Calculations in Nuclear Evaporation Theory*

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Methods are developed for analytic treatment of problems in nuclear evaporation theory using the level density formula $\exp\{2[a(E^*-\epsilon)]^{\frac{1}{2}}\}$. Several useful expansions are given, with their ranges of validity. Comparisons made with existing calculations indicate the validity of this approach.

I. INTRODUCTION

WHEN a nucleus is excited by means of a nuclear reaction or through energy made available in the process of fission, it may de-excite by ejecting one or several nucleons. If the nuclear excitation is less than 50 Mev, one treats the probability of ejecting various numbers of nucleons by means of evaporation theory.[1] In the most commonly used version of the theory, the energy spectrum of the emitted nucleons depends exponentially on the square root of the excitation energy of the parent nucleus. Because there are difficulties in treating the consecutive emission of several nucleons analytically, people have resorted to the use of Monte Carlo calculations.[2] It is the purpose of this paper to show that such calculations may be done with much greater convenience through the use of suitable approximations in an analytic treatment of the problem, rather than through the use of Monte Carlo techniques. Through the use of these techniques, one may examine the parameters involved in the theory with considerably more ease.

The analysis presented here may be conveniently divided into two parts: (1) the relative probabilities of various numbers of particles being emitted, when only one type of particle is emitted, e.g., neutrons in the heavy element region; and (2) the competition between different modes of evaporation when more than one mode is energetically possible, e.g., protons and neutrons. The analysis described here is used to calculate the probability of finding various final products from the evaporation process rather than for calculating spectra of emitted particles. Some comparisons have been made with available Monte Carlo calculations.

II. TECHNIQUES OF CALCULATION FOR ONE EVAPORATION MODE

Before proceeding with the details of the calculation, we define the symbols to be used. a is the nuclear level density parameter; ϵ_i is the kinetic energy of the ith emitted nucleon. E^* is the initial excitation of the first nucleus in the evaporation chain and Q_i is the binding energy of the ith emitted particle. V_0 is the effective Coulombic barrier for charged particle emission. We also define

$$E_1=E^*-Q_1, \tag{1}$$

and

$$E_i=E_{i-1}-Q_i, \tag{2}$$

to complete the listing.

In the form of evaporation theory to be treated here, the probability of emitting a neutron with kinetic energy ϵ, is

$$\int_0^E P(\epsilon)d\epsilon=\int_0^E \epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon. \tag{3}$$

For protons, the situation is analogous, we obtain

$$\int_0^{E'} P(\epsilon)d\epsilon=\int_0^{E'} \epsilon\exp\{2[a(E'-\epsilon)]^{\frac{1}{2}}\}d\epsilon, \tag{4}$$

where

$$E'=E^*-Q-V_0; \tag{5}$$

i.e., the proton may be treated as a neutron if we add V_0 to Q in order to obtain an effective binding energy.

With all of the definitions in hand, we analyze the case in which only one type of particle is emitted. The competition between two- and three-neutron evaporation illustrates the methods involved in the calculations and there are no fundamental difficulties in extending this approach to the evaporation of more particles.

The probability of emitting three neutrons is given as

$$P(3)=\frac{\displaystyle\int_0^{E_3}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]\}\left[\int_0^{E_3-\epsilon_1}\epsilon_2\exp\{2[a(E_2-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2\Big/\int_0^{E_2-\epsilon_1}\epsilon_2\exp\{2[a(E_2-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2\right]d\epsilon_1}{\displaystyle\int_0^{E_1}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1}, \tag{6}$$

when three neutrons at most may be emitted.

* Based on work performed under the auspices of the U. S. Atomic Energy Commission.

[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), pp. 365–374.

[2] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. **116**, 683 (1959). This article is an excellent introduction to the notions of evaporation theory and the Monte Carlo methods applicable to evaporation problems.

