Magnetic Moments of the Λ and Σ Hyperons*

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A relation among the magnetic moments of Σ^+, Σ^0, Σ^-, and Λ is obtained as a consequence of the proposed symmetries of strong interactions, a minimal electromagnetic coupling for the electromagnetic interactions being assumed. The magnetic moments of the Λ and Σ hyperons are calculated with the aid of mass spectral representations in which only the contributions of the bound states are taken into account. The present calculation of these magnetic moments are compared with various other calculations. Remarks are made on the possible experimental values.

1. INTRODUCTION

THE measurement of the Λ magnetic moment μ_Λ and the Σ magnetic moments μ_+, μ_-, and μ_0 will offer insight into the symmetries of the strong interactions within the framework of current field theory. We shall first discuss μ_Λ. So far all estimates on the values of μ_Λ based on perturbation theory point to a small value.[1,2] In particular, it has been shown rigorously under very general assumptions that $\mu_\Lambda=0$ in the limit in which the mass differences among the baryons are neglected.[3]

The analysis of the nucleon magnetic moment by use of spectral representations has revealed that perturbation theoretical results are unreliable.[4,5] This treatment suggests that the necessary modifications of the perturbation treatment (which are also valid for the μ_Λ) are that the baryon-current contribution to μ_Λ should be neglected, and that the structure of the emitted boson and the rescattering terms (contributions other than that from the one-baryon poles) of the K current be taken into account. The K-current contribution is formulated in terms of ΛK scattering. Further, when the K-current contribution from the baryon poles is expressed as an integral over the square of some mass variable over the range from $(2\mu)^2$ to ∞, μ being the mass of the K meson, the integration should be carried from $(2\mu)^2$ to the physical threshold $(2M)^2$, where M is the mass of the Λ hyperon. These suggestions based on arguments of unitarity have been discussed in detail by Federbush *et al.*[5]

When the mass variable exceeds $(2M)^2$ one can formulate the K-current contribution in terms of the Λ pair annihilation into two K mesons. The Λ pair of the matrix element for the pair annihilation are in $^3S_1$ and $^3D_1$ states, in analogy to the case of the magnetic moment of the nucleon. The magnitude of the contributions from these states is limited by unitarity, so that one can set an upper limit to the K-current contribution for the range from $(2M)^2$ to ∞. If one considers the contribution from the baryon poles, the unitarity condition is violated. The violation for the contribution from the π-meson and pseudoscalar K-meson currents to the magnetic moments of Λ and Σ is smaller than for that to the magnetic moment of the nucleon, and the violation for the scalar K meson is approximately the same as for the latter.

The Λ and Σ hyperons involved in a strong interaction may be considered as a bound state of two particles. In the loosely bound case, there is an anomalous threshold that gives rise to an additional contribution to the magnetic moment. There is no anomalous threshold for the cases $\Lambda\rightarrow\Xi+K$, $\Sigma\rightarrow\Xi+K$, and $\Sigma\rightarrow\Sigma+\pi$, but there is one for the cases $\Lambda\rightarrow N+K$, $\Sigma\rightarrow N+K$, and $\Sigma\rightarrow\Lambda+\pi$. Rough estimates of these additional contributions to the magnetic moments are made.

Let us examine how we may make maximum use of the information gained above in the calculation of the nucleon magnetic moment and effect an evaluation of μ_+, μ_-, μ_0, and μ_Λ on the basis of spectral representations. The largeness of the rescattering terms for the nucleon magnetic moment is due to a resonance in the pion-nucleon scattering. In the present case, one needs the dispersion relations for (Λ,K) scattering. Since no reliable method of evaluating the rescattering terms has been found, and no possible resonance of (Λ,K) scattering is known, we shall henceforth ignore this contribution. Further, no information is available on the structure of the K meson, although such structure may increase or decrease the value of μ_Λ obtained for a point K meson, so that any detailed analysis with some arbitrarily assumed structure for the K meson does not appear to be meaningful.

