# Λ Binding in Hypernuclei by Nonlocal Interaction

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The characteristics of the Λ-$N$ interaction at low energy have been obtained assuming that the Λ-$N$ potential is nonlocal but separable and similar to that suggested by Yamaguchi in the case of $N$-$N$ potential. The unknown parameters entering in the proposed potential are determined on the basis of the global symmetry hypothesis of the strongly interacting particles. Our model predicts, in agreement with Dalitz and Downs' phenomenological findings of the nature of the Λ-$N$ potential based on hyperfragment data, that there is no bound Λ-nucleon system and that the singlet Λ-$N$ potential is stronger than the triplet potential, both being attractive. Binding energies of the Λ particle in light hypernuclei based on the present model are, however, much too high compared with the experimental data. It is further pointed out that although the global symmetry hypothesis ($g_{\Lambda\pi}=g_{\Sigma\pi}=g_{N\pi}$) supplemented by the Yamaguchi type nonlocal Λ-$N$ potential is incompatible with the presently existing data, the restricted symmetry ($g_{\Lambda\pi}=g_{\Sigma\pi}\neq g_{N\pi}$) model is certainly admissible.

## 1. INTRODUCTION

THE study of the Λ binding in light hypernuclei affords us a good insight into the hyperon-nucleon problem at low energy. From the knowledge of the lightest hyperfragment $_\Lambda H^3$, we conclude that it has a very loose structure as the total binding energy of the system is of the order of 2.3 Mev. This implies that when any pair of particles are very close together the third particle is relatively far away from them so that it has no important effect on the mutual interaction; which shows that the effect of three-body forces may be neglected. For the same reason Λ-nucleon interaction in hypernuclei such as $_\Lambda H^3$, say, may very well be represented by an interaction potential required to describe free Λ-nucleon collisions at low energy of the relative motion. As to the strong interaction from which the hyperon-nucleon force may originate, one can speculate that the force may be due to one $K$-meson exchange or two pion exchanges or both $K$ and $\pi$ exchanges. The range of the interaction thus introduced is $\gtrsim\hbar/2m_\pi c$ which is small compared to the nuclear force range. Further the $K$-meson exchange leads to an exchange type of force while the $2\pi$ exchange is of ordinary type. As the binding energy of the system is small it implies that the system consists predominantly of the $S$ state; hence the difference in the two types of forces will not be exhibited, as the parity of the state is even. It is well known that the effect of an ordinary force is independent of the sign of the parity of the state while that of the exchange force is dependent on this sign.

It may be reasonably expected that the properties of light hypernuclei could be derived from the over-all features of the Λ-$N$ potential such as the scattering length and effective range irrespective of the shape of this potential function. Dalitz and Downs[1] have analyzed the hypernuclear binding energies looking for a phenomenological potential and restricting themselves only to two-body forces. With a spin-dependent Gaussian-shaped central potential they were able to fit the observed binding energies of light hypernuclei. The important conclusions are that the singlet Λ-$N$ potential must be much more attractive than the triplet one and that there does not exist any hyperdeuteron. Instead of choosing a phenomenological potential, Lichtenberg and Ross[2] calculated the hyperon-nucleon potential due to pion exchanges only, assuming that the coupling constants $g_{\Lambda\pi}$ and $g_{\Sigma\pi}$ are equal but not necessarily equal to the $g_{N\pi}$ coupling constant, and using a static model similar to that of Brueckner and Watson.[3] The hard-core radius is assumed to be the same as in the case of the $N$-$N$ potential of Brueckner and Watson. These authors have calculated the $S$-wave hyperon-nucleon scattering parameters for small values of the energy, by noting that in treating the Λ-$N$ scattering one must take into account the transition $\Lambda N\leftrightarrow\Sigma N$, and hence in addition to the $\Lambda N\leftrightarrow\Lambda N$ and $\Sigma N\leftrightarrow\Sigma N$ potentials one should have to define a $\Lambda N\leftrightarrow\Sigma N$ potential and the whole scattering problem will be described by a coupled set of Schrödinger equations. They find that their scattering parameters are in agreement with those of Dalitz and Downs. Further, on introducing $K$-meson interactions, they find[4] that a large coupling constant for $K$-meson interactions is incompatible with hypernuclear data. Ferrari and Fonda,[5] using a Tamm-Dancoff approximation, have derived the Λ-$N$ potential and compared the implications of their results with those of Dalitz and Downs, and they have noticed that $K$-meson interaction is not capable of giving agreement with experiment.

In the present paper we have introduced the viewpoint that the Λ-$N$ interaction could be nonlocal and may have a separable form as suggested and used by

[1] R. H. Dalitz and B. W. Downs, Phys. Rev. **110**, 958 (1958); **111**, 967 (1958); **114**, 593 (1959).

[2] D. B. Lichtenberg and Marc Ross, Phys. Rev. **107**, 1714 (1957).

[3] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

[4] D. B. Lichtenberg and Marc Ross, Phys. Rev. **109**, 2163 (1958).

[5] F. Ferrari and L. Fonda, Nuovo cimento **9**, 842 (1958).

Yamaguchi[6] for the $N$-$N$ case. The necessity of introducing a nonlocal potential has already been felt in analyzing the high-energy nucleon-nucleon data; however, there is no compelling reason to take it into consideration for low-energy data; nevertheless, Yamaguchi has found that the two-nucleon low-energy data could be well-fitted by means of a special type of nonlocal potential. Furthefmore, the use of separable potential is very attractive as it affords a completely soluble model.

