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Abrupt-Kink Model of Dislocation Motion

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A new model of dislocation motion is presented. The behavior of a dislocation in the presence of an applied stress is described in terms of a redistribution of kinks along its length. In contrast with previous models, in which a kink is envisaged as a smooth step extending over many lattice constants, we suppose a kink to be abrupt. Consequently, kink diffusion is considered to be a thermally activated process. Transport equations are formulated which include the effects of generation, diffusion, and collision of kinks. General results obtained from these equations show that a dislocation does not behave like an extensible string in this model. Particular application to small harmonically-time-dependent stresses leads naturally to a new theory of the Bordoni anelastic peak. The characteristic relaxation time depends on line length as well as the attempt frequency and activation energy for diffusion. As a result the decrease in the peak height and slight lowering of the peak temperature upon alloying or neutron irradiation are explained. Assuming an exponential distribution of line lengths, the results of the theory are used to evaluate the merit of different published values of the activation energy. Calculated attenuation peaks for different frequencies are shown to account for the experimentally observed large half-widths in pure cold-worked metals. The absence of a peak in well-annealed metals is explained if dislocations are then arranged parallel to the close-packed directions, thereby eliminating the kink density. The process by which cold-working annealed materials can give rise to kinks is discussed. Experiments are suggested which might further test the theory.

I. INTRODUCTION

In this paper a new theory of dislocation motion is developed. The response of a dislocation to an applied stress is expressed in terms of a redistribution of kinks along its length. We consider here only dislocation loops which are firmly pinned. This first application of the model is shown to yield a new theory of the Bordoni anelastic peak, and provides a natural explanation of the shape of the peak and its behavior, both in magnitude and temperature, upon alloying, neutron irradiation, cold work, and annealing.

Although, in a general sense, some of the ideas contained within the model are not original with us, in detail we differ substantially from previous authors upon certain basic points. Accordingly in Sec. II we shall give an outline of the concepts which are later developed, expressed particularly from our point of view. In Sec. III we then formulate the mathematical problem and consider some of the general consequences of our treatment. Section IV is devoted to a study of internal friction and to a detailed discussion of the experimental properties of the Bordoni peak in terms of the theory we derive. Finally Sec. V contains a brief recapitulation of this research.

II. DESCRIPTION OF THE MODEL

The model is based upon the hypothesis that an isolated dislocation in a given slip plane would preferentially be oriented along a single close-packed crystal direction. In real crystals, containing many dislocations, few can be expected to obtain this most favorable configuration. Because of their mutual interactions within the network and with other lattice imperfections, almost of necessity there will be many lying at some average inclination relative to a particular close-packed direction. We assume that at 0°K a possible steady state of a dislocation of this type is one in which, microscopically, equal segments are arranged parallel to the close-packed direction, each displaced laterally from its preceding neighbor by the same unit lattice vector, so that the average line direction is preserved.[1] This arrangement is illustrated schematically in Fig. 1. It is of course only one of many possible decompositions based upon our original premise and indeed we believe that others can also be important (see Sec. IV-3).

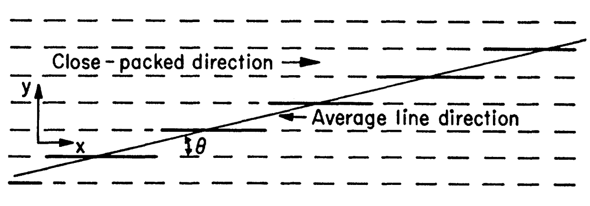

Fig. 1. A schematic illustration of the composition of a dislocation containing kinks. The dislocation segments are represented by the heavy lines.

[1] W. Shockley (private communication), quoted by W. T. Read, Jr., *Dislocations in Crystals* (McGraw-Hill Book Company, Inc., New York, 1953), p. 46.

However, for the present we shall ignore the alternatives and pursue the consequences of the example we have cited.

The transition region between two adjacent segments is known as a kink.[1] In contrast with previous models, in which kinks are considered to extend over many lattice distances, we shall suppose, as indicated in Fig. 1, that they are abrupt and well localized. It will be necessary to distinguish between two types of kinks, namely "left" and "right." These may be defined operationally as follows: looking along a segment of the line, if the adjacent segment is displaced one lattice vector to the left (right), the two are connected by a left (right) kink. In the dislocation drawn in Fig. 1, five left kinks are shown; a dislocation drawn at $(-\theta)$ would contain five right kinks in the same interval.

A kink produces additional atomic displacements in its environment. It constitutes a source of extra energy of the dislocation. We assume that kinks are independent and that the total energy of the dislocation is determined by the sum of the lengths of the different segments and the number of kinks.

At any finite temperature, kinks may be generated by thermal activation.[2] If the dislocation is constrained by pinning points, the generation process must involve the simultaneous production of a right and left kink. Assuming that they are mobile, any nonuniform distribution of kinks along the dislocation will produce kink diffusion[3] which, since our kinks are abrupt, is also assumed to be thermally activated. As a result of this motion, collisions between right and left kinks will occur, causing their mutual annihilation. Thermal equilibrium is attained when the diffusion currents are zero and the generation and recombination rates of right and left kinks are equal at each point in the dislocation.

