Method of Determining the Spin and Parity of a Pion-Hyperon Resonance

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The reaction sequence, $M_1+N_1\rightarrow Y^*+M_2$; $Y^*\rightarrow Y+\pi_1$; $Y\rightarrow N_2+\pi_2$ is considered, where M_1 and M_2 are spin-zero mesons, N_1 and N_2 are nucleons, Y is a Λ or Σ^+ particle, and Y^* is a pion-hyperon resonance of spin $\frac{1}{2}$ or $\frac{3}{2}$. The general form of the angular distribution of the particles π_1 and N_2 is written down under the assumption that final state interactions between the meson M_2 and the Y^* decay particles may be neglected. If any polarization of the hyperon Y exists, the spin and parity of the resonance Y^* may be determined from this angular distribution. The structure of the spin density matrix of the Y^* is discussed.

THERE is now strong experimental evidence for the existence of a pion-lambda particle resonance at a center-of-mass kinetic energy of about 130 Mev.[1] Since this energy is below the $\bar{K}+N$ rest mass, the resonance can be produced in the laboratory only in conjunction with other particles; the simplest producible final state including the resonance is of the type involved in the experiment of Alston *et al.*,[1] i.e., $M+Y^*$, where Y^* is the resonance and M is a spin-zero meson. It is likely that one or more other pion-hyperon resonances will be found in the low-energy region; it may be necessary to investigate these resonances also from a reaction sequence of the type,

$$M_1+N_1\rightarrow Y^*+M_2, \tag{1a}$$

$$Y^*\rightarrow Y+\pi_1, \tag{1b}$$

$$Y\rightarrow N_2+\pi_2, \tag{1c}$$

where M_1 and M_2 are spin-zero mesons, N_1 and N_2 are nucleons, and Y is a Σ or Λ particle, assumed to have spin $\frac{1}{2}$.

The experimental determination of the spin and parity of a resonance Y^* from the interaction sequence of Eqs. (1) is a difficult problem because the orbital and total angular momenta of the state Y^*+M_2 are unknown. Several types of measurements that might reveal the spin or parity have been suggested.[2,3] The method of Adair can be used to determine the spin (but not the parity) if enough data are available.[2] Several spin measurements have already been attempted for the Y^* of Alston *et al.*, but no definite results have been obtained yet.[1]

The purpose of this note is to point out certain angular distribution and correlation measurements that might reveal the parity and spin of a pion-hyperon resonance from the interaction sequence of Eq. (1). We use the following set of unit vectors for describing the event: $\mathbf{k}_i$ and $\mathbf{k}_f$ denote the momentum directions of the mesons M_1 and M_2 of Eq. (1a) in the center-of-mass system; $\mathbf{x}$ and $\mathbf{y}$ represent any orthonormal linear combinations of $\mathbf{k}_i$ and $\mathbf{k}_f$, while $\mathbf{z}=\mathbf{x}\times\mathbf{y}$ is the normal to the plane of the production event, Eq. (1a). The momentum direction of the Y^* decay pion π_1, *measured in the center-of-mass system of the* Y^*, is denoted by $\mathbf{k}$, and the momentum direction of the proton from the Y decay, *measured in the center-of-mass system of the* Y, is denoted by $\mathbf{p}$.

For the time being, we neglect the possible effects of final state interactions between the meson M_1 and the particles resulting from the "fast" Y^* decay. We may then write down the most general form for the distribution $\boldsymbol{\Sigma}$ of the momentum unit vectors $\mathbf{k}$ and $\mathbf{p}$ corresponding to the four simplest possibilities for the spin and parity of the Y^*, i.e., that it is an $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, $P_{\frac{3}{2}}$, or $D_{\frac{3}{2}}$ combination of the Y and π particles. The formulas for $\boldsymbol{\Sigma}$ are written below in cylindrical coordinates defined with respect to the direction normal to the production plane, i.e., $k_r=(k_x^2+k_y^2)^{\frac{1}{2}}$, $\mathbf{k}_r=k_r\mathbf{r}/r$, $\tan\phi_k=k_y/k_x$, $k_z=k_z$, etc.