We may do the integrations over ϵ_2 in closed form, and obtain

$$\int_0^{E_3-\epsilon_1}\epsilon_2\exp\{2[a(E_2-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2\Big/\int_0^{E_2-\epsilon_1}\epsilon_2\exp\{2[a(E_2-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2$$

$$=1-\exp\{-2[a(E_2-\epsilon_1)]^{\frac{1}{2}}\}\exp\{2[a(E_2-E_3)]^{\frac{1}{2}}\}f(\epsilon_1), \quad (7)$$

where

$$f(\epsilon_1)=\frac{[2E_2-3E_3+2E_3[a(E_2-E_3)]^{\frac{1}{2}}-3[(E_2-E_3)/a]^{\frac{1}{2}}+3/2a]-\epsilon_1\{2[a(E_2-E_3)]^{\frac{1}{2}}-1\}}{[2E_2-3(E_2/a)^{\frac{1}{2}}+3/2a]-\epsilon[2-3/2(aE_2)^{\frac{1}{2}}]}. \quad (8)$$

This expression may be simplified to give

$$f(\epsilon_1)=\frac{1+(E_3-\epsilon_1)\{2[a(E_2-E_3)]^{\frac{1}{2}}-3+3/[2(aE_2)^{\frac{1}{2}}]\}}{1-(\epsilon_1/E_2)\{1+3/[4(aE_2)^{\frac{1}{2}}][2E_2-3(E_2/a)^{\frac{1}{2}}+3/2a]\}}. \quad (9)$$

In obtaining Eq. (9) we assume that the lower integral in Eq. (7) can be neglected when it is evaluated at its upper limit. We also assume that the radical $(E_2-\epsilon_1)^{\frac{1}{2}}$ may be expanded to give

$$(E_2-\epsilon_1)^{\frac{1}{2}}=(E_2)^{\frac{1}{2}}(1-\epsilon_1/2E_2), \quad (10)$$

when this radical does not appear in an exponential. The first approximation introduces no appreciable inaccuracy ($<1\%$) and the second approximation will cause errors of at most 3%, remembering that $\epsilon_1<E_3$ and the term for which we make the expansion is not the largest term in the denominator. We should also note that if we had been considering an evaporation process involving more than three neutrons the effect would be to replace ϵ_1 by $\epsilon_1+\epsilon_2+\epsilon_3$ etc.

Next, we must integrate the upper integral in Eq. (6) over ϵ_1. The difficulties appear in the evaluation of

$$I_0=\int_0^{E_3}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}\exp\{-2[a(E_2-\epsilon_1)]^{\frac{1}{2}}\}\times\exp\{2[a(E_2-E_3)]^{\frac{1}{2}}\}f(\epsilon_1)d\epsilon_1. \quad (11)$$

At this point we make further approximation in order to continue with the problem. We must be very exact in the expansion of the radicals which occur in the exponentials, as these will be our biggest source of error. For the exponential terms, we use

$$(1-x)^{\frac{1}{2}}=1-\tfrac{1}{2}x-\tfrac{1}{8}x^2-\tfrac{1}{8}x^3. \quad (12)$$

This expansion is very accurate for $0<x<0.7$. In the evaluation of I_0, we will be taking the difference of two such expansions so the magnitude of the exponential dependence will be cut down. We obtain

$$\exp\{-2[a(E_2-\epsilon_1)]^{\frac{1}{2}}\}\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}=\exp[2a^{\frac{1}{2}}(\sqrt{E_1}-\sqrt{E_2})]\exp\{[(a/E_2)^{\frac{1}{2}}-(a/E_1)^{\frac{1}{2}}]\epsilon_1\}\times\exp\{[(a/E_2^3)^{\frac{1}{2}}-(a/E_1^3)^{\frac{1}{2}}]\epsilon_1^2/4\}\times\exp\{[(a/E_2^5)^{\frac{1}{2}}-(a/E_1^5)^{\frac{1}{2}}]\epsilon^3/4\}. \quad (13)$$

The exponentials in ϵ_1, ϵ_1^2, and ϵ_1^3 may then be expanded in Taylor sums, as the coefficients of the powers of ϵ in the exponentials are in general small. In most cases three terms will suffice for each of these expansions. Before doing this expansion, it is convenient to expand the denominator of Eq. (9) using the relation

$$1/(1-x)=\exp(x+\tfrac{1}{2}x^2+\tfrac{3}{4}x^3), \quad (14)$$

then combine all of the exponentials and finally expand the exponentials in Taylor sums. The approximation made in Eq. (14) is good for $0<x<0.75$. We also note that the probability for two-neutron emission is given by

$$P(2)=\frac{\int_{E_3}^{E_2}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1}{\int_0^{E_1}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1}+\frac{\int_0^{E_3}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}\left[\int_{E_3-\epsilon_1}^{E_2-\epsilon_1}\epsilon_2\exp\{2[a(E_1-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2\Big/\int_0^{E_2-\epsilon_1}\epsilon_2\exp\{2[a(E_1-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2\right]d\epsilon_1}{\int_0^{E_1}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1}. \quad (15)$$

This calculation involves no new difficulties.

