 4.04086 | 14.37820 |
| 3.10 | −5.47149 | 4.16976 | 4.17325 | 15.53204 |
| 3.20 | −6.23279 | 4.30276 | 4.30576 | 16.77742 |
| $T=4.0$ | | | | |
| −0.95 | 1.68691 | 0.20302 | 0.29097 | 0.10321 |
| −0.80 | 1.68667 | 0.20765 | 0.29550 | 0.14537 |
| −0.60 | 1.68597 | 0.22116 | 0.30873 | 0.22842 |
| −0.40 | 1.68429 | 0.25209 | 0.33900 | 0.35479 |
| −0.20 | 1.68050 | 0.31573 | 0.40127 | 0.53830 |
| 0.00 | 1.67266 | 0.42636 | 0.50944 | 0.78555 |
| 0.10 | 1.66644 | 0.50014 | 0.58149 | 0.93179 |
| 0.20 | 1.65820 | 0.58455 | 0.66382 | 1.09134 |
| 0.30 | 1.64753 | 0.67764 | 0.75447 | 1.26290 |
| 0.40 | 1.63400 | 0.77753 | 0.85155 | 1.44557 |
| 0.50 | 1.61709 | 0.88265 | 0.95352 | 1.63899 |
| 0.60 | 1.59625 | 0.99184 | 1.05923 | 1.84328 |
| 0.70 | 1.57082 | 1.10428 | 1.16792 | 2.05899 |
| 0.80 | 1.54008 | 1.21940 | 1.27905 | 2.28700 |
| 0.90 | 1.50321 | 1.33682 | 1.39232 | 2.52843 |
| 1.00 | 1.45933 | 1.45622 | 1.50749 | 2.78462 |
| 1.10 | 1.40743 | 1.57740 | 1.62441 | 3.05703 |
| 1.20 | 1.34646 | 1.70013 | 1.74295 | 3.34728 |
| 1.30 | 1.27524 | 1.82425 | 1.86301 | 3.65710 |
| 1.40 | 1.19253 | 1.94960 | 1.98447 | 3.98833 |
| 1.50 | 1.09696 | 2.07604 | 2.10723 | 4.34297 |
| 1.60 | 0.98710 | 2.20342 | 2.23119 | 4.72311 |
| 1.70 | 0.86138 | 2.33163 | 2.35625 | 5.13103 |
| 1.80 | 0.71815 | 2.46056 | 2.48229 | 5.56914 |
| 1.90 | 0.55562 | 2.59012 | 2.60922 | 6.04003 |
| 2.00 | 0.37188 | 2.72020 | 2.73696 | 6.54649 |
| 2.10 | 0.16490 | 2.85076 | 2.86540 | 7.09152 |
| 2.20 | −0.06750 | 2.98170 | 2.99448 | 7.67832 |
| 2.30 | −0.32767 | 3.11299 | 3.12411 | 8.31036 |
| 2.40 | −0.61811 | 3.24458 | 3.25423 | 8.99138 |
| 2.50 | −0.94151 | 3.37641 | 3.38478 | 9.72538 |
| 2.60 | −1.30075 | 3.50846 | 3.51571 | 10.51669 |
| 2.70 | −1.69893 | 3.64069 | 3.64696 | 11.36998 |
| 2.80 | −2.13939 | 3.77308 | 3.77850 | 12.29029 |
| 2.90 | −2.62570 | 3.90561 | 3.91028 | 13.28302 |
| 3.00 | −3.16170 | 4.03825 | 4.04228 | 14.35404 |
| 3.10 | −3.75155 | 4.17099 | 4.17447 | 15.50966 |
| 3.20 | −4.39970 | 4.30382 | 4.30681 | 16.75670 |
| $T=5.0$ | | | | |
| −0.60 | 2.59271 | 0.64555 | 0.70820 | 0.13255 |
| −0.40 | 2.59229 | 0.65282 | 0.71532 | 0.20915 |
| −0.20 | 2.59128 | 0.67016 | 0.73232 | 0.32797 |
| 0.00 | 2.58887 | 0.70897 | 0.77036 | 0.50720 |
| 0.20 | 2.58355 | 0.78568 | 0.84554 | 0.76347 |
| 0.40 | 2.57291 | 0.91210 | 0.96940 | 1.10202 |
| 0.50 | 2.56466 | 0.99325 | 1.04884 | 1.29970 |
| 0.60 | 2.55390 | 1.08410 | 1.13770 | 1.51414 |
| 0.70 | 2.54017 | 1.18263 | 1.23398 | 1.74399 |
| 0.80 | 2.52297 | 1.28705 | 1.33592 | 1.98852 |
| 0.90 | 2.50173 | 1.39598 | 1.44215 | 2.24761 |