The application of a stress is an additional force for kink motion.[3] Right and left kinks are driven towards opposite ends of the dislocation. The pinning points are assumed to be rigid and therefore act as perfect reflectors. Consequently, if the stress is time independent, there results a pile-up of kinks of predominantly one type at different ends of the dislocation. On a macroscopic scale, this process corresponds to the familiar bowing-out of a dislocation in the presence of an applied stress. Again, in the steady state, the kink currents are zero and the generation and recombination rates are everywhere equal. For since we regard kinks as regions of high energy density, this conclusion is assured by the second law of thermodynamics.

III. TRANSPORT EQUATIONS FOR KINKS

The model can be formulated by methods already familiar from kinetic theory, for example. Coordinate axes are chosen as shown in Fig. 1. We divide the projected length of the dislocation on the x axis into elementary regions of length δx such that δx is small compared with L but larger than the average spacing between kinks. On this scale, we may define the number of left and right kinks in the element of length δx at x as $n(x)\delta x$ and $p(x)\delta x$, respectively, and consider the densities $n(x)$ and $p(x)$ to be continuous functions of a continuous variable. The kink densities are related to other coarse-grained variables which describe the configuration and energy of the dislocation. The position of the dislocation, $y(x)$, is

$$y(x)=a\int^{x}(n-p)dx, \tag{1}$$

and the energy density, $E(x)$, is

$$E(x)=E_0+(n+p)\epsilon_k. \tag{2}$$

Here a is the spacing, in the y direction, between lattice planes; ϵ_k is the energy of a kink, and E_0 is the energy per unit length of a dislocation lying parallel to the x axis.

Equations governing the kink distributions may be derived from the principle of continuity. Consider the right kink density, $p(x)$, in a region of length δx at x. In general, $p(x)$ will vary with time as a result of kink motion, thermal generation, and recombination with left kinks. The generation rate per unit length along the x direction is taken to be independent of x and of the total kink density and is denoted by a temperature-dependent parameter, g. Recombination occurs only if kinks of both species are present. We assume the rate of recombination is simply proportional to the product of the local kink densities. Then the continuity equation for right kinks becomes

$$\frac{\partial p}{\partial t}+\frac{\partial I_p}{\partial x}-g+rnp=0, \tag{3}$$

where I_p is the right kink current and r is a constant. Similarly, for left kinks, we obtain

$$\frac{\partial n}{\partial t}+\frac{\partial I_n}{\partial x}-g+rnp=0. \tag{4}$$

It will be noted that, at any "point" in the dislocation, the generation and recombination rates of right and left kinks are set equal. This must be the case since the processes involve either the simultaneous appearance or disappearance of one kink of each type.

The kink currents, I_p and I_n, consist of a "convection" current arising from the applied stress and a diffusion current. For right and left kinks, each characterized by the same mobility μ and diffusion coefficient D, they are, respectively,

$$I_p=F\mu p-D\partial p/\partial x, \tag{5}$$

[2] A. Seeger, Phil. Mag. **1**, 651 (1956).
[3] J. Lothe and J. P. Hirth, Phys. Rev. **115**, 543 (1959).

and

$$I_n = -F\mu n - D\partial n/\partial x. \qquad (6)$$

F denotes the effective force exerted by the stress on a right kink. We assume that it may be obtained from the increment in strain-energy per unit lateral displacement of a kink. Thus we set

$$F = \sigma ab, \qquad (7)$$

where σ is the component of shear stress along the direction of the Burgers vector of magnitude b. As stated in II, we consider D to be of the form

$$D = D_0 \exp(-W/kT), \qquad (8)$$

and in addition we shall suppose μ and D satisfy the Einstein relation

$$\mu = D/kT. \qquad (9)$$

Finally, we note that the coefficient of recombination r, which appears in (3) and (4), is not an independent parameter. In the absence of any stress, the kink currents must vanish. Hence, from (3) or (4),

$$r = g/n_0 p_0, \qquad (10)$$

where n_0 and p_0 are the kink densities when $\sigma = 0$. By elementary statistics one finds that at low temperatures ($kT < \epsilon_k$) their product is given by

$$n_0 p_0 \approx b^{-2}\{1-(b/a)|\tan\theta|\}\cdot\exp(-2\epsilon_k/kT). \qquad (11)$$

The relatively larger probability of collisions in dislocations with $\theta \neq 0$ which contain kinks at 0°K is therefore automatically included in (10).

We shall consider, exclusively, the motion of a dislocation which is firmly fixed by pinning points at $x=0$ and $x=L$. The problem is then completely defined if Eqs. (3) and (4) are supplemented by the following boundary conditions:

$$\left.\begin{array}{l} I_n = 0 \\ I_p = 0 \end{array}\right\} \quad \text{at} \quad x=0 \quad \text{and} \quad x=L, \qquad (12)$$

which insure that the pinning points are immobile, and

$$\int_0^L (n-p)dx = (L/a)\tan\theta, \qquad (13)$$

which defines their relative orientation in the slip plane.