On account of the absence of any experimental information on the scattering of Λ particles by nucleons at low energy, we will make use of Gell-Mann's[7] global symmetry model of strong interactions.[7a] With the help of this assumption it would be easy to determine the unknown parameters occurring in the proposed Λ-$N$ potential by making use of low-energy nucleon-nucleon data, since the Λ-$N$ potential would be explicitly expressible in terms of various spin-dependent $N$-$N$ potentials. (We must take into consideration that no Pauli principle is applicable for a Λ-$N$ system.) Such an approach will be free from the approximations involved in the field-theoretic determinations of the Λ-$N$ potential by the above authors. With these assumptions, the coupled Schrödinger equations describing the hyperon-nucleon scattering in the $T=\frac{1}{2}$ state have been solved exactly and the well-depth parameter, scattering length, and effective range have been determined. Following Dalitz,[1] the binding energies of the Λ particle in some of the light hypernuclei have been calculated by means of a central local potential having the same characteristics for low-energy scattering. The main conclusion is that the global symmetry hypothesis supplemented by a nonlocal separable potential is in qualitative agreement with the empirical results of Dalitz and Downs but it fails to reproduce the observed binding energies of the light hypernuclei considered in this paper.

Finally, we have abandoned the global symmetry hypothesis (the failure of such hypothesis has been pointed out by many authors in analyzing other strange-particle data) and instead we have assumed in our approach the restricted symmetry[8] of pion-hyperon interactions (i.e., the Σ, Λ couplings with pions are equal but not the same as the pion-nucleon coupling). We find that a nonlocal potential of the form used here for the hyperon-nucleon interaction, satisfying the restricted symmetry of the pion-baryon strong interaction, is quite tenable and the parameters of the interaction potential can be so chosen as to reproduce the observed binding energies of the Λ hyperfragment.

[6] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. **95**, 1628, 1635 (1954).

[7] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[7a] *Note added in proof.* The global symmetry hypothesis is not necessarily restricted to Yukawa type of couplings. One can certainly generalize it for more complicated couplings between the baryons and the mesons and in this paper it is used in this generalized sense.

[8] J. Prentki and B. d'Espagnat, Nuovo cimento **15**, 130 (1960).

## 2. FORM OF THE POTENTIAL BASED ON GLOBAL SYMMETRY HYPOTHESIS

We define the potential for describing the hyperon-nucleon ($Y$-$N$) scattering in $T=\frac{1}{2}$ state in the following way: $V^{\Lambda}$ describes the transition $\Lambda+N\leftrightarrow\Lambda+N$, and $V^{\Sigma}$ the transition $\Sigma+N\leftrightarrow\Sigma+N$ only in $T=\frac{1}{2}$ state. In addition to these, we should have two more potentials $V^{\Lambda\Sigma}$ and $V^{\Sigma\Lambda}$ in order to describe the transition $\Lambda+N\leftrightarrow\Sigma+N$. Assuming global symmetry of strong interactions, we can easily connect these potentials by means of the nucleon-nucleon potential. We have then[9]

$$V^{\Lambda}=\tfrac{1}{4}(3V_1^N+V_0^N), \quad (1)$$

$$V^{\Sigma}=\tfrac{1}{4}(3V_0^N+V_1^N), \quad (2)$$

$$V^{\Lambda\Sigma}=V^{\Sigma\Lambda}=\tfrac{1}{4}\sqrt{3}(V_1^N-V_0^N), \quad (3)$$

where $V_T^N$ (with $T=1$ and 0) is the nucleon-nucleon potential in the isotopic spin state $T$.

Since we are primarily interested in the Λ-nucleon scattering in the nucleus allowing the transition $\Lambda N\leftrightarrow\Sigma N$, it would be necessary to solve the following system of coupled Schrödinger equations which describe completely the $T=\frac{1}{2}$ state hyperon-nucleon scattering:

$$\left(\frac{1}{2\mu_\Lambda}\nabla^2+E_\Lambda\right)\psi_\Lambda(r)=V^\Lambda(r)\psi_\Lambda(r)+V^{\Lambda\Sigma}(r)\psi_\Sigma(r), \quad (4)$$

$$\left(\frac{1}{2\mu_\Sigma}\nabla^2+E_\Sigma\right)\psi_\Sigma(r)=V^\Sigma(r)\psi_\Sigma(r)+V^{\Sigma\Lambda}(r)\psi_\Lambda(r), \quad (5)$$

where $\psi_\Lambda$ and $\psi_\Sigma$ denote the Λ nucleon and Σ nucleon (in $T=\frac{1}{2}$) system and $E_\Lambda$, $\mu_\Lambda$, $E_\Sigma$, and $\mu_\Sigma$ denote the respective energies and reduced masses of the system. (We have chosen $\hbar=c=1$.) $V^\Lambda$, etc., are defined in (1), (2), and (3) above. The energies $E_\Lambda$ and $E_\Sigma$ are related by

$$E_\Lambda=E_\Sigma+\delta M, \quad (4a)$$

where $\delta M$ is the mass difference $M_\Sigma-M_\Lambda$.

With quite arbitrary types of potentials (e.g., field-theoretic, etc.) Eqs. (4) and (5) can only be solved by means of numerical methods which are no doubt involved. A way out of this difficulty would be to assume suitable type of potential forms for the $V$'s so that one may obtain exact analytical solutions whose advantages can easily be realized. To this end, one may assume the well-known Yamaguchi potential which is nonlocal but separable. The implications of these assumptions in the theory will be completely discussed in the following section.