When all of the expansions involved in solving Eq. (6) or Eq. (15) are done, one has 7 to 10 terms of polynomial series in ϵ_1, and the problem may be done quite feasibly by hand. The various expansions may be made more exact by the inclusion of more terms, and the problem will still be far easier to do than the equivalent Monte Carlo calculation.

Finally, we would like to point out a few things about the extension of this method to more than three neutrons being evaporated. We may use the evaporation of four neutrons to point out the applicability of this technique.

$$P(4)=\frac{\displaystyle\int_0^{E_4}\epsilon_1\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}\left[\frac{\displaystyle\int_0^{E_4-\epsilon_1}\epsilon_2\exp\{2[a(E_1-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}\times\left[\int_0^{E_4-\epsilon_1-\epsilon_2}\epsilon_3\exp\{2[a(E_3-\epsilon_1-\epsilon_2-\epsilon_3)]^{\frac{1}{2}}\}d\epsilon_3\Big/\int_0^{E_3-\epsilon_1-\epsilon_2}\epsilon_3\exp\{2[a(E_3-\epsilon_1-\epsilon_2-\epsilon_3)]^{\frac{1}{2}}\}d\epsilon_3\right]d\epsilon_2}{\displaystyle\int_0^{E_2-\epsilon_1}\epsilon_2\exp\{2[a(E_2-\epsilon_1-\epsilon_2)]^{\frac{1}{2}}\}d\epsilon_2}\right]d\epsilon_1}{\displaystyle\int_0^{E_1}\epsilon\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1}. \quad (16)$$

The point to note is that the integration over ϵ_2 will be comparable in difficulty with the integration over ϵ_1 in Eq. (6), but the integration over ϵ_1 will not require too many terms in the expansion because $\exp\{2[a(E_1-\epsilon_1)]^{\frac{1}{2}}\}\exp\{-2[a(E_2-\epsilon_1)]^{\frac{1}{2}}\}$ will have very little ϵ_1 dependence, remembering that $\epsilon_1\leq E_4$. Also the denominator equivalent to Eq. (9) will be easier to expand as ϵ_1/E_2 will be quite small ($\sim$0.4 at most).

III. EXTENSION TO COMPETITION BETWEEN TWO EVAPORATION MODES

When we consider the evaporation process from lighter nuclei, we observe competition between proton and neutron emission. In the formalism of evaporation theory, the effect of this competition is to cause us to make the following change:

$$\frac{\displaystyle\int_0^{E_n}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}{\displaystyle\int_0^{E}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}\rightarrow\frac{\displaystyle\int_0^{E_n}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}{\displaystyle\int_0^{E}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon+\int_0^{E'}\epsilon'\exp\{2[a(E'-\epsilon')]^{\frac{1}{2}}\}d\epsilon'}, \quad (17)$$

where ϵ may be a neutron kinetic energy and ϵ', a proton kinetic energy. A useful technique for dealing with this new complication is to divide numerator and denominator of the right-hand side of (17) by $\int_0^E\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon$. We then obtain:

$$\text{Right-hand side of (17)}=\frac{\displaystyle\int_0^{E_n}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}{\displaystyle\int_0^{E}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}\times\left[\frac{1}{1+\displaystyle\int_0^{E'}\epsilon'\exp\{2[a(E'-\epsilon')]^{\frac{1}{2}}\}d\epsilon'\Big/\int_0^{E}\epsilon\exp\{2[a(E-\epsilon)]^{\frac{1}{2}}\}d\epsilon}\right]. \quad (18)$$

The first term on the right-hand side of Eq. (18) gives us the probability of a nucleon being in some range of interest and the bracketed term gives the probability of that type of nucleon being emitted. Quantities of the type of the first term have been calculated in the consideration of competition between two and three neutron emission in Sec. II, and we see that they may be calculated quite accurately for widely differing ratios of final products, from Table I. For the second term, we must develop a slightly different approach, but from Eq. (18) it should be clear that the calculation of a given product will be accurate, when we can treat the bracketed term accurately. The approach of Sec. II was that the $\sum_i\epsilon_i$ must be less than E_{n-1} by at least 30%, and usually the sum will be only 60% of E_{n-1}, and this justified the expansions. In the