Two general results can be established without too much analysis. For example, subtracting (3) from (4), one obtains

$$\partial(n-p)/\partial t = \partial(I_p - I_n)/\partial x. \qquad (14)$$

Upon integrating (14) from 0 to x and multiplying by σb, with the aid of (1), one finds

$$\sigma b \partial y/\partial t = \sigma ab(I_p - I_n). \qquad (15)$$

The interpretation of (15) is simply that the power supplied by the external source is dissipated in driving the kink currents.

Furthermore, integrating the sum of (3) and (4), we find, with the use of (12),

$$\frac{\partial}{\partial t}\int_0^L (n+p)dx - \int_0^L (g - rnp)dx = 0. \qquad (16)$$

When departures from the steady state are small, (16) may be put in a more transparent form by means of (2) and (10), namely,

$$\frac{\partial}{\partial t}\Delta E + \frac{r(n_0+p_0)}{2}\Delta E = \frac{\epsilon_k r}{4a^2}\int_0^L \left(\frac{\partial \Delta y}{\partial x}\right)^2 dx. \qquad (17)$$

where ΔE is the increment in the total energy of the dislocation associated with the (variable) displacement Δy. Thus, for a given displacement, the energy relaxes in a time τ_E given by

$$\tau_E = 2/r(n_0+p_0), \qquad (18)$$

to a steady-state value where

$$\Delta E = [\epsilon_k/(n_0+p_0)a^2]\Delta l, \qquad (19)$$

Δl being the increase in the macroscopic line length.

A proportionality between changes in energy and line length has been derived previously from continuum elasticity theory.[4] It has been interpreted as illustrating that a dislocation is the analog of an extensible string. However, as we shall now demonstrate, our model provides a counter-example showing that such a conclusion cannot be general. For consider the dislocation displacement produced by a static stress. It may be verified by direct substitution, with the aid of (5) and (6), that the following are time-independent solutions of (3) and (4):

$$p = p_\lambda \exp(\lambda x), \quad n = n_\lambda \exp(-\lambda x), \qquad (20)$$

where

$$\lambda = \sigma ab\mu/D, \qquad (21)$$

and n_λ and p_λ are functions of the stress which are related by

$$n_\lambda p_\lambda = n_0 p_0. \qquad (22)$$

Both of the densities, (20), correspond to zero current flow and, for reasons stated in Sec. II, are the only steady-state solutions of physical interest. The boundary conditions, (12), are automatically satisfied. Imposing (13), together with Eq. (22), we derive

$$n_\lambda = \{(\lambda L \tan\theta)/a \pm [(\lambda L\tan\theta)^2/a^2 + n_0 p_0 \sinh^2(\lambda L/2)]^{\frac{1}{2}}\}/2[1-\exp(-\lambda L)]. \qquad (23)$$

The choice of sign in (23) is governed by the sign of $(\lambda L \tan\theta)$, being + or − according to $(\lambda L\tan\theta) > 0$ or < 0. In both cases p_λ may be obtained from (22) and (23).

[4] N. F. Mott and F. R. N. Nabarro, *Report on Strength of Solids* (The Physical Society, London, 1948), p. 1.

According to our model then, one finds from (1), (20) and (23) that if $\theta=0$, the displacement of the dislocation is described by the catenary

$$y=(2n_0a/\lambda)[\cosh(\tfrac{1}{2}\lambda L)-\cosh\lambda(\tfrac{1}{2}L-x)], \quad (24)$$

which in the limit of small stress reduces to

$$y=n_0a\lambda x(L-x). \quad (25)$$

In contrast with (25), if a string analog existed, the dislocation configuration would be equivalent to that of a string of line tension, S, defined by

$$\Delta E=S\Delta l, \quad (26)$$

subject to a force σb, per unit length, normal to the line direction. Then the displacement of the line would be, for small stresses,

$$y_S=\sigma bx(L-x)/2S. \quad (27)$$

For (19) and (25) to be compatible with (26) and (27) would require setting (D/μ) equal to ϵ_k. Consequently, an analogy with an extensible string is alien to the concepts contained in the present model.

IV. INTERNAL FRICTION

The dissipation of power in a sample in some mode of mechanical vibration is known as internal friction. Experimental studies of this phenomena have evidenced several interesting effects which have been attributed to the presence of dislocations. An investigation of the anelastic behavior to be predicted upon the basis of a dislocation model is therefore of direct practical interest. This section is devoted to such a study in the light of our abrupt-kink model. We are thus led to a new theory of the Bordoni attenuation peak observed in lightly cold-worked metals. A detailed comparison with experiment is made and the theory shown to account for several hitherto unexplained properties of the peak.