## 3. CONSEQUENCES OF THE CHOICE OF YAMAGUCHI POTENTIAL

The Yamaguchi potential has an important characteristic which should be noted. Ignoring noncentral

[9] D. Amati and B. Vitale, Fortschr. Physik **7**, 375 (1959).

forces for a moment, consider the Yamaguchi potential[6] of the form:

$$(\mathbf{p}|V_l|\mathbf{p}') = \sum_{m=-l}^{+l} Y_l^{m*}(\theta_p,\varphi_p) Y_l^m(\theta_{p'},\varphi_{p'}) g(|\mathbf{p}|)g(|\mathbf{p}'|), \quad (6)$$

where the symbols are self-explanatory. If this potential $V_l$ acts on a wave function $\psi$, we have

$$V_l\psi = \int (\mathbf{p}|V_l|\mathbf{p}')\psi(\mathbf{p}')d^3p'. \quad (7)$$

From (6) and (7), it is clear that because of the orthogonal properties of the spherical harmonics the potential $V_l$ acts as a projection operator on $\psi$; it picks out from $\psi$ only that particular state having an orbital angular momentum $l$. One can show in an analogous manner that a Yamaguchi potential which includes noncentral forces will act as a projection operator for a state having a particular *total* angular momentum and parity.

The above property leads to a significant simplification in the hyperon-nucleon potentials as used in this paper. Consider, for example, the spin singlet Λ-$N$ potential, which according to Eq. (1) is

$$V_s^{\Lambda} = \tfrac{1}{4}(3V_{1s}^N + V_{0s}^N). \quad (8)$$

Since the nucleons obey the Pauli principle, $V_{1s}^N$ is an even-parity potential while $V_{0s}^N$ is an odd-parity potential. Equation (8) therefore implies that in general, information on both the parity states of the $N$-$N$ system is necessary for a discussion of the $Y$-$N$ system in any one parity state. However, if we choose the Yamaguchi potential, then either $V_{1s}^N$ or $V_{0s}^N$ alone will contribute according as the $Y$-$N$ system is in an even- or odd-parity state, because of the projection property mentioned above.

In the present discussion we will assume that in the ground states of light hypernuclei, the relative motion between the hyperon and nucleon will be predominantly in the $S$ state and so we are interested only in the even-parity hyperon-nucleon potentials. Thus, because of the assumptions of the Yamaguchi potential and the predominance of the $S$ states in light hypernuclei, the following simplified potentials will alone contribute to our calculations (we have dropped the superfluous isospin index):

$$V_s^{\Lambda} = \tfrac{3}{4}V_s^N, \quad (9)$$

$$V_s^{\Sigma} = \tfrac{1}{3}V_s^{\Lambda}, \quad (10)$$

$$(V_s^{\Lambda\Sigma})^2 = V_s^{\Lambda}V_s^{\Sigma}, \quad (11)$$

$$V_t^{\Lambda} = \tfrac{1}{4}V_t^N, \quad (12)$$

$$V_t^{\Sigma} = 3V_t^{\Lambda}, \quad (13)$$

$$(V_t^{\Lambda\Sigma})^2 = V_t^{\Lambda}V_t^{\Sigma}. \quad (14)$$

Evidently, such a choice is unrealistic, as $P$ states of $Y$-$N$ system may probably be present even in light hypernuclei; this is certainly the case when we go to heavier hypernuclei.

## 4. DETERMINATION OF THE PARAMETERS OF THE POTENTIAL

The well-known nonlocal separable $N$-$N$ potential can be written in the momentum space as[6]

$$(\mathbf{p}|V^N|\mathbf{p}') = -\frac{\lambda^N}{2\mu_N}g(\mathbf{p})g(\mathbf{p}'),$$

where $\lambda^N$ is the interaction strength and $\mu_N$ denotes the reduced mass of the nucleon-nucleon system. For the singlet spin state, where the potential has to be central, we assume

$$g_s(\mathbf{p}) = g_s(p) = 1/(p^2+\beta_s^2).$$

We fix the values of $\lambda_s^N$ and $\beta_s$, the parameters for the singlet potential from the low-energy $n$-$p$ singlet scattering. We require $\lambda_s^N = 0.1467\ \mathrm{f}^{-3}$ and $\beta_s = 1.158$ f to fit the scattering length and effective range: $a_s = -23.69$ f and $r_{0s} = 2.7$ f. In the triplet state (where tensor forces can occur) we assume the following form for the function $g(\mathbf{p})$:

$$g_t(\mathbf{p}) = C(p) + \frac{1}{\sqrt{8}}S(\mathbf{p})T(p),$$

where

$$S(\mathbf{p}) = \frac{3}{p^2}(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2),$$

$$C(p) = 1/(p^2+\beta_t^2), \quad T(p) = -tp^2/(p^2+\gamma^2)^2.$$

The parameters $\lambda_t$, $\beta_t$, $t$, and $\gamma$ could easily be determined to fit the following triplet-state properties of the two-nucleon system, namely $\alpha_D[=(2\mu_N\times \mathrm{B.E.})^{\frac{1}{2}}] = 0.2316\ \mathrm{f}^{-1}$, triplet scattering length $a_t = 5.378$ f, deuteron quadrupole moment $Q = 0.274\ \mathrm{f}^2$, and $D$-state probability in deuteron $P_D = 4\%$. These require

$$\lambda_t^N = 0.249\ \mathrm{f}^{-3},$$
$$\beta_t = 1.334\ \mathrm{f}^{-1},$$
$$\gamma = 1.568\ \mathrm{f}^{-1},$$
$$t = 1.784.$$