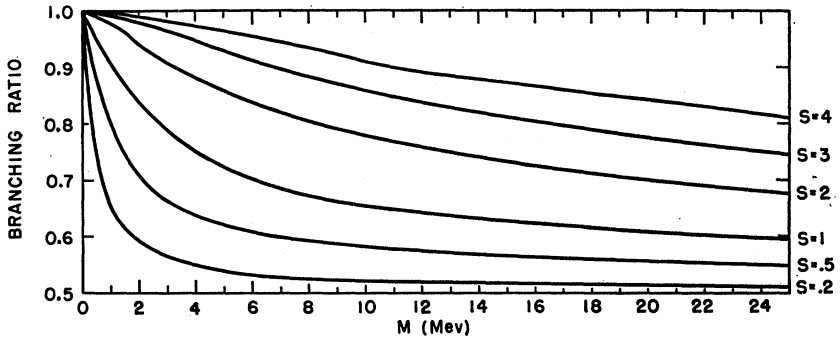

FIG. 1. Branching ratio as a function of energy.

TABLE I. Evaporation of neutrons from U^{239}.

E^* (Mev)	Monte Carlo $P(2n)$	Monte Carlo $P(3n)$	Analytic $P(2n)$	Analytic $P(3n)$
21	0.19±0.01	0.81 ±0.02	0.178	0.822
19	0.56±0.01	0.44 ±0.01	0.577	0.423
18	0.81±0.02	0.19 ±0.01	0.826	0.174
17	0.97±0.02	0.027±0.003	0.980	0.020

bracketed term of Eq. (18) we have ratios of integrals over entire spectra, so the approach of Sec. II will not be valid.

We treat the problem of the branching ratio by developing an easily integrated function, which depends on the difference in energy available for the two branches and the amount of energy available at the time that the branching occurs. The branching ratios are calculated for a particular value of the nuclear level density parameter, but should not be too sensitive to small changes in this parameter. We define

$$S=|Q-Q'| \tag{19}$$

and

$$M_n=(E^*-\sum_{i=1}^{n-1}(Q_i+\epsilon_i)-Q_n), \tag{20}$$

using as the final Q value the larger of the two possibilities, at the branch of interest.

In Fig. 1, we give plots of the branching ratio (B.R.) different values of S, as a function of M. The ratios given here are for an a value of 2.85 $(\text{Mev})^{-1}$. From Fig. 1, we note that the branching ratios are straight lines, with a slight M dependence for $M>10$ Mev; this means that at all stages but the last, the effect of competition will be to put in a term of the form

$$\text{B.R.}=A-B\textstyle\sum_i \epsilon_i, \tag{21}$$

as the Q values are $\simeq$10 Mev when there is competition. At the last stage of the evaporation, we will have to use some other approximation. We have found that the branching ratios are well represented over the entire interval of Fig. 1 by analytic expressions of the form

$$\text{B.R.}=1-\frac{1}{2}\left(\frac{M-\alpha}{(1+\beta)M+\gamma}\right), \tag{22}$$

where B.R. is the branching ratio to the favored mode of evaporation and α, β, and γ depend only on S.

For $a=2.85$ $(\text{Mev})^{-1}$, we have

$$\gamma=1.7S+1.95S^2 \tag{23}$$

$$\alpha=0.4(S-1), \quad S>1 \tag{24}$$

$$\alpha=0, \quad S<1$$

$$\beta=0.08S, \quad S<1 \tag{25}$$

$$\beta=0, \quad S>1.$$

The existence of nonzero values for α in this approximation means that the branching ratio to the unfavored product is negligible ($<0.3\%$) for $M<\alpha$, and is considered to be zero for convenience in making the approximations. For values of S larger than one, the value of γ will be sufficiently large so that the denominator in Eq. (22) can be expanded using the methods of Eq. (14), i.e.,

$$1/(M+\gamma)=1/(E+\gamma-\epsilon)=\frac{1}{(E+\gamma)[1-\epsilon/(E+\gamma)]}, \tag{26}$$

and we identify

$$x=\epsilon/(E+\gamma), \tag{27}$$

for making the expansion of Eq. (14).