Dislocations influence the vibrational characteristics of a solid by virtue of the inelastic strains which result from their motion in the presence of the internal stress. Hence the preliminary burden of this section is the calculation of the contribution made by a single dislocation to the total strain. The internal friction is then calculated by standard methods. We consider only effects which occur at extremely low stress levels and thus are concerned entirely with amplitude-independent internal friction phenomena. Also, for the sake of simplicity, the sample is supposed to be in a state of homogeneous strain. Although this is hardly ever the case in practice, it has been shown elsewhere that results such as we derive are generally applicable to conditions of inhomogeneous strain.[5]

If a dislocation is displaced from its steady-state position, the average shear strain, ϵ_d, which is produced in a sample of volume V is

$$\epsilon_d=(b/V)\int_0^L(y-y_0)dx. \quad (28)$$

In general, the position of the dislocation, and hence the strain, as a function of time is to be found by solution of (3) and (4) for the individual kink densities. However, when, as in the present case, only a first order approximation is required, the same result may be derived more directly. For if as independent variables we use the total kink density and the dislocation displacement itself, we find after integrating (14) and substituting for the currents that

$$\frac{\partial y}{\partial t}-D\frac{\partial^2 y}{\partial x^2}=\sigma ba^2\mu(n+p). \quad (29)$$

To first order in the stress, σ, given by

$$\sigma=\sigma_0\exp(i\omega t), \quad (30)$$

then, the solution we require is

$$y=y_0+y_1\exp(i\omega t)\sum_{n=0}^{\infty}\frac{\sin[(2n+1)\pi x/L]}{(2n+1)[i\omega\tau_L+(2n+1)^2]}, \quad (31)$$

where

$$y_1=4\sigma_0ba^2L^2(n_0+p_0)/kT\pi^3, \quad (32)$$

and

$$\tau_L=L^2/D\pi^2. \quad (33)$$

Substitution of (31) in (28) yields ϵ_d. It develops that terms in the sum in (31) give contributions decreasing like $(2n+1)^{-2}$ with increasing n and so for all practical purposes we need consider only the term $n=0$. Therefore, the contribution of each dislocation to the total strain is found to be

$$\epsilon_d=\left(\frac{2by_1L}{\pi V}\right)\frac{\exp(i\omega t)}{(i\omega\tau_L+1)}. \quad (34)$$

The amplitude of ϵ_d being complex, stress and strain are out of phase and there results a net energy dissipation during one period of the stress. The fundamental measure of this effect, Δ, is defined as the ratio of energy lost per radian to the maximum stored energy. It is related to experimentally determined quantities such as the Q factor of a sample in forced vibration or the logarithmic decrement, δ, of free vibrations as follows: $\Delta=Q^{-1}=\delta/\pi$.[5] Thus the internal friction arising from one dislocation in a sample subject to a homogeneous strain is

$$\Delta_d=(G/2\pi\sigma_0^2)\,\mathrm{Re}\int_0^{2\pi/\omega}\sigma^*\cdot\dot{\epsilon}_d dt, \quad (35)$$

which upon substitution from (32) and (34) yields

$$\Delta_d=\Delta(L,T)\omega\tau_L/(1+\omega^2\tau_L^2), \quad (36)$$

[5] A. S. Nowick, Progr. Metal Phys. 4, 1 (1953).

where

$$\Delta(L,T)=8Ga^2b^2L^3(n_0+p_0)/VkT\pi^4, \qquad (37)$$

G being the shear modulus.

Only the imaginary or out-of-phase component of ϵ_d contributes to Δ_d. The component in phase with the stress constitutes a strain in addition to the purely elastic strain and thereby gives rise to an apparent lowering of the modulus. This has been called the "ΔM effect."[5] It is usually measured in terms of the fractional decrease in modulus—the modulus defect—ΔM. Thus the dislocation contribution, ΔM_d, to the modulus defect is by definition

$$\Delta M_d=G\,\mathrm{Re}(\epsilon_d/\sigma), \qquad (38)$$

and in particular, from (34), is found to be

$$\Delta M_d=\Delta(L,T)/(1+\omega^2\tau_L^2). \qquad (39)$$

The total internal friction and modulus defect of a sample containing a network of dislocations is found, in the present approximation, by summing the contributions appropriate to each mobile length. By virtue of (33) and (37) the net results evidently will depend upon the distributions, both in length and in orientation relative to the close-packed directions, of dislocations in the network. However we anticipate that in general their behavior will be qualitatively similar to that of the individual contributions, (36) and (39), to which we restrict the discussion for the present. The latter become particularly simple if provisionally we exclude the possibility of thermal generation of kinks. For then, the total kink density in (37) being constant, Δ_d and ΔM_d describe a simple relaxation process in which, with increasing temperature, Δ_d passes through a maximum at a temperature T_0 given by

$$\exp(-W/kT_0)\simeq\omega L^2/D_0\pi^2, \qquad (40)$$

and ΔM_d exhibits a step-wise increase. Such behavior is characteristic of the Bordoni attenuation peak and accompanying modulus change, commonly attributed to the presence of dislocations, which is observed in lightly cold-worked metals.[6] This suggests, and it is later confirmed by other experimental evidence, that we identify the above relaxation mechanism with the Bordoni peak. We conclude then that the thermal generation of kinks is indeed negligible at temperatures near T_0. Thus we find that dislocations lying exactly along close-packed directions play a negligible role in determining the internal friction in the cold-worked state. In contrast with Seeger's theory,[2] our model requires that the dislocations which make the major contribution to the Bordoni peak are oriented in some random fashion relative to the close-packed directions, thereby containing a sequence of "built-in" kinks at 0°K.