$$\boldsymbol{\Sigma}(S_{\frac{1}{2}})=\mathcal{A}+\mathcal{B}p_z, \tag{2a}$$

$$\boldsymbol{\Sigma}(P_{\frac{1}{2}})=\mathcal{A}'+\mathcal{B}'[2(p_zk_z+\mathbf{p}_r\cdot\mathbf{k}_r)k_z-p_z], \tag{2b}$$

$$\boldsymbol{\Sigma}(P_{\frac{3}{2}};D_{\frac{3}{2}})=\boldsymbol{\Sigma}_1(P_{\frac{3}{2}};D_{\frac{3}{2}})+\boldsymbol{\Sigma}_2(P_{\frac{3}{2}};D_{\frac{3}{2}}),$$

$$\boldsymbol{\Sigma}_1(P_{\frac{3}{2}})=A+Bk_z^2+(\tfrac{5}{2}D-\tfrac{3}{2}C)p_zk_z^2+(\tfrac{3}{2}C-\tfrac{1}{2}D)p_z-2D\mathbf{p}_r\cdot\mathbf{k}_rk_z, \tag{3a}$$

$$\boldsymbol{\Sigma}_2(P_{\frac{3}{2}})=Ek_r^2\cos(2\phi_k+\alpha)+F[p_zk_r^2\cos(2\phi_k+\beta)+2p_rk_rk_z\cos(\phi_k+\phi_p+\beta)], \tag{3b}$$

$$\boldsymbol{\Sigma}_1(D_{\frac{3}{2}})=A'+B'k_z^2+(9D'-3C')(p_zk_z+\mathbf{p}_r\cdot\mathbf{k}_r)k_z^3+[(9/2)C'-(15/2)D']p_zk_z^2+(\tfrac{1}{2}D'-\tfrac{3}{2}C')p_z+3(C'-D')\mathbf{p}_r\cdot\mathbf{k}_rk_z, \tag{3c}$$

$$\boldsymbol{\Sigma}_2(D_{\frac{3}{2}})=E'k_r^2\cos(2\phi_k+\alpha')+F'[p_z(6k_z^2-1)k_r^2\cos(2\phi_k+\beta')-p_rk_rk_z(3k_z^2-1)\cos(\phi_k+\phi_p+\beta')+3p_rk_r^3k_z\cos(3\phi_k-\phi_p+\beta')], \tag{3d}$$

where the coefficients $\mathcal{A}$ and $\mathcal{B}$, or A, B, C, D, E, F, α, and β are real functions of the production angle $\mathbf{k}_i\cdot\mathbf{k}_f$, and are unknown for $\mathbf{k}_i\cdot\mathbf{k}_f\neq\pm1$ if the dynamics of

[1] M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **5**, 521 (1960).
[2] R. K. Adair, Phys. Rev. **100**, 1540 (1955).
[3] Ph. Meyer, J. Prentki, and Y. Yamaguchi, Phys. Rev. Letters **5**, 442 (1955).

the process $M_1+N_1 \rightarrow Y^*+M_2$ are unknown.[4] When $\mathbf{k}_i \cdot \mathbf{k}_f = \pm 1$ the distribution must reduce to the form given by Adair.[2] All terms linear in $\mathbf{p}$ are proportional to the asymmetry parameter of the Y decay. It is seen that there are many kinds of correlations that may be present for $j(Y^*)=\frac{3}{2}$ but cannot be present for $j=\frac{1}{2}$. Terms proportional to $(v/c)^2$ have been neglected, where v is the velocity of the Y in the center-of-mass system of the Y^*.

The quantities $\mathcal{A}$, $\mathcal{B}$ or A, B, C, D, $E\cos\alpha$, $E\sin\alpha$, $F\cos\beta$, $F\sin\beta$ are linear combinations of elements of the spin density matrix ρ of the Y^* produced in reaction (1a); we may make the general structure of $\mathbf{\Sigma}$ more transparent by discussing the general structure of ρ. The density matrix is Hermitean and so, in general, contains $(2j+1)^2$ independent real numbers, but parity conservation together with the lack of any polarization in the initial state reduces this by a factor of two, as may be seen from the following argument. The matrix ρ may be written as a sum of products of J_x, J_y, and J_z, where $\mathbf{J}$ is the spin operator for the Y^*. Invariance to rotations of the entire interaction scheme, together with the lack of any initial polarization, implies that the coefficients of this sum are functions only of $\mathbf{k}_i \cdot \mathbf{k}_f$ and energy. Since $\mathbf{x}$ and $\mathbf{y}$ are normalized linear combinations of the momenta $\mathbf{k}_i$ and $\mathbf{k}_f$, they are odd under reflections, while $\mathbf{z}=\mathbf{x}\times\mathbf{y}$ is even under reflections. Hence, reflection invariance implies that ρ is unchanged by the simultaneous substitutions $J_x \rightarrow -J_x$, $J_y \rightarrow -J_y$. This is equivalent to invariance under rotations of 180 degrees around the $\mathbf{z}$ axis, implying that $[\rho, e^{i\pi J_z}]=0$. In the representation where J_z is diagonal, this commutator condition implies that $\rho_{\alpha\beta}=0$ if $|\alpha-\beta|$ is odd, so that ρ has a "checker-board" pattern.