| 1.00 | 2.47579 | 1.50840 | 1.55171 | 2.52172 |
| 1.10 | 2.44445 | 1.62362 | 1.66395 | 2.81176 |
| 1.20 | 2.40689 | 1.74114 | 1.77842 | 3.11899 |
| 1.30 | 2.36221 | 1.86062 | 1.89484 | 3.44495 |
| 1.40 | 2.30944 | 1.98179 | 2.01300 | 3.79141 |
| 1.50 | 2.24750 | 2.10446 | 2.13273 | 4.16030 |
| 1.60 | 2.17522 | 2.22845 | 2.25390 | 4.55375 |
| 1.70 | 2.09134 | 2.35361 | 2.37640 | 4.97406 |
| 1.80 | 1.99450 | 2.47980 | 2.50012 | 5.42369 |
| 1.90 | 1.88324 | 2.60692 | 2.62494 | 5.90528 |
| 2.00 | 1.75600 | 2.73485 | 2.75077 | 6.42166 |
| 2.10 | 1.61110 | 2.86349 | 2.87750 | 6.97588 |
| 2.20 | 1.44675 | 2.99276 | 3.00505 | 7.57121 |
| 2.30 | 1.26102 | 3.12258 | 3.13333 | 8.21116 |
| 2.40 | 1.05186 | 3.25287 | 3.26225 | 8.89949 |
| 2.50 | 0.81708 | 3.38358 | 3.39175 | 9.64027 |
| 2.60 | 0.55431 | 3.51465 | 3.52175 | 10.43787 |
| 2.70 | 0.26103 | 3.64603 | 3.65219 | 11.29699 |
| 2.80 | −0.06546 | 3.77769 | 3.78302 | 12.22268 |
| 2.90 | −0.42808 | 3.90958 | 3.91419 | 13.22041 |
| 3.00 | −0.82994 | 4.04167 | 4.04565 | 14.29606 |
| 3.10 | −1.27439 | 4.17393 | 4.17737 | 15.45596 |
| 3.20 | −1.76504 | 4.30634 | 4.30931 | 16.70696 |
| $T=6.0$ | | | | |
| −0.50 | 3.32282 | 0.99139 | 1.03816 | 0.10915 |
| −0.20 | 3.32241 | 0.99812 | 1.04477 | 0.21693 |
| 0.00 | 3.32161 | 1.01117 | 1.05758 | 0.34117 |
| 0.10 | 3.32085 | 1.02322 | 1.06942 | 0.42641 |
| 0.20 | 3.31968 | 1.04119 | 1.08708 | 0.53095 |
| 0.30 | 3.31792 | 1.06724 | 1.11268 | 0.65761 |
| 0.40 | 3.31530 | 1.10360 | 1.14841 | 0.80862 |
| 0.50 | 3.31152 | 1.15212 | 1.19609 | 0.98516 |
| 0.60 | 3.30620 | 1.21363 | 1.25653 | 1.18713 |
| 0.70 | 3.29895 | 1.28773 | 1.32933 | 1.41324 |
| 0.80 | 3.28934 | 1.37295 | 1.41303 | 1.66156 |
| 0.90 | 3.27691 | 1.46728 | 1.50564 | 1.93020 |
| 1.00 | 3.26117 | 1.56870 | 1.60515 | 2.21774 |
| 1.10 | 3.24158 | 1.67550 | 1.70989 | 2.52352 |
| 1.20 | 3.21753 | 1.78639 | 1.81860 | 2.84761 |
| 1.30 | 3.18833 | 1.90044 | 1.93040 | 3.19071 |
| 1.40 | 3.15322 | 2.01702 | 2.04468 | 3.55405 |
| 1.50 | 3.11133 | 2.13569 | 2.16105 | 3.93924 |
| 1.60 | 3.06173 | 2.25613 | 2.27923 | 4.34822 |
| 1.70 | 3.00338 | 2.37811 | 2.39903 | 4.78318 |
| 1.80 | 2.93514 | 2.50145 | 2.52027 | 5.24656 |
| 1.90 | 2.85579 | 2.62599 | 2.64284 | 5.74100 |
| 2.00 | 2.76402 | 2.75161 | 2.76661 | 6.26937 |
| 2.10 | 2.65840 | 2.87818 | 2.89148 | 6.83474 |
| 2.20 | 2.53741 | 3.00560 | 3.01735 | 7.44043 |
| 2.30 | 2.39941 | 3.13378 | 3.14411 | 8.08999 |
| 2.40 | 2.24265 | 3.26262 | 3.27169 | 8.78725 |
| 2.50 | 2.06525 | 3.39205 | 3.39998 | 9.53629 |
| 2.60 | 1.86522 | 3.52200 | 3.52891 | 10.34157 |
| 2.70 | 1.64041 | 3.65240 | 3.65842 | 11.20779 |