[6] D. H. Niblett and J. Wilks, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 1. This is a review article containing a general survey of all the properties of the Bordoni peak to which we refer later in the text.

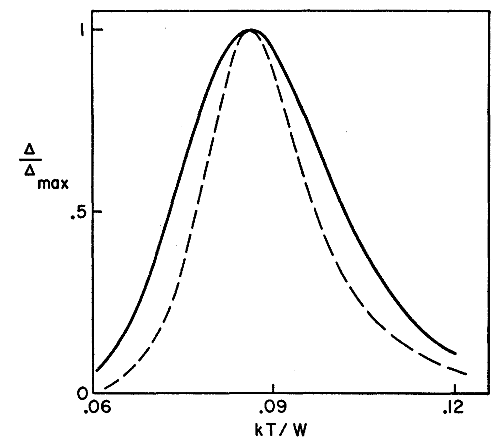

FIG. 2. The internal friction as a function of temperature. The full curve was calculated from the theory, for a value of the frequency given by $(\omega l_0^2/D_0\pi^2)=10^{-6}$. The broken curve corresponds to a single relaxation process with the same peak temperature.

In view of the preceding discussion, at the temperatures of current interest, either n_0 or p_0 is negligible in a given dislocation. Then

$$n_0+p_0\approx a^{-1}|\tan\theta| \qquad (41)$$

in (37). No other parameters in Δ_d depend critically upon angle within the largest possible range of θ ($|\theta|<\pi/6$, for fcc). Consequently, beyond the stipulation of randomness, the total internal friction is insensitive to the detailed distribution in orientation. Therefore, for a network of N dislocations, the total internal friction is essentially

$$\Delta=N\int\bar{\Delta}(l,T)\frac{\omega\tau_l}{(1+\omega^2\tau_l^2)}P(l)dl, \qquad (42)$$

where $\bar{\Delta}$ is obtained from (37) by substituting an average kink density (assumed $\sim1/10a$) and l is the macroscopic line length, to which we ascribe the probability distribution, $P(l)$.

It is evident from (42) that Δ is the synthesis of a weighted sequence of relaxation peaks. Therefore the width in temperatures of the internal friction must be greater than that of a single relaxation process. Naturally any estimate of this increase in width depends upon the form of $P(l)$, which in reality is not known. Thus we resort to the most plausible choice, namely

$$P(l)dl=\exp(-l/l_0)dl/l_0, \qquad (43)$$

corresponding to a random distribution with mean length, l_0.

With $P(l)$ given by (43) we have computed Δ as a function of temperature over several decades of frequency. One typical result which illustrates the broadening is plotted in Fig. 2 where for comparison we show Δ and also a single relaxation peak. The respective widths are in the ratio 1.5:1. Also, as a consequence of the broadening, the change in modulus defect near the peak is enhanced relative to the maximum value of Δ. Thus, whereas in a single relaxation process the ratio of modulus change to maximum internal friction is 2, the same ratio, with ΔM found by the same arguments as those leading to (42), yielded the value 2.7. Furthermore, we found that at all frequencies the maximum value of Δ was given by

$$\Delta_{\max}=2.1N\bar{\Delta}(l_0,T_0) \quad (44)$$

and occurred at a temperature such that

$$\omega\tau_e=1, \quad (45)$$

where τ_e defines an effective length, l_e, given by $l_e^2=10l_0^2$. Again, (44) and (45) are a consequence of our choice of $P(l)$. With the wisdom of hindsight they are easy to understand. Since $\bar{\Delta}(l,T)\propto l^3$, the most effective lengths are those for which $l^3P(l)$ is a maximum, namely $l_e=3l_0$. Hence the internal friction has its maximum near $9\omega\tau_{l0}=1$ and there has the value $\sim(N/2)\bar{\Delta}(l_0,T_0)\times$[average of $(l/l_0)^3$], or $3N\bar{\Delta}(l_0,T_0)$.

The above discussion completes the mathematical analysis. Henceforth we shall be concerned with the comparison of theory with experiment. Since the majority of work in the latter field has been devoted to studies of the Bordoni peak exhibited by pure Cu or dilute alloys with Cu as solvent, we shall be dealing mainly with this metal. Many different properties of the peak have been established. We shall discuss each separately together with its interpretation in terms of our model.