If $j=\frac{1}{2}$ only the diagonal elements of the density matrix may be nonzero in the J_z representation, so that the Y^* may be regarded as a statistical mixture of particles with spins in the $\mathbf{z}$ and $(-\mathbf{z})$ directions. It may be shown that for any Y^* spin those correlation and angular distribution measurements that are invariant to rotations about the $\mathbf{z}$ axis involve only diagonal elements of $\mathbf{\Sigma}$. For $j=\frac{3}{2}$ these terms are represented by $\mathbf{\Sigma}_1$ in Eqs. (3). The $\mathbf{\Sigma}_2$ terms vary harmonically, with period π, under rotations about the $\mathbf{z}$ axis. The Y^* may be regarded as a statistical mixture of the four eigenstates of J_z only for computing the ϕ invariant terms $\mathbf{\Sigma}_1$.[5]

We now limit our consideration to the possibilities, $j=\frac{1}{2}$ or $\frac{3}{2}$, and assume that the Y particle resulting from the Y^* decay is a Λ or Σ^+, and therefore can decay asymmetrically. Then if any Y polarization exists, so that some term linear in $\mathbf{p}$ of the distribution $\mathbf{\Sigma}$ exists, the parity and spin may be determined unambiguously. For illustration we will show how the $P_{\frac{3}{2}}$ and $D_{\frac{3}{2}}$ cases may be distinguished by means of the polarization angular distribution (defined as the distribution in k_z of all events in which p_z is positive, minus the corresponding distribution for p_z negative) together with the "transverse correlation" represented by the $\mathbf{p}_r \cdot \mathbf{k}_r k_z$ terms of $\mathbf{\Sigma}$. The presence of k_z^4 terms in the polarization angular distribution would indicate a $D_{\frac{3}{2}}$ resonance. On the other hand, these k_z^4 terms can vanish for the $D_{\frac{3}{2}}$ case only if $C'=3D'$, in which case the terms linear in $\mathbf{p}$ of Eq. (3c) become $2D'[p_z(3k_z^2-2)+3\mathbf{p}_r\cdot\mathbf{k}_r k_z]$. Such a distribution is impossible for the $P_{\frac{3}{2}}$ case, however, for the $P_{\frac{3}{2}}$ polarization angular distribution can be of the form $3k_z^2-2$ only if $C=-(7/3)D$, in which case the terms linear in $\mathbf{p}$ of Eq. (3a) become

$$2D[p_z(3k_z^2-2)-\mathbf{p}_r\cdot\mathbf{k}_r k_z].$$

Hence, if the polarization angular distribution happens to be of the form $3k_z^2-2$, one of the possibilities $P_{\frac{3}{2}}$ or $D_{\frac{3}{2}}$ may be eliminated from the sign and magnitude of the transverse correlation relative to the polarization angular distribution.

It may be shown easily, by extending the above analysis, that there is no nonzero form for the terms of the distribution linear in $\mathbf{p}$ that is possible for more than one of the four spin and parity cases under consideration. Hence, the spin and parity may be determined unless all Y polarization terms are small at all production angles, not a very likely occurrence if the energy of the incident meson is sufficiently high so that several Y^*+M_2 orbital angular momenta are present.

Even if all the polarization terms do vanish, the theorem of Eberhard and Good[6] can be used to show that one of the two Y^* spin cases can be eliminated from data at any production angle. The theorem states that if the spin wave function of a particle is a statistical mixture of Q pure polarization states, the trace of the square of the spin density matrix satisfies the inequality $\mathrm{Tr}\rho^2 \geqq (\mathrm{Tr}\rho)^2/Q$. In reaction (1a), Q is equal to two (the number of polarization states of N_1); hence, the Y^* spin density matrix cannot be a multiple of the unit matrix if $j(Y^*)\geqq\frac{3}{2}$. This implies that at least one of the constants B, C, D, E, and F in Eqs. (3) must be nonzero, so that $\mathbf{\Sigma}(j=\frac{3}{2})$ cannot have a form that is possible for $j=\frac{1}{2}$.

The preceding results are based on the assumption that the final-state interactions between the meson M_2 and the Y^* decay particles are negligible. Such interaction effects should not be important if, when the distribution of all $M_1+N\rightarrow Y+M_2+\pi_1$ events in the center-of-mass energy of the $Y+\pi_2$ is plotted, the

[4] These coefficients are not quite independent. They must satisfy certain inequalities and, for $j=\frac{3}{2}$, one complicated quadratic equality. This equality is not significant for the arguments of this paper.