TABLE I (*continued*)

| $\text{Log}_{10}(N_0)$ | $\text{Log}_{10}(-dU/dt)$ | $\text{Log}_{10}(P)$ | $\text{Log}_{10}(E)$ | $\mu$ |
|---|---|---|---|---|
| $T=6.0$ | | | | |
| 2.80 | 1.38852 | 3.78319 | 3.78842 | 12.14006 |
| 2.90 | 1.10709 | 3.91433 | 3.91887 | 13.14389 |
| 3.00 | 0.79348 | 4.04577 | 4.04970 | 14.22519 |
| 3.10 | 0.44486 | 4.17747 | 4.18087 | 15.39033 |
| 3.20 | 0.05820 | 4.30940 | 4.31233 | 16.64617 |
| $T=7.0$ | | | | |
| −0.30 | 3.93542 | 1.27902 | 1.31516 | 0.12241 |
| −0.20 | 3.93535 | 1.28025 | 1.31637 | 0.15400 |
| −0.10 | 3.93523 | 1.28218 | 1.31826 | 0.19366 |
| 0.00 | 3.93504 | 1.28520 | 1.32124 | 0.24338 |
| 0.10 | 3.93474 | 1.28992 | 1.32589 | 0.30556 |
| 0.20 | 3.93427 | 1.29722 | 1.33308 | 0.38304 |
| 0.30 | 3.93355 | 1.30837 | 1.34405 | 0.47907 |
| 0.40 | 3.93243 | 1.32507 | 1.36051 | 0.59712 |
| 0.50 | 3.93074 | 1.34945 | 1.38451 | 0.74060 |
| 0.60 | 3.92822 | 1.38376 | 1.41832 | 0.91238 |
| 0.70 | 3.92455 | 1.43000 | 1.46388 | 1.11419 |
| 0.80 | 3.91937 | 1.48925 | 1.52225 | 1.34630 |
| 0.90 | 3.91226 | 1.56135 | 1.59328 | 1.60749 |
| 1.00 | 3.90279 | 1.64499 | 1.67568 | 1.89565 |
| 1.10 | 3.89048 | 1.73818 | 1.76748 | 2.20849 |
| 1.20 | 3.87486 | 1.83886 | 1.86662 | 2.54422 |
| 1.30 | 3.85537 | 1.94519 | 1.97131 | 2.90184 |
| 1.40 | 3.83141 | 2.05580 | 2.08019 | 3.28124 |
| 1.50 | 3.80230 | 2.16967 | 2.19229 | 3.68309 |
| 1.60 | 3.76727 | 2.28613 | 2.30696 | 4.10869 |
| 1.70 | 3.72547 | 2.40468 | 2.42374 | 4.55983 |
| 1.80 | 3.67596 | 2.52501 | 2.54232 | 5.03871 |
| 1.90 | 3.61771 | 2.64687 | 2.66251 | 5.54785 |
| 2.00 | 3.54960 | 2.77007 | 2.78412 | 6.09005 |
| 2.10 | 3.47039 | 2.89448 | 2.90703 | 6.66838 |
| 2.20 | 3.37877 | 3.01995 | 3.03111 | 7.28617 |
| 2.30 | 3.27332 | 3.14637 | 3.15625 | 7.94700 |
| 2.40 | 3.15251 | 3.27365 | 3.28236 | 8.65473 |
| 2.50 | 3.01471 | 3.40168 | 3.40934 | 9.41351 |
| 2.60 | 2.85817 | 3.53039 | 3.53711 | 10.22781 |