We now write the various hyperon-nucleon potentials as

$$V^{\Lambda} = -\frac{\lambda^{\Lambda}}{2\mu_{\Lambda}}g(\mathbf{p})g(\mathbf{p}'), \quad (14a)$$

$$V^{\Sigma} = -\frac{\lambda^{\Sigma}}{2\mu_{\Sigma}}g(\mathbf{p})g(\mathbf{p}'), \quad (14b)$$

$$V^{\Lambda\Sigma} = -\frac{\lambda^{\Lambda\Sigma}}{2\mu_{\Lambda}}g(\mathbf{p})g(\mathbf{p}') = -\frac{\lambda^{\Sigma\Lambda}}{2\mu_{\Sigma}}g(\mathbf{p})g(\mathbf{p}'). \quad (14c)$$

We note that $\mu_\Sigma\lambda^{\Lambda\Sigma}=\mu_\Lambda\lambda^{\Sigma\Lambda}$. The $g(\mathbf{p})$'s have the same form as written above. Using Eqs. (9) and (12), we get

$$\lambda_s^\Lambda=\tfrac{3}{4}(\mu_\Lambda/\mu_N)\lambda_s^N=0.1193\ \text{f}^{-3}, \tag{15}$$

$$\lambda_t^\Lambda=\tfrac{1}{4}(\mu_\Lambda/\mu_N)\lambda_t^N=0.0676\ \text{f}^{-3}. \tag{16}$$

From (10), (11), and (13), (14) we determine

$$\lambda_s^\Sigma=\tfrac{1}{3}(\mu_\Sigma/\mu_\Lambda)\lambda_s^\Lambda, \tag{17}$$

$$\lambda_t^\Sigma=3(\mu_\Sigma/\mu_\Lambda)\lambda_t^\Lambda, \tag{18}$$

and finally

$$(\lambda^{\Lambda\Sigma}\lambda^{\Sigma\Lambda})_{s,t}=(\lambda^\Lambda/\lambda^\Sigma)_{s,t}. \tag{19}$$

### 5. BOUND-STATE SOLUTIONS OF COUPLED SCHRÖDINGER EQUATIONS AND THE NONEXISTENCE OF HYPERDEUTERON

We write the coupled Schrödinger Eqs. (4) and (5) in momentum space as follows:

$$(p^2+\alpha_\Lambda^2)\psi_\Lambda(\mathbf{p})=\lambda^\Lambda g(\mathbf{p})\int g(\mathbf{p}')\psi_\Lambda(\mathbf{p}')d^3p' +\lambda^{\Lambda\Sigma}g(\mathbf{p})\int g(\mathbf{p}')\psi_\Sigma(\mathbf{p}')d^3p', \tag{20}$$

$$(p^2+\alpha_\Sigma^2)\psi_\Sigma(\mathbf{p})=\lambda^\Sigma g(\mathbf{p})\int g(\mathbf{p}')\psi_\Sigma(\mathbf{p}')d^3p' +\lambda^{\Sigma\Lambda}g(\mathbf{p})\int g(\mathbf{p}')\psi_\Lambda(\mathbf{p}')d^3p', \tag{21}$$

where $E_\Lambda=-\alpha_\Lambda^2/2\mu_\Lambda$ and $E_\Sigma=-\alpha_\Sigma^2/2\mu_\Sigma$. From Eq. (4a), using $\alpha_0=(2\mu_\Sigma\delta M)^{\frac{1}{2}}$ and $\mu=\mu_\Sigma/\mu_\Lambda\approx1.03$, we can write

$$\alpha_\Sigma^2=\alpha_0^2+\mu\alpha_\Lambda^2. \tag{20a}$$

Equation (20) can be easily solved in terms of $\psi_\Sigma$, giving

$$\psi_\Lambda(\mathbf{p})=\frac{\lambda^{\Lambda\Sigma}}{[1-\lambda^\Lambda I(\alpha_\Lambda)]}\frac{g(\mathbf{p})}{(p^2+\alpha_\Lambda^2)}\int g(\mathbf{p}')\psi_\Sigma(\mathbf{p}')d^3p', \tag{22}$$

and similarly $\psi_\Sigma$ can be obtained as

$$\psi_\Sigma(\mathbf{p})=\frac{\lambda^{\Sigma\Lambda}}{[1-\lambda^\Sigma I(\alpha_\Sigma)]}\frac{g(\mathbf{p})}{(p^2+\alpha_\Sigma^2)}\int g(\mathbf{p}')\psi_\Lambda(\mathbf{p}')d^3p', \tag{23}$$

where

$$\begin{aligned} I(\alpha)&=\int\frac{g^2(\mathbf{p}')d^3p'}{(p'^2+\alpha^2)} \\ &=\frac{\pi^2}{\beta_s(\alpha+\beta_s)^2}\ \text{for singlet state} \\ &=\pi^2\left[\frac{1}{\beta_t(\alpha+\beta_t)^2}+\frac{t^2}{8}\frac{(5\alpha^2+4\alpha\gamma+\gamma^2)}{(\alpha+\gamma)^4}\right]\ \text{for triplet state.}\end{aligned} \tag{22a}$$