[5] If the relative populations of the four states are denoted by $\rho(j_z)$, then the coefficients C and D (or C' and D') in Eqs. (3) are proportional to $\rho(\frac{3}{2})-\rho(-\frac{3}{2})$ and $\rho(\frac{1}{2})-\rho(-\frac{1}{2})$, respectively.

[6] Philippe Eberhard and M. L. Good, Phys. Rev. **120**, 1442 (1960). See also, Murray Peshkin (to be published) for an extension of this theorem.

resonance peak is many times higher than the average of the distribution outside the resonance. This is not always the case, however. One precautionary check that may easily be made is to plot the angular distribution of all the events under the resonance peak with respect to the direction $\mathbf{k}_f$ of the M_2. This distribution must be symmetric in the absence of a final-state interaction, but is likely to be appreciably asymmetric if the M_2-Y or M_2-π_1 interaction is sufficiently important to alter the general distribution of Eq. (2) or (3). One should make sure that there is no large asymmetry in $\mathbf{k}\cdot\mathbf{k}_f$ present in the data.

It has come to the author's attention that the results of this paper have been derived independently by R. Gatto and H. P. Stapp [see Phys. Rev. **121**, 1555 (1961)].

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High-Energy Potential Scattering with Short-Range Forces

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An attempt is made to separate out long- and short-range effects for high-energy elastic scattering. Within the context of a high-energy approximation, expressions for the scattering amplitudes are obtained for the cases $kR\gg ka\gg 1$ and $kR\gg 1>ka$, where R and a denote the long and short ranges, respectively. For the latter case, the entire short-range effect is included in a phenomenological S-wave term while the long-range contributions are written explicitly.

I. INTRODUCTION

With the increase in energy of scattering experiments, we see more details of the interaction of particles at short ranges. Such experiments are complicated by the presence of the longer range interactions generally employed to interpret experimental results at lower energies. Thus, it is of interest to see to what extent we can separate these two effects, hopefully in such a manner that will enable us to utilize our previous knowledge about the long-range interactions in some relatively simple way.[1] As a first approach, it seems convenient to work within the context of a high-energy small-angle approximation for elastic scattering based on the work of Molière[2] and developed in some detail by Glauber[3] and others.[4–6]

II. HIGH-ENERGY APPROXIMATION

We are interested in the case where the scattering amplitude for a high-energy particle of reduced mass m with an incident momentum propagation vector $\mathbf{k}$ and a final propagation vector $\mathbf{k}'$ is given by[3]

$$f(\mathbf{k}',\mathbf{k})=\frac{k}{2\pi i}\int \exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}]\times\{\exp[i\chi(\mathbf{b})]-1\}d^{(2)}b, \quad (1)$$

where

$$\chi(\mathbf{b})=-\frac{m}{\hbar^2 k}\int_{-\infty}^{\infty}V(\mathbf{b}+\boldsymbol{\kappa}z)dz, \quad (2)$$

with

$$\boldsymbol{\kappa}=(\mathbf{k}+\mathbf{k}')/|\mathbf{k}+\mathbf{k}'|. \quad (3)$$

Here $V(\mathbf{r})=V(\mathbf{b}+\boldsymbol{\kappa}z)$ is assumed to be a potential which represents the interaction between the projectile and target particles. For simplicity, we will further assume the potential to be azimuthally symmetric, that is $V(\mathbf{b}+\boldsymbol{\kappa}z)=V(b,z)$. The extension to other interesting cases is fairly direct.[3,4] Equation (1) can then be written

$$f_0(\theta)=\frac{k}{i}\int_0^{\infty}J_0(2kb\sin\tfrac{1}{2}\theta)\{e^{i\chi(b;R)+i\chi(b;a)}-1\}bdb, \quad (4)$$

where $J_0(2kb\sin\frac{1}{2}\theta)$ is the Bessel function of order zero. In writing the above expression, we have now explicitly assumed that the potential can be split into a long-range part characterized by a range R and a short-range part characterized by a range a, where $V(b,z)=V(b,z;R)$

[1] B. J. Malenka and H. S. Valk, Bull. Am. Phys. Soc. **5**, 269 (1960). This abstract contains a preliminary report of some of the work in this paper and reference 7.
[2] G. Molière, Z. Naturforsch. **2A**, 133 (1947).
[3] R. J. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.
[4] B. J. Malenka, Phys. Rev. **95**, 522 (1956).
[5] I. I. Shapiro, thesis, Harvard University, Cambridge, Massachusetts, 1955 (unpublished).
[6] L. I. Schiff, Phys. Rev. **103**, 443 (1956).