| 2.70 | 2.68102 | 3.65970 | 3.66557 | 11.10241 |
| 2.80 | 2.48125 | 3.78953 | 3.79465 | 12.04245 |
| 2.90 | 2.25672 | 3.91983 | 3.92428 | 13.05348 |
| 3.00 | 2.00513 | 4.05053 | 4.05439 | 14.14145 |
| 3.10 | 1.72402 | 4.18159 | 4.18493 | 15.31277 |
| 3.20 | 1.41076 | 4.31295 | 4.31585 | 16.57434 |
| 3.30 | 1.06251 | 4.44458 | 4.44709 | 17.93360 |
| $T=8.0$ | | | | |
| −0.20 | 4.46376 | 1.52375 | 1.55247 | 0.11515 |
| 0.00 | 4.46363 | 1.52588 | 1.55457 | 0.18227 |
| 0.10 | 4.46350 | 1.52793 | 1.55660 | 0.22920 |
| 0.20 | 4.46329 | 1.53115 | 1.55977 | 0.28802 |
| 0.30 | 4.46297 | 1.53617 | 1.56473 | 0.36156 |
| 0.40 | 4.46246 | 1.54392 | 1.57238 | 0.45315 |
| 0.50 | 4.46167 | 1.55574 | 1.58404 | 0.56657 |
| 0.60 | 4.46046 | 1.57339 | 1.60146 | 0.70586 |
| 0.70 | 4.45863 | 1.59905 | 1.62679 | 0.87491 |
| 0.80 | 4.45591 | 1.63498 | 1.66226 | 1.07691 |
| 0.90 | 4.45196 | 1.68309 | 1.70977 | 1.31372 |
| 1.00 | 4.44641 | 1.74432 | 1.77023 | 1.58545 |
| $T=8.0$ | | | | |
| 1.10 | 4.43881 | 1.81835 | 1.84333 | 1.89056 |
| 1.20 | 4.42873 | 1.90370 | 1.92763 | 2.22659 |
| 1.30 | 4.41569 | 1.99836 | 2.02110 | 2.59095 |
| 1.40 | 4.39918 | 2.10021 | 2.12168 | 2.98167 |
| 1.50 | 4.37864 | 2.20750 | 2.22761 | 3.39772 |
| 1.60 | 4.35345 | 2.31887 | 2.33758 | 3.83909 |
| 1.70 | 4.32291 | 2.43337 | 2.45065 | 4.30662 |
| 1.80 | 4.28623 | 2.55032 | 2.56617 | 4.80187 |
| 1.90 | 4.24255 | 2.66929 | 2.68374 | 5.32696 |
| 2.00 | 4.19089 | 2.78996 | 2.80305 | 5.88446 |
| 2.10 | 4.13022 | 2.91210 | 2.92389 | 6.47730 |
| 2.20 | 4.05936 | 3.03554 | 3.04610 | 7.10875 |
| 2.30 | 3.97709 | 3.16014 | 3.16955 | 7.78239 |
| 2.40 | 3.88204 | 3.28577 | 3.29412 | 8.50209 |
| 2.50 | 3.77278 | 3.41233 | 3.41971 | 9.27202 |
| 2.60 | 3.64776 | 3.53972 | 3.54622 | 10.09668 |
| 2.70 | 3.50530 | 3.66785 | 3.67355 | 10.98090 |
| 2.80 | 3.34362 | 3.79664 | 3.80163 | 11.92988 |
| 2.90 | 3.16083 | 3.92601 | 3.93037 | 12.94920 |