Substituting (23) in (22) we get the homogeneous integral equation for $\psi_\Lambda$:

$$\psi_\Lambda(\mathbf{p})=\frac{\lambda^{\Lambda\Sigma}\lambda^{\Sigma\Lambda}I(\alpha_\Sigma)}{[1-\lambda^\Lambda I(\alpha_\Lambda)][1-\lambda^\Sigma I(\alpha_\Sigma)]}\times\frac{g(\mathbf{p})}{(p^2+\alpha_\Lambda^2)}\int g(\mathbf{p}')\psi_\Lambda(\mathbf{p}')d^3p'. \tag{24}$$

The bound-state solution $\psi_\Lambda$ for Λ-$N$ system with real $\alpha_\Lambda$ will exist only if the Fredholm determinant of (24) vanishes. Hence $\alpha_\Lambda$ must satisfy

$$\frac{\lambda^{\Lambda\Sigma}\lambda^{\Sigma\Lambda}I(\alpha_\Lambda)I(\alpha_\Sigma)}{[1-\lambda^\Lambda I(\alpha_\Lambda)][1-\lambda^\Sigma I(\alpha_\Sigma)]}=1, \tag{25}$$

and the corresponding normalized solutions are

$$\psi_\Lambda(\mathbf{p})=\frac{N_\Lambda g(\mathbf{p})}{p^2+\alpha_\Lambda^2},\quad \psi_\Sigma(\mathbf{p})=N_\Sigma\frac{g(\mathbf{p})}{p^2+\alpha_\Sigma^2}. \tag{26}$$

where $N_\Lambda$ and $N_\Sigma$ are obtained from

$$\frac{N_\Sigma}{N_\Lambda}=\frac{\lambda^{\Sigma\Lambda}I(\alpha_\Lambda)}{1-\lambda^\Sigma I(\alpha_\Sigma)},$$

and

$$N_\Lambda^2J(\alpha_\Lambda)+N_\Sigma^2J(\alpha_\Sigma)=1,$$

with

$$J(\alpha)=\int\frac{g^2(\mathbf{p}')}{(p'^2+\alpha^2)^2}d^3p'.$$

The compatibility Eq. (25) reduces to

$$\lambda^\Lambda I(\alpha_\Lambda)+\lambda^\Sigma I(\alpha_\Sigma)=1, \tag{27}$$

in view of the fact that $\lambda^{\Sigma\Lambda}\lambda^{\Lambda\Sigma}=\lambda^\Lambda\lambda^\Sigma$. Equation (27) further reduces to the following by using (17) and (18):

$$\lambda_s^\Lambda[I_s(\alpha_\Lambda)+\tfrac{1}{3}\mu I_s(\alpha_\Sigma)]=1, \tag{28}$$

$$\lambda_t^\Lambda[I_t(\alpha_\Lambda)+3\mu I_t(\alpha_\Sigma)]=1, \tag{29}$$

corresponding to singlet and triplet states.

Using the parameters of the potential it is found that neither (28) nor (29) is compatible for any real value of $\alpha_\Lambda$. This shows that there does not exist any bound system of Λ and nucleon.[10] We can check this result by calculating the well-depth parameter of the Λ-$N$ interaction defined by[11]

$$s=\lambda^\Lambda/\lim_{\alpha_\Lambda\to0}\lambda^\Lambda(\alpha_\Lambda), \tag{30}$$

[10] It may be noted that the above implies also that there does not exist any Σ-$N$ bound system in $T=\frac{1}{2}$ state. However, even if it exists it can never be observed because of its quick transition to the Λ-$N$ channel and such a system is not bound. In our present case we have not discussed the hyperon-nucleon interaction in $T=\frac{3}{2}$ state at all.

[11] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

where $\lambda^{\Lambda}(\alpha_{\Lambda})$ is the value of $\lambda^{\Lambda}$ calculated from Eqs. (28) and (29). If there is no bound state, $s$ should be less than unity. The calculated values are

$$s_s=0.81, \quad s_t=0.69,$$

where the suffixes $s$ and $t$ denote singlet and triplet nature. The above values confirm that with the present choice of the potential supplemented by the global symmetry point of view there is no hyperdeuteron and they also show that the singlet state is more attractive than the triplet one. Thus our hypothesis leads to results which agree qualitatively with those of Dalitz and Downs.[1]

In the next section we calculate the binding energies of the light hypernuclei by means of a suitable local central potential supposed to be capable of producing the parameters as predicted by our model.

## 6. BINDING ENERGIES OF LIGHT HYPERNUCLEI

Since the problem of the calculation of the binding energies of hypernuclei directly using our nonlocal, noncentral, and nondiagonal (because of the transition $\Lambda N \leftrightarrow \Sigma N$) is very involved, we proceed as follows. First we solve the Schrödinger equations for $\Lambda N$ scattering below the threshold for $\Sigma$ production, and from this we determine the scattering length $a$ and effective range $r_0$ for $\Lambda$-$N$ scattering. In light hypernuclei, one expects that the $\Lambda$-$N$ scattering will occur with low relative momenta and hence the scattering length $a$ and effective range $r_0$ would be the only important characteristics of the interaction. Therefore, for the purpose of calculating the binding energy, we choose a local central $\Lambda$-$N$ potential having the same $a$ and $r_0$. (For comments on this procedure, see Sec. 8.) For heavy hypernuclei, the $\Lambda$-$N$ scattering can occur with large relative momenta in which case the effective range formalism will not do. Further, if relative momenta larger than the threshold are involved, $\Sigma$ will be produced in the $\Lambda$-$N$ scattering process and then this $\Sigma$ can again be scattered by some other nucleon. Thus, the problem becomes complicated and it will be necessary to treat the problem by means of Brueckner's $G$-matrix formalism. However, we shall restrict ourselves to light hypernuclei in which case these complications do not arise.