This procedure will be necessary at the last stage of the evaporation process; at the earlier stages, the branching ratio will be a linear function of M. We may use this expansion for $E/(E+\gamma)\leq 0.75$. The only difficulty which now remains is when $S\ll 1$ at the last stage of the evaporation process. Interestingly, this is the situation for which one would expect to obtain the best results from the Monte Carlo techniques. Here we use a different expansion,

$$M/(M+\gamma)=A+Be^{-CM}+De^{-FM}, \tag{28}$$

and obtain the coefficients B, C, D, and F by analyzing the function $M/(M+\gamma)$ as a decay curve with two components. A will be the value of the function at the largest M value under consideration. For $S=0$, the problem is quite amenable to treatment. Because the experimental data are often in violent disagreement with calculations, one often redefines the Q value for a particular step, i.e.,

$$Q=Q_0+\delta, \tag{29}$$

in order to obtain better agreement with experiment. This can be readily handled and leads to a new value of S to be used in calculating the branching ratio curve.

The main point of this section is that simple analytic functions (simple from the point of view of integration) for the branching ratios make it possible to treat competition between two modes of evaporation with relative ease.

IV. COMPARISON WITH MONTE CARLO CALCULATIONS

The treatment described in Secs. II and III has been applied to the calculation of neutron evaporation from U^{239} and compared with the Monte Carlo calculations of Miller, Huizenga, and Vandenbosch.[3] Calculations have also been done for proton neutron competition in the evaporation from Ni^{58} and compared with the calculations of Dostrovsky, Fraenkel, and Friedlander.[2]

In the Monte Carlo calculations on U^{239}, the parameters of interest are $a=10.5$, $Q_1=4.76$, $Q_2=6.11$, and $Q_3=5.27$. In Table I, we compare the relative probabilities for emitting two and three neutrons as calculated using both methods.

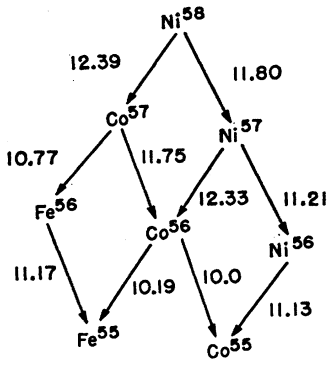

FIG. 2. Evaporation of nucleons from Ni^{58}.

The agreement between the two sets of results is quite good. The results of the analytic treatment may be improved by taking a few more terms in the various expansions.

The comparison with the Ni^{58} data of Dostrovsky, Fraenkel, and Friedlander[2] is used as a test of the methods for comparing competition between two modes. The data which were used in the calculation are given in Fig. 2. The numbers beside each arrow are the Q values; the Q values for proton emission contain an effective Coulombic barrier.

For comparison with the Monte Carlo values, we use $a=2.85$. Also neutron spectra are multiplied by a factor of 1.34 and proton spectra by 1.23. These factors lead to slight changes in the branching ratio formula.

[3] W. Miller, J. R. Huizenga, and R. Vandenbosch (private communication).

The effect is that

$$1-\frac{1}{2}\frac{(M-\alpha)}{[(1+\beta)M+\gamma]}\rightarrow 1$$

$$-\frac{(1+\Delta)}{2(1+\beta+\frac{1}{2}\Delta)}\frac{[M-\alpha]}{[M+\gamma/(1+\beta+\frac{1}{2}\Delta)]}. \quad (30)$$

When neutron emission is favored, $\Delta=-0.082$; for the case of proton emission being favored, $\Delta=+0.089$. The branching ratio functions may then be easily modified for comparison with Dostrovsky, Fraenkel, and Friedlander. The comparison is given in Table II.

TABLE II. Evaporation from Ni^{58}.

E^* (Mev)	Ratio calculated	D.F.F.[a]	Analytic
25	Co^{57}/Ni^{57}	0.79	0.75
35	Co^{56}/Ni^{56}	0.51	0.52
40	Fe^{55}/Co^{55}	0.61	0.62

[a] See Reference 2.

The analytic results were obtained after discovering many numerical errors.