| 3.00 | 2.95488 | 4.05590 | 4.05969 | 14.04485 |
| 3.10 | 2.72358 | 4.18624 | 4.18954 | 15.22330 |
| 3.20 | 2.46460 | 4.31698 | 4.31984 | 16.49147 |
| 3.30 | 2.17543 | 4.44807 | 4.45055 | 17.85685 |
| $T=10.0$ | | | | |
| 0.00 | 5.34331 | 1.92720 | 1.94653 | 0.11344 |
| 0.10 | 5.34328 | 1.92771 | 1.94704 | 0.14277 |
| 0.20 | 5.34323 | 1.92852 | 1.94784 | 0.17966 |
| 0.30 | 5.34314 | 1.92979 | 1.94910 | 0.22602 |
| 0.40 | 5.34301 | 1.93179 | 1.95108 | 0.28422 |
| 0.50 | 5.34281 | 1.93494 | 1.95419 | 0.35719 |
| 0.60 | 5.34248 | 1.93984 | 1.95905 | 0.44843 |
| 0.70 | 5.34197 | 1.94742 | 1.96656 | 0.56213 |
| 0.80 | 5.34118 | 1.95899 | 1.97801 | 0.70301 |
| 0.90 | 5.33997 | 1.97630 | 1.99515 | 0.87617 |
| 1.00 | 5.33812 | 2.00151 | 2.02012 | 1.08658 |
| 1.10 | 5.33538 | 2.03692 | 2.05519 | 1.33841 |
| 1.20 | 5.33140 | 2.08447 | 2.10230 | 1.63419 |
| 1.30 | 5.32579 | 2.14519 | 2.16246 | 1.97427 |
| 1.40 | 5.31810 | 2.21882 | 2.23542 | 2.35692 |
| 1.50 | 5.30788 | 2.30394 | 2.31978 | 2.77908 |
| 1.60 | 5.29464 | 2.39849 | 2.41350 | 3.23750 |
| 1.70 | 5.27786 | 2.50036 | 2.51447 | 3.72960 |
| 1.80 | 5.25699 | 2.60773 | 2.62090 | 4.25398 |
| 1.90 | 5.23139 | 2.71920 | 2.73141 | 4.81049 |
| 2.00 | 5.20037 | 2.83380 | 2.84503 | 5.40011 |
| 2.10 | 5.16312 | 2.95084 | 2.96111 | 6.02473 |
| 2.20 | 5.11878 | 3.06987 | 3.07920 | 6.68697 |
| 2.30 | 5.06638 | 3.19057 | 3.19901 | 7.39001 |
| 2.40 | 5.00485 | 3.31273 | 3.32031 | 8.13752 |
| 2.50 | 4.93303 | 3.43617 | 3.44294 | 8.93361 |
| 2.60 | 4.84968 | 3.56075 | 3.56677 | 9.78275 |
| 2.70 | 4.75343 | 3.68635 | 3.69169 | 10.68982 |
| 2.80 | 4.64283 | 3.81287 | 3.81758 | 11.66007 |
| 2.90 | 4.51631 | 3.94022 | 3.94436 | 12.69916 |
| 3.00 | 4.37220 | 4.06831 | 4.07194 | 13.81319 |
| 3.10 | 4.20872 | 4.19705 | 4.20023 | 15.00868 |
| 3.20 | 4.02393 | 4.32638 | 4.32915 | 16.29266 |
| 3.30 | 3.81578 | 4.45623 | 4.45864 | 17.67269 |