We therefore replace the coupled Schrödinger equations by the following pair:

$$\psi_{\Lambda\mathbf{k}}(\mathbf{p})=\delta(\mathbf{p}-\mathbf{k})+\frac{\lambda^{\Lambda}g(\mathbf{p})}{p^2-k^2-i\epsilon}\int g(\mathbf{p}')\psi_{\Lambda\mathbf{k}}(\mathbf{p}')d^3p' + \frac{\lambda^{\Lambda\Sigma}g(\mathbf{p})}{p^2-k^2-i\epsilon}\int g(\mathbf{p}')\psi_{\Sigma\alpha}(\mathbf{p}')d^3p', \quad (31)$$

and

$$\psi_{\Sigma\alpha}(\mathbf{p})=\frac{\lambda^{\Sigma}g(\mathbf{p})}{p^2+\alpha^2}\int g(\mathbf{p}')\psi_{\Sigma\alpha}(\mathbf{p}')d^3\mathbf{p}' + \frac{\lambda^{\Sigma\Lambda}g(\mathbf{p})}{p^2+\alpha^2}\int g(\mathbf{p}')\psi_{\Lambda\mathbf{k}}(\mathbf{p}')d^3p', \quad (32)$$

where $\alpha^2=\alpha_0^2-\mu k^2$ [actually the $\delta$ function in (31) should be multiplied by a constant; but we conveniently choose it to be unity]. The solutions of (31) and (32) are immediately obtained as

$$\psi_{\Lambda\mathbf{k}}(\mathbf{p})=\delta(\mathbf{p}-\mathbf{k}) + \frac{\lambda^{\Lambda}}{[1-\lambda^{\Lambda}K(k)-\lambda^{\Sigma}I(\alpha)]}\frac{g(\mathbf{p})g(\mathbf{k})}{p^2-k^2-i\epsilon}, \quad (33)$$

and

$$\psi_{\Sigma\alpha}(\mathbf{p})=\frac{\lambda^{\Sigma\Lambda}}{[1-\lambda^{\Lambda}K(k)-\lambda^{\Sigma}I(\alpha)]}\frac{g(\mathbf{p})g(\mathbf{k})}{\alpha^2+p^2}, \quad (34)$$

where

$$K(k)=\int\frac{g^2(\mathbf{p}')d^3p'}{p'^2-k^2-i\epsilon}$$

and $I(\alpha)$ is defined in Eq. (22a). In writing (33) and (34) we have made use of (19).

The $T$-matrix element for $\Lambda$-$N$ scattering is

$$(\Lambda\mathbf{k}'|T|\Lambda\mathbf{k})=\int\Phi_{\Lambda\mathbf{k}'}{}^*(\mathbf{p})(\mathbf{p}|\mathbf{V}|\mathbf{p}')\Psi_{\Lambda\mathbf{k}}(\mathbf{p}')d^3pd^3p', \quad (35)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the incident and final momentum of the $\Lambda$ particle in the c.m. system. We denote by $(\mathbf{p}|V|\mathbf{p}')$ a $2\times2$ matrix and $\Psi$, $\Phi$ are column matrices. We have

$$(\mathbf{p}|\mathbf{V}|\mathbf{p}')=\begin{pmatrix}(\mathbf{p}|V^{\Lambda}|\mathbf{p}') & (\mathbf{p}|V^{\Lambda\Sigma}|\mathbf{p}')\\(\mathbf{p}|V^{\Sigma\Lambda}|\mathbf{p}') & (\mathbf{p}|V^{\Sigma}|\mathbf{p}')\end{pmatrix},$$

$$\Psi_{\Lambda\mathbf{k}}(\mathbf{p})=\begin{pmatrix}\Psi_{\Lambda\mathbf{k}}(\mathbf{p})\\\Psi_{\Sigma\alpha}(\mathbf{p})\end{pmatrix},\quad \Phi_{\Lambda\mathbf{k}'}(\mathbf{p})=\begin{pmatrix}\delta(\mathbf{k}'-\mathbf{p})\\0\end{pmatrix}.$$

Obviously $\Phi$ and $\Psi$ are the eigenfunctions of the free and total Hamiltonians, respectively.

Using (33), (34), and (35), we have

$$(\Lambda\mathbf{k}'|T|\Lambda\mathbf{k})=\frac{1}{2\mu_{\Lambda}}\frac{\lambda^{\Lambda}}{[1-\lambda^{\Lambda}K(k)-\lambda^{\Sigma}I(\alpha)]}g(\mathbf{k}')g(\mathbf{k}). \quad (36)$$

The wave functions (33) and (34) can now be written, by using (36), in the following way:

$$\psi_{\Lambda\mathbf{k}}(\mathbf{p})=\delta(\mathbf{p}-\mathbf{k})+2\mu_{\Lambda}\frac{(\Lambda\mathbf{p}|T|\Lambda\mathbf{k})}{p^2-k^2-i\epsilon}, \quad (37)$$

$$\psi_{\Sigma\alpha}(\mathbf{p})=\frac{\lambda^{\Sigma\Lambda}}{\lambda^{\Lambda}}2\mu_{\Lambda}\frac{(\Lambda\mathbf{p}|T|\Lambda\mathbf{k})}{p^2+\alpha^2}. \quad (38)$$