1. Effect of Frequency

The Bordoni peak has been observed in cold-worked Cu at various frequencies between 380 cps and 10 Mc/sec. It has generally been assumed that the frequency, f, and temperature of the peak satisfy a relation of the type

$$f=f_0\exp(-W/kT_0), \quad (46)$$

where f_0 is some intrinsic parameter, but attempts to fit the data obtained from all specimens, irrespective of the amount or nature of the pre-strain, on the basis of (46) have not met with great success. While (46) does indeed correlate the data in a rough manner, individual deviations far exceed the quoted experimental errors and there appear to be no universally accepted values of W and f_0. For example, in order to cover a wide frequency range with specimens of the same purity Bordoni *et al.*[7] have investigated the peak in a set of plates machined from the same bulk sample, and quote values of $W=0.12$ ev for $f_0=3.8\times10^{11}$. On the other hand, Niblett and Wilks[6] have suggested that, since the peak temperature has been found to be affected by the amount and nature of the pre-strain, a better way to determine these parameters is to consider only data from specimens deformed in the same manner by comparable amounts and thereby they obtain the values $W=0.08$ ev and $f_0\simeq10^8$.

The discrepancies find a natural explanation in our theory. For, identifying (46) with (45), one finds

$$f_0=(\pi D_0/2l_e^2). \quad (47)$$

Thus, in contrast with what has previously been supposed, f_0, by virtue of its dependence upon the line length, is evidently a structure-sensitive parameter. In every instance, its value will depend upon the previous history of a specimen. It is hardly surprising then that the experimental data as a whole, obtained from samples of differing purity, prepared by various modes of deformation, do not correspond to (46) with a unique value of f_0. One might hope, however, that the variations in effective line length, and hence f_0, are not too extreme in specimens which have been similarly prepared. Accordingly the procedure adopted by Niblett and Wilks appears to be the best method of estimating W from the data presently available. In fact an analysis based upon their criteria shows that the data of Bordoni *et al.* are also not incompatible with an activation energy of about 0.08 ev. The extreme value of $f_0\approx3\times10^9$ so found differs from that quoted by Niblett and Wilks but this is no longer surprising. It simply implies that in the specimens considered by the latter authors the mean line length was roughly five times larger than in the machined samples of relatively low purity which were used by Bordoni and his co-workers.

We have demonstrated that variations in f_0 are to be anticipated. It remains to be shown, however, that the magnitude of f_0 can be accounted for with reasonable values of the fundamental parameters. To this end we must first estimate the pre-exponential term, D_0, of the diffusion coefficient. As usual, this will depend upon the jump length, ξ, and the attempt frequency ν in the following way:

$$D_0\propto\nu\xi^2, \quad (48)$$

where in our case ξ is a lattice spacing and ν might typically be of the order of the Debye frequency. The only question is whether the entropy term, i.e., the factor of proportionality relevant to (48), is substantially different from unity. We believe this not to be the case for two reasons. First because the changes in atomic configuration accompanying the lateral displacement of a kink are small, so that variations in the vibrational frequencies of the lattice should not be too great and secondly, moreover, because T_0 is so much smaller than the Debye frequency that any changes in the lattice modes that do occur will not be fully evinced at temperatures of interest to us. We consider then that a reasonable estimate is $D_0\approx10^{13}b^2$. Consequently l_e can now be determined from (47) and the only other unknown

[7] P. G. Bordoni, M. Nuovo, and L. Verdini, Nuovo cimento **14**, 273 (1959).

parameter, N/V, obtained from a comparison of (37) and (44) with the observed magnitude of the peak. Thus we find from the data considered by Niblett and Wilks an effective line length, $l_e \approx 4\times10^2 b$ and $N/V \approx 6 \times 10^{12}$ cm^{-3}, corresponding to a planar dislocation density $\Lambda(=Nl_0/V) \approx 2\times10^7$ cm^{-2}.

2. Effects of Impurities

Caswell[8] has made a systematic study of the effects of impurities upon the Bordoni peak. He found that after addition of small amounts of gold or nickel to copper the peak was appreciably reduced in height and also moved to slightly lower temperature.

These properties are but further ramifications of the dependence of f_0 and Δ_{max} upon l_0. It is generally believed that impurities have the effect of decreasing the line length. By (37) then, a reduction of Δ_{max} with increasing impurity is to be expected, and the accompanying decrease in T_0 follows from (46) and (47). In particular, combining these equations we find that

$$(\Delta_{max}/\Lambda)\exp(W/kT_0) = \text{const.} \qquad (49)$$

(We neglect the slight dependence of $\bar{\Delta}$ upon T_0.) Although (49) is strictly valid only for measurements in the same specimen, it should form a fair basis for comparison of internal friction in different samples provided all are deformed in the same manner, as was the case in Caswell's experiments. Assuming then that all his specimens had the same value of Λ, the temperature shifts can be calculated from a knowledge of the relative peak heights. Thus for two specimens containing 0.065 and 0.25 at. % Au, respectively, we calculate from (49), with $W=0.08$ ev, and Caswell's data, temperature shifts of 4°K and 10°K below the peak position in pure Cu, which should be compared with the experimental values of 2°K and 6°K. In view of the uncertainties involved, this agreement to within a factor of two is probably all that can be expected. For example, since essentially perfect agreement is obtained if we assume instead that the number of dislocation loops is the same in all specimens, further refinements are inappropriate until more is known about the variations in distribution of dislocations with alloying.