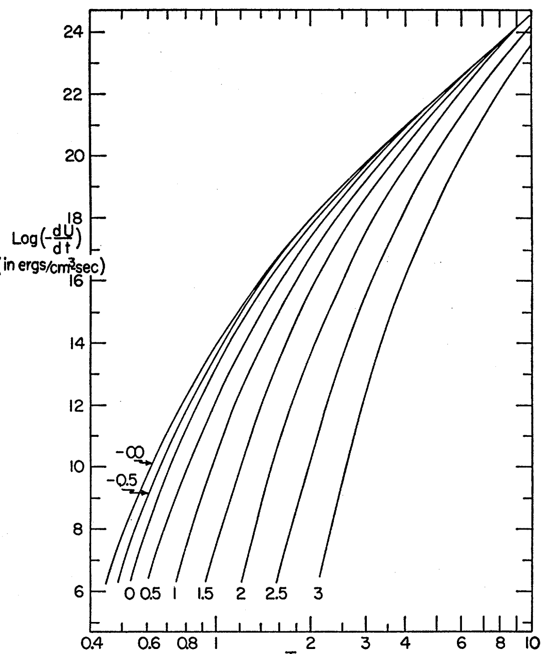


FIG. 1. Log($-dU/dt$) is plotted against $T$ for different values of $\log_{10}N_0$. $-dU/dt$ is measured in ergs/cm³-sec and $T$ is measured in units of $10^9$ °K. Numbers attached to curves are values of $\log_{10}N_0$. $\text{Log}_{10}N_0=0$ corresponds to a density of $3\times10^6\ \mu_e$ g/cm³.

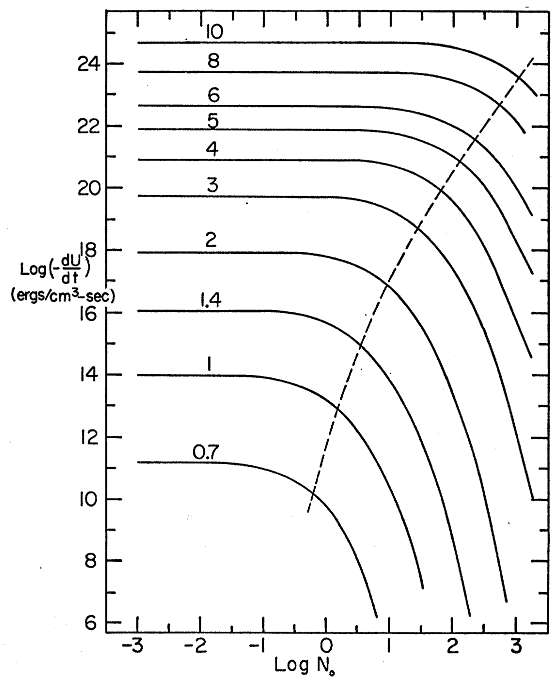


FIG. 2. $\text{Log}_{10}(-dU/dt)$ is plotted against $\log_{10}N_0$ for different values of $T$. $T$ is measured in units of $10^9$ °K.

## III. DISCUSSION

We have based our calculations on the following assumptions:

(1) The process $e^-+e^+\rightarrow\nu+\bar{\nu}$ is a first-order weak process. We remark that this is the case if one assumes the validity of, for example, Feynman and Gell-Mann's theory.

(2) The electron (or positron) gas is a perfect Fermi gas.

(3) All radiative corrections are neglected.

That the process $e^-+e^+\rightarrow\nu+\bar{\nu}$ is a first-order process has never been subjected to experimental tests. Thus the consequences of this process in stellar evolution may be used as a confirmation of the existence of direct electron-neutrino interaction, although at present it is not possible to determine the coupling constant accurately in this way. The justification for treating electron and positron as perfect Fermi gas has been given by a number of authors.[11] The Coulomb interaction has been demonstrated to be small when the density is high.[8] Plasma effect is expected to affect the equilibrium among pair production and annihilation only when the energy involved in such plasma is comparable to the total energy of the electron gas. This is not likely to be true.

Radiative correction is expected to be of the order $(137)^{-1}\sim10^{-2}$ in the temperature regime we consider ($T\lesssim10^{10}$ °K $\rightleftharpoons 2mc^2$). Formation of positronium will be suppressed by high density.

Finally, other competing processes like $\gamma+\gamma\rightarrow\gamma+\nu+\bar{\nu}$ may be quite important. $\gamma+\gamma\rightarrow\nu+\bar{\nu}$ may not occur if Fermi interaction is strictly local.[12]

[11] D. F. DuBois, Ann. Phys. 8, 24 (1959).

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## APPENDIX

The function

$$\langle E_{\pm}{}^m\rangle=\int_0^\infty (E^m p^2 dp/\exp[\beta(E\pm\mu)]+1) \equiv F_m{}^{\pm}(\beta,\mu) \quad (19)$$

[12] See, for example, M. Gell-Mann, Phys. Rev. Letters 6, 70 (1961).