We express the $T$-matrix element (36) in terms of the phase-shift $\delta$ and then from the effective-range expansion of $k \cot\delta$ we can easily determine the scattering length and effective range. The results are

$$\frac{1}{a_s}=\frac{\beta_s}{2}-\frac{\beta_s^4}{2\pi^2\lambda_s^\Lambda}+\frac{\lambda_s^\Sigma\beta_s^3}{2\lambda_s^\Lambda(\alpha_0+\beta_s)^2}, \tag{39}$$

$$r_{0s}=\frac{1}{\beta_s}+\frac{2\beta_s^2}{\pi^2\lambda_s^\Lambda}-\frac{2\lambda_s^\Sigma\beta_s}{\lambda_s^\Lambda(\alpha_0+\beta_s)^2}-\frac{\lambda_s^\Sigma\mu\beta_s^3}{\lambda_s^\Lambda\alpha_0(\alpha_0+\beta_s)^3}, \tag{40}$$

$$\frac{1}{a_t}=\frac{\beta_t}{2}-\frac{\beta_t^4}{2\pi^2\lambda_t^\Lambda}+\frac{\beta_t^4t^2}{16\gamma^3}+\frac{\lambda_t^\Sigma}{2\lambda_t^\Lambda}\frac{\beta_t^3}{(\alpha_0+\beta_t)^2} +\frac{t^2}{16}\frac{\lambda_t^\Sigma}{\lambda_t^\Lambda}\frac{\beta_t^4(5\alpha_0^2+4\alpha_0\gamma+\gamma^2)}{\gamma(\alpha_0+\gamma)^4}, \tag{41}$$

$$r_{0t}=\frac{1}{\beta_t}+\frac{2\beta_t^2}{\pi^2\lambda_t^\Lambda}-\frac{t^2\beta_t^2(\beta_t^2+2\gamma^2)}{8\gamma^5}-\frac{\lambda_t^\Sigma\beta_t^3}{\lambda_t^\Lambda(\alpha_0+\beta_t)^2}\times\left(\frac{2}{\beta_t^2}+\frac{\mu}{\alpha_0(\alpha_0+\beta_t)}\right)-\frac{\lambda_t^\Sigma t^2\beta_t^4}{8\lambda_t^\Lambda\gamma(\alpha_0+\gamma)^4}\times\left[(5\alpha_0^2+4\alpha_0\gamma+\gamma^2)\left(\frac{2}{\beta_t^2}+\frac{2\mu}{\alpha_0(\alpha_0+\gamma)}\right)-\left(5\mu+\frac{2\gamma\mu}{\alpha_0}\right)\right], \tag{42}$$

where $a$ and $r_0$ are the scattering length and effective range and the suffixes denote singlet and triplet, respectively.

Using the numerical values of the various parameters from Sec. 4 in the right-hand side of (39)–(42), we get the values of $a$ and $r_0$ given in Table I. We are now in a position to calculate the binding energies of the light hypernuclei. Equivalent local and central potentials which would regenerate the above $a$ and $r_0$ parameters are here assumed to be of exponential or Gaussian types:

$$V(r)=-V_0e^{-r/c}, \qquad \text{(exponential)}$$

$$V(r)=-V_0\exp(-r^2/c^2), \qquad \text{(Gaussian).}$$

The parameters $V_0$ and $c$ should be such as to predict the values of $a$ and $r_0$ given in Table I. The numerical results for these are given in Table II. We note here the longer range of the singlet potential compared to that of the triplet.

TABLE I. Calculated values of scattering length and effective range.

| | Singlet | Triplet |
|---|---|---|
| $a$ | −6.9 f | −1.4 f |
| $r_0$ | 3.0 f | 3.4 f |

TABLE II. Parameters of equivalent potentials.

| | Exponential potential Singlet | Exponential potential Triplet | Gaussian potential Singlet | Gaussian potential Triplet |
|---|---|---|---|---|
| $V_0$ (in Mev) | 90.46 | 90.23 | 25.87 | 25.20 |
| $c$ (in fermis) | 0.60 | 0.52 | 1.76 | 1.37 |

Adopting the same method as used by Dalitz and Downs and Lichtenberg and Ross we calculate the binding energies and the results are given in Table III. The potential shapes used in respective nuclei have been mentioned by the side of the calculated column. Our results (see the last column) clearly indicate a large binding energy compared to the experimental values. We thus see that a nonlocal potential in conjunction with global symmetry is not a valid model for hypernuclei. In the next section we abandon the global symmetry hypothesis but retain the nonlocal Yamaguchi potential as the effective Λ-$N$ potential.

## 7. COMPATIBILITY OF THE RESTRICTED SYMMETRY MODEL WITH HYPERNUCLEAR DATA

Under the restricted symmetry hypothesis of the strong interactions it is easy to see that the various potentials $V^\Lambda$, $V^\Sigma$, and $V^{\Lambda\Sigma}$ defined in Sec. 2 for $\Lambda N\leftrightarrow\Lambda N$, $\Sigma N\leftrightarrow\Sigma N$ (in $T=\frac{1}{2}$ state only), and $\Lambda N\leftrightarrow\Sigma N$ transitions, respectively, satisfy the following relations:

$$V_s^\Sigma=\tfrac{1}{3}V_s^\Lambda,\quad V_t^\Sigma=3V_t^\Lambda,\quad V_s^{\Lambda\Sigma}=\sqrt{3}V_s^\Sigma,\quad V_t^{\Lambda\Sigma}=-\sqrt{3}V_t^\Lambda. \tag{43}$$

The nonlocal shape of the potentials are still the same as has been assumed in Eq. (14a)–(14c). However, the tensor part is now dropped from the triplet potentials, since it introduces too many unknown parameters. In this model the parameters $\lambda$ and $\beta$ occurring in the potential function can no longer be determined from nucleon-nucleon data. Hence, to decide whether the present model is compatible with the hypernuclear data, it is necessary to see whether the scattering length and effective range as required by Dalitz and Downs to fit the binding energies of light hypernuclei with a suitable over-all central potential, could be obtained with admissible values of the interaction strength $\lambda$ and a range parameter $1/\beta$ of the nonlocal potential.