Under this same category we might mention the effects of neutron irradiation. Unfortunately as far as we are aware only two samples showing well developed peaks have been investigated before and after irradiation.[9,10] Both exhibited a decrease in peak height but in one instance[10] it was apparently too small to produce a noticeable change in T_0. In the other specimen a reduction of the peak by roughly a factor of six was accompanied by a lowering of T_0 by about 4°K.[9] This again is somewhat lower than we calculate with $W=0.08$ ev. Clearly, further experimental work in this field would be of great interest since potentially it offers the best means of testing (49) without reservations as to changes in Λ.

3. Effects of Annealing and Cold Work

The Bordoni peak is absent in well-annealed specimens. It appears upon cold-working and, with increasing deformation, initially increases in height and moves to slightly higher temperatures. These effects do not continue indefinitely but appear to saturate after $\sim$2% pre-strain. Further cold work does not produce any significant change. Conversely, annealing plastically-deformed specimens usually reduces the height of the peak (although there is a range of annealing temperatures in which an increase is observed at first) and moves it to slightly lower temperatures.[6]

We have already pointed out that the peak arises from dislocations which have built-in kinks. Therefore its *absence* in well-annealed materials implies that dislocations are then oriented along close-packed directions, which essentially eliminates the kink density. One could then envisage the experimentally observed behavior being a manifestation of the following process (see also Fig. 3). In the annealed state all the kinks are condensed to form one or more large steps, presumably thereby attaining the lowest self-energy configuration. Under plastic deformation, kinks are broken off these steps, which act as barriers to kink motion. The steps continue to disintegrate with increasing cold work until the final metastable state is attained when kinks are distributed uniformly along the entire length. In this manner one can synthesize the increase in peak height, its shift in temperature, and the subsequent saturation.

The annealing properties of the peak we attribute to the reverse of the above process. The observation of an initial increase at certain temperatures suggests that there may be preliminary changes in the dislocation network as a whole which activate more loops.[6] However, we emphasize that the lowering of the peak temperature generally is strong indication that the dominant mechanism, whatever its nature in detail, involves the reduction of the individual line lengths.

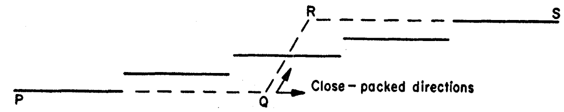

FIG. 3. A schematic illustration of the dislocation configuration which we propose for the annealed state. This is represented by the step PQRS. The corners at Q and R act as sources of the kinks which are produced by cold-work.

[8] H. L. Caswell, J. Appl. Phys. **29**, 1210 (1958). Further details of this work are available in Technical Report No. 3, Cornell University, 1957, AFOSR-TR-57-69 (unpublished).

[9] D. H. Niblett and J. Wilks, Phil. Mag. **2**, 1427 (1957).

[10] D. O. Thompson and D. K. Holmes, J. Appl. Phys. **30**, 525 (1959).

4. Structure of the Peak

Experimental studies have established that the peak cannot be associated with a single relaxation process. With even the most favorable values of W and f_0 the half-width of the peak is roughly twice as large as would obtain if this were the case. Since we have found by other computations based upon (42) that a suitable choice of $P(l)$ will yield almost any half-width, it would appear that this problem could be dispensed with immediately. However, in the present crude state of the art, we repeat that the exponential distribution (43) appears to be the most plausible. Moreover, in view of preliminary achievements, one is encouraged to explore the consequences of the abrupt-kink hypothesis to the full. Thereby an additional feature is revealed: namely, that in the mixed dislocations lying near to the close-packed directions at 60° to the Burgers vector, right and left kinks are not simply transformed into each other by rotation about the close-packed directions but are instead quite distinct configurations. These are illustrated schematically in Fig. 4, the dislocation segments having been connected

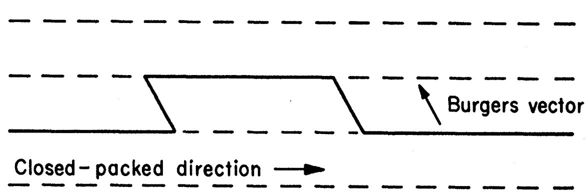

FIG. 4. An acute kink (on the left) and an obtuse kink (on the right), in a mixed dislocation oriented at 60° to the Burgers vector.

at the kink sites by lines drawn parallel to the Burgers vector. Because of the resulting geometry, we shall use the names "acute kink" and "obtuse kink" for the two types of configuration. This terminology also conveys the invariance property of the different kinks under rotation about the close-packed direction.

We would expect an acute kink and an obtuse kink to have different self-energies and more particularly different activation energies for diffusion. Therefore the model predicts that the main peak should have at least two components characterized by different values of W. Such is indeed the case in Al,[11] where two components are clearly resolved and even in Cu there is substantial evidence that the peak is not a singlet. In several instances the data show some fine structure near the attenuation maximum. We believe therefore that the peak half-width results not only from a distribution of line lengths but also from the near superposition of the components of an as yet incompletely resolved doublet. That the fine structure is reproducible in repeated experiments on the same specimen and yet varies from one specimen to another[10]

[11] L. J. Bruner, Phys. Rev. **118**, 399 (1960).