(where $E^2=1+p^2$) possesses series expansions in $K_n(Z)$ only in limits of $\mu$. $K_n(Z)$ is defined as

$$K_n(Z)=\int_0^\infty \exp(-Z\cosh\theta)\cosh n\theta d\theta, \qquad (20)$$

and is the modified Bessel function of the second kind. Analytical properties of $K_n(Z)$ may be found in existing literature.[13]

It is easily verified that

$$\int_0^\infty \exp(-\beta E)p^2dp=K_2(\beta)/\beta. \qquad (21)$$

By differentiating Eq. (21) with respect to $(-\beta)$ $m$ times, one obtains

$$\int_0^\infty E^m \exp(-\beta E)p^2dp=\left[\frac{\partial}{\partial(-\beta)}\right]^m \frac{K_2(\beta)}{\beta}. \qquad (22)$$

Equation (22) may be further simplified by using the well-known recurrence relations for $K_n(Z)$. Useful recurrence relations are

$$ZK_n'=-nK_n-ZK_{n-1}=nK_n-ZK_{n+1}, \qquad (23)$$

$$(2n/Z)K_n=K_{n+1}-K_{n-1}. \qquad (24)$$

From Eq. (23), $K_0'=-K_1$. For $0\leq m\leq 2$, we have [letting $f_m(\beta)=\int_0^\infty E^m \exp(-\beta E)p^2dp$]:

$$f_0(\beta)=[K_2(\beta)/\beta], \qquad (25)$$

$$f_1(\beta)=[3K_2(\beta)/\beta^2]+\frac{1}{\beta}K_1(\beta), \qquad (26)$$

$$f_2(\beta)=\left[\frac{12}{\beta^3}+\frac{1}{\beta}\right]K_2(\beta)+\frac{3}{\beta^2}K_1(\beta). \qquad (27)$$

[13] G. Watson, *Theory of "Bessel Functions"* (The Macmillan Company, New York, 1948).

The case $m=-1$ may be worked out as follows:

$$\begin{aligned} f_{-1}(\beta)&=-\int d\beta\, f_0(\beta)=-\int_m \frac{K_2(\beta)}{\beta}d\beta \\ &=\tfrac{1}{2}\int (K_2'+K_1)d\beta=\tfrac{1}{2}\int (K_2'-K_0')d\beta \\ &=\tfrac{1}{2}(K_2-K_0). \end{aligned} \qquad (26)$$

The constant of integration is zero since $f_{-1}$, $K_2$, $K_0$ all tend to zero when $\beta\rightarrow\infty$.

When $Z\rightarrow\infty$, the behavior of $K_n(Z)$ is

$$K_n(Z) \xrightarrow{Z\to\infty} (\pi/2Z)^{\frac{1}{2}}\exp(-Z). \qquad (27)$$

Thus, by using the series expansion for $1/(1+x)=\sum_0^\infty(-)^n x^n$, we may write, for $\mu<1$,

$$\begin{aligned} F_m^{\pm}(\beta,\mu)&=\int_0^\infty E^m \sum_0^\infty (-)^{n+1}\exp[-n\beta(E\pm\mu)]p^2dp \\ &=\sum_0^\infty (-)^{n+1}\exp(\pm n\beta\mu)f_m(n\beta). \end{aligned} \qquad (28)$$

For the case $\mu\leq 1$, the above expansion for $F_m^+$ is still absolutely convergent while that for $F_m^-$ is divergent. Unfortunately no series expansions of any virtue are known for $F_m^-$. Chandrasekhar[14] has worked out asymptotic expansions for functions of the same type as $F_m^-$. Their expansion are accurate to the order $\exp(-\beta\mu)$, when $\beta\mu\gg 1$. Physically, interesting cases occur at around $\mu\beta\simeq 1$. This corresponds to the region around the dashed curve in Fig. 2. The evolution of a perfect gas sphere follows this dashed curve closely, when the initial conditions are taken from the latest red giant model[15] ($\rho_0=5\times 10^4$ g/cm$^3$, $T_0=2\times 10^8$ °K, $\rho_0$ is the density) and the law of evolution is $\rho/\rho_0=(T/T_0)^3$.[4]

[14] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (Dover Publications, New York, 1957), p. 389.

[15] M. Schwarzschild, private communication (to be published).