The ratio of Fe^{55}/Co^{55} was calculated by making several approximations in order to simplify the calculation. The major difficulty is in calculating the branching from Co^{56} to Fe^{55} and Co^{55}. This calculation was done exactly for a case which was an average of the two ways to get to Co^{56} from Ni^{58}. The branching ratios for all steps before the last were made energy independent by assuming 3-Mev kinetic energy carried off per nucleon. As these branching ratios have little energy dependence, this assumption will introduce little error. Finally, the assumption was made that the integrals over particle kinetic energies would be about the same, i.e., if we can remove the branching ratios, the rest of the integrals will be a common factor. The justification for this step is that we are only considering particles emitted with a small fraction of the available kinetic energy. This final assumption will be useable only when the products being compared cannot evaporate another nucleon and when a is only a function of the nuclear mass. This procedure allows one to make quick, fairly accurate estimates of ratios, when the $\Sigma_i Q_i$ are about the same. There is no real need for this final approximation.

For the purpose of clarifying the techniques used here, it is instructive to set up a sample calculation. We shall consider the probability of forming Ni^{56} as the evaporation product of the compound nucleus Ni^{58}, having 35-Mev excitation energy.

From a consideration of Fig. 2, we see that

$$P(\mathrm{Ni}^{56})=\mathrm{B.R.}(\mathrm{Ni}^{58}\rightarrow\mathrm{Ni}^{57})\int_0^{10.9}\epsilon_1\exp\{2[a(23.2-\epsilon_1)]^{\frac{1}{2}}\}\mathrm{B.R.}(\mathrm{Ni}^{57}\rightarrow\mathrm{Ni}^{56})d\epsilon_1\Big/\int_0^{23.2}\epsilon_1\exp\{2[a(23.2-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1$$

$$+\mathrm{B.R.}(\mathrm{Ni}^{58}\rightarrow\mathrm{Ni}^{57})\int_{10.9}^{12}\epsilon_1\exp\{2[a(23.2-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1\Big/\int_0^{23.2}\epsilon_1\exp\{2[a(23.2-\epsilon_1)]^{\frac{1}{2}}\}d\epsilon_1. \quad (31)$$

In order to make the calculation, we need the values of the two B.R. functions.

From Eq. (30), we have

$$\mathrm{B.R.}(\mathrm{Ni}^{58}\rightarrow\mathrm{Ni}^{57})$$

$$=1-\frac{1}{2}\frac{(1-0.082)}{(1+\beta-0.041)}\frac{(M-\alpha)}{[M+\gamma/(1+\beta-0.041)]}. \quad (32)$$

Next we use Eq. (19) and Eq. (20) to obtain

$$M=35-12.39=22.61, \quad (33)$$

$$S=0.59. \quad (34)$$

We then use Eqs. (23), (24), and (25) to obtain α, β, and γ:

$$\alpha=0, \quad \beta=0.048, \quad \gamma=1.68. \quad (35)$$

We then obtain

$$\mathrm{B.R.}(\mathrm{Ni}^{58}\rightarrow\mathrm{Ni}^{57})=0.574. \quad (36)$$

To calculate $\mathrm{B.R.}(\mathrm{Ni}^{57}\rightarrow\mathrm{Ni}^{56})$, we follow the same procedure and have

$$M=10.9-\epsilon_1, \quad S=1.12. \quad (37)$$

In this case M depends on ϵ_1, because ϵ_1 determines the excitation of Ni^{57}.

Substituting the values of α, β, and γ, we obtain from Eq. (30)

$$\mathrm{B.R.}(\mathrm{Ni}^{57}\rightarrow\mathrm{Ni}^{56})=1-\frac{0.478(10.9-\epsilon_1)}{(15.41-\epsilon_1)}. \quad (38)$$

Since $\epsilon_1\,\mathrm{max}/15.4<0.75$ we may use the expansion of Eq. (14) for the denominator of Eq. (38). Using Eq. (12) to expand $\exp\{2[a(23.2-\epsilon_1)]^{\frac{1}{2}}\}$, we may then proceed to evaluate the integrals in Eq. (31).

V. CONCLUSION

With the methods described here, it becomes possible to treat problems in evaporation theory analytically. There are special advantages in using these techniques for calculating the probability of forming unlikely products, as Monte Carlo methods must treat many events in order to obtain good statistics. The analytic techniques described here make it possible to treat the parameters of evaporation theory with variations over wide ranges of values.

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