Using the potential stated in (43), we easily obtain in the same way as before, the following algebraic equa-

TABLE III. Binding energies of Λ in light hypernuclei.

| Hypernucleus | Experimental B.E. (in Mev) | Calculated value (in Mev) |
|---|---|---|
| $_\Lambda H^3$ | 0.12±0.26 | 1.06 (exponential) |
| $_\Lambda H^4$ | 2.20±0.14 | 4.46 (Gaussian) |
| $_\Lambda He^4$ | 2.36±0.12 | 4.46 (Gaussian) |
| $_\Lambda He^5$ | 3.08±0.09 | 8.17 (Gaussian) |

TABLE IV. Semiempirical parameters of Dalitz and Downs.

| | Singlet | Triplet |
|---|---|---|
| $a$ | $-2.72$ f | $-0.53$ f |
| $r_0$ | 1.92 f | 3.67 f |

tions for $\beta$ and $\lambda$:

$$\left.\begin{aligned} \frac{1}{a}&=\frac{\beta}{2}-\frac{\beta^4}{2\pi^2\lambda^\Lambda}+\frac{\lambda^\Sigma\beta^3}{2\lambda^\Lambda(\alpha_0+\beta)^2},\\ r_0&=\frac{1}{\beta}+\frac{2\beta^2}{\pi^2\lambda^\Lambda}-\frac{2\lambda^\Sigma\beta}{\lambda^\Lambda(\alpha_0+\beta)^2}-\frac{\lambda^\Sigma\beta^3\mu}{\lambda^\Lambda\alpha_0(\alpha_0+\beta)^3},\end{aligned}\right\} \quad (44)$$

where $\lambda^\Sigma/\lambda^\Lambda$ is $\mu/3$ and $3\mu$ for singlet and triplet states, respectively. The values of $a$ and $r_0$ corresponding to the Gaussian central potential determined[12] by Dalitz and Downs from hypernuclear data are given in Table IV.

Using the numerical values of $a$ and $r_0$ from Table IV, we can solve (44) for $\beta$ and $\lambda^\Lambda$. Obviously there again exists a number of possible solutions. A set of acceptable solutions are given in Table V.

We thus conclude that restricted symmetry hypothesis is consistent with the present hypernuclear data. It may be remarked here that a similar conclusion was also reached by Lichtenberg and Ross[2,4] for the present problem; however, their discussion is based on a potential obtained from the field-theoretic treatment. We may further point out that recently in analyzing the $K^-$-nucleon interaction data, Prentki and d'Espagnat[8] and also Gupta[13] have shown that the restricted symmetry model is quite successful in explaining the data where one knows in advance that global symmetry fails.[14]

TABLE V. Potential parameters fitted to give the $a$ and $r_0$ of Dalitz and Downs.

| | Singlet | Triplet |
|---|---|---|
| $\beta$ | 1.92 $f^{-1}$ | 1.80 $f^{-1}$ |
| $\lambda^\Lambda$ | 0.478 $f^{-3}$ | 0.145 $f^{-3}$ |

## 8. FINAL REMARKS

It should be remarked that the conclusions drawn in this paper are not completely free from objections because of the number of assumptions made in the calculations. First, the low-energy $\Lambda$-$N$ scattering parameters deduced on the basis of global symmetry will depend also on the high-energy behavior of the $N$-$N$ scattering because of the possibility of the transitions $\Lambda N\leftrightarrow\Sigma N$ which involve the rather high-energy change $\approx 75$ Mev; therefore, before giving a final judgment, one has to see whether the separable nonlocal $N$-$N$ potentials fitted to low-energy $N$-$N$ data can also satisfy high-energy $N$-$N$ data. Secondly, it is necessary to establish that the shape-independent parameters such as scattering length and effective range alone are significant for the calculation of the binding energies of light hypernuclei. It has long been known that in the case of the normal triton, the results for the ground-state energy are highly well-shape dependent. Most of the present-day calculations on hyperfragments suffer from this uncertainty. Next, the effects of three- and many-body forces between $\Lambda$ and nucleons have been neglected. The calculations of Weitzner[15] and Spitzer[16] suggest that there could be an appreciable effect of three-body forces. However, it has recently been shown by Liul'ka and Filimonov[17] that such effects are quite small and negligible. Finally, for a rigorous demonstration of the validity of the restricted symmetry hypothesis we must calculate the stimulated decay of $\Lambda$ in the hyperfragment, realizing that the decay can go partially through $\Sigma$-decay channel. This evidently requires the knowledge of the relative probabilities of $\Lambda$ and $\Sigma$ in the hypernucleus, which can be calculated from the solution of the system of Schrödinger equations using the potentials (43).

[12] In fact, Dalitz has assumed the ranges of the potentials from semitheoretical considerations and determined only the strengths of the potential from hypernuclear data. Hence, the values of $\lambda^\Lambda$ and $\beta$ which we have determined should be regarded as only one of the possible sets which reproduce the observed binding energies.

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