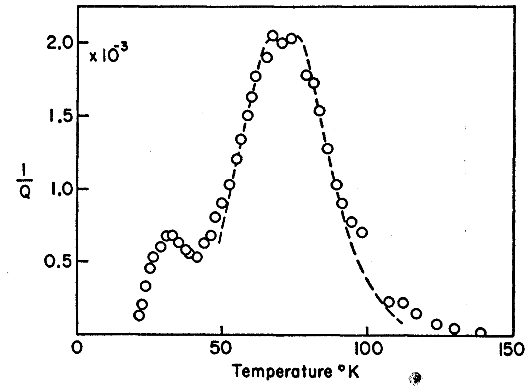

FIG. 5. A comparison of the calculated internal friction (broken line) with the data of Niblett and Wilks (open circles). The high-temperature background attenuation has been subtracted from the experimental points.

is not too surprising since the two types of kinks, having different self-energies, could well be produced in varying numbers depending upon the mechanical history.

In view of these conclusions one should in general return to the original equations (3) and (4) and reformulate the problem for kinks having different transport coefficients, etc. However, for our application this is not really necessary since we are concerned with such low temperatures that kinks of only one type are present in a given dislocation. In this instance, as we have verified in detail, the only result of these generalizations is that instead of (42) the total internal friction becomes the sum of two such terms with different values of W.

In order to determine how these ideas might further compare with experiment we have calculated the internal friction which results from two peaks derived from (42) of equal strength and with activation energies of 0.07 and 0.09 ev (thereby retaining the same mean value we have used previously). In Fig. 5 is shown the result of a comparison with data of Niblett and Wilks[9] taken at 1.1 kc/sec on a specimen deformed 8% in tension. The doublet structure in the calculated curve is here partially resolved because the separation between individual peaks was slightly greater than their half-width. Figure 6 on the other hand shows how the resolution rapidly disappears at higher frequencies, due to the increase in the broadening of each component. The experimental data are those of Caswell,[8] obtained at a frequency of 40 kc/sec, on a specimen which had been cross-rolled. Incidentally, it is interesting to note that in this case we had to assume an effective line length one half that for the tensile specimen, perhaps because of more intersections between dislocations in different slip systems, in order to obtain the peak at the correct temperature. Although the numerical values we have used are naturally speculative, the over-all agreement demonstrates that the theory can account

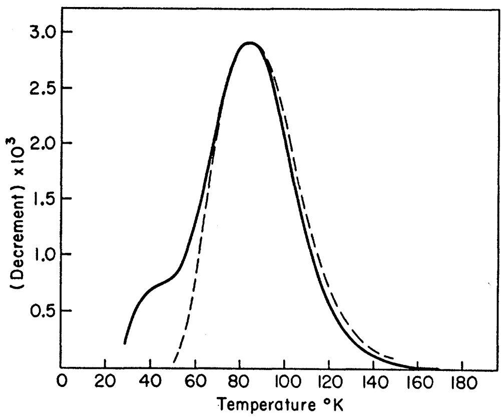

FIG. 6. A comparison of the calculated internal friction (broken line) with the experimental data of Caswell (full curve). The high-temperature background has been subtracted from the experimental data.

for the shape of the peak without too much refinement. We would suggest, then, that experiments at very low frequencies (≈ 1 cps), despite their difficulty, would be very valuable. In this region the individual components, if they really exist, would be much narrower and their resolution would provide a further test of our model.

5. The Subsidiary Peak

In addition to the main Bordoni peak, a subsidiary peak of smaller height has been observed at lower temperatures both in Cu[6] and Al.[12] Since we have already attributed the main peak to the mixed dislocations, the subsidiary peak we associate with kinked dislocations which are approximately pure screw type. Systematic studies of the properties of this peak are difficult since it is also influenced by the main peak. However, one piece of evidence, the relative insensitivity of the peak temperature to impurity content, is compatible with our assignment.

This concludes our discussion of internal friction.

[12] E. Lax and D. H. Filson, Phys. Rev. **114**, 1273 (1959).

V. SUMMARY

A new model of dislocation motion has been developed. It is based upon the following assumptions concerning kinks:

1. They exist and are abrupt.
2. They are mobile, diffusion being a thermally activated process.
3. They can be manufactured by cold work; thermally generated, or annihilated as a result of collisions.
4. In fcc lattices, there are two distinct species, namely acute kinks and obtuse kinks, relevant to the mixed dislocations lying close to 60° to the Burgers vector.

Transport equations for kinks have been formulated. It has been shown that a dislocation does not behave like an extensible string in this model. A new theory of the Bordoni anelastic peak observed in cold-worked metals has been presented. A detailed comparison with experiment has demonstrated that this theory can account for (1) the large peak half-width in pure Cu, (2) the decrease in the maximum decrement and accompanying shift in peak temperature with alloying, (3) the annealing properties of the peak, and (4) the initial growth of the peak, and its subsequent saturation with increasing amounts of cold-work. On this basis, subjects believed worthy of further experimental study have been suggested.

ACKNOWLEDGMENT

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