This leaves only the possible modification of the K-current contribution or the contribution from the $|2K\rangle$ state. This contribution arising from the poles of the cascade particle Ξ and the nucleon N is integrated over (mass)^2 from $(2\mu)^2$ to $(2M)^2$. The analysis will necessarily be exploratory because of lack of detailed

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[1] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **18**, 375 (1957).
[2] W. G. Holladay, Phys. Rev. **115**, 1331 (1959). This article has references to previous articles.
[3] G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959).
[4] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 265 (1958).
[5] P. Federbush, M. L. Goldberger, and S. R. Treiman, Phys. Rev. **112**, 642 (1958) (hereafter referred to as FGT).

experimental data. The essential features that emerge from the analysis, however, are expected to survive.

In Sec. 2, a relation among the magnetic moments of Λ and Σ hyperons is obtained from symmetry arguments. In Sec. 3, the method is outlined and an expression is obtained for μ_Λ, which is evaluated in Sec. 4. In Sec. 5, the magnetic moments of Σ hyperons are evaluated and finally some remarks are made in Sec. 6.

2. MAGNETIC MOMENTS OF Λ AND Σ HYPERONS

On the basis of charge independence, Marshak *et al.*[6,1] have shown rigorously that the sum of the magnetic moments of Σ^+ and Σ^- is equal to twice that of Σ^0, i.e., $\mu_+ + \mu_- = 2\mu_0$. Using the method of Feinberg and Behrends[3] and the doublet representation of Gell-Mann[7] and Pais[8] in which $I=\frac{1}{2}$ is assigned to all baryons and $I=0$ to K^+ and K^0, we shall show rigorously that

$$\mu_+ + \mu_- = 2\mu_0 = 2\mu_\Lambda.$$

The interaction Hamiltonian adopted for π and K mesons is

$$H_\pi = i[G_1\bar{N}_1\boldsymbol{\tau}\gamma_5 N_1 + G(\bar{N}_2\boldsymbol{\tau}\gamma_5 N_2 + \bar{N}_3\boldsymbol{\tau}\gamma_5 N_3) + G_4\bar{N}_4\boldsymbol{\tau}\gamma_5 N_4]\boldsymbol{\pi}, \quad (1)$$

and

$$H_K = F_I\sqrt{2}[(\bar{N}_1\eta_I N_2)K^0 + (\bar{N}_1\eta_I N_3)K^+] + F_{II}\sqrt{2}[(\bar{N}_4\eta_{II} N_2)\bar{K}^+ - (\bar{N}_4\eta_{II}N_3)\bar{K}^0] + \text{H.c.}, \quad (2)$$

where $\hbar = c = 1$,

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix},\ N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix},\ N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix},\ N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},$$
$$Z^0 = (\Lambda + \Sigma^0)/\sqrt{2}, \quad Y^0 = (\Lambda - \Sigma^0)/\sqrt{2}, \quad (3)$$

the symbol of a particle denotes the field operator that destroys it, and H.c. stands for Hermitian conjugate. The assumptions underlying Eqs. (1) and (2) are that all strong interactions are charge independent, the present baryon spectrum and its isotopic spin assignments are correct, the baryon spins are $\frac{1}{2}$ and the K spin is zero, the (Σ,Λ) parity and (K^0,K^+) parity are even, and the (Σ,Λ) mass difference can be neglected. The factors η_I and η_{II} stand for 1 or $i\gamma_5$.

The minimal electromagnetic interactions H_{em} of the baryons and mesons to order e are

$$H_{\text{em}} = H_a + H_b + H_c, \quad (4)$$

where

$$H_a = \tfrac{1}{2}ie[\bar{N}_1\gamma_\mu N_1 - \bar{N}_4\gamma_\mu N_4]A_\mu + \tfrac{1}{2}ie[\bar{K}\partial_\mu K - \partial_\mu\bar{K}K]A_\mu,$$
$$H_b = \tfrac{1}{2}ie[\bar{N}_2\gamma_\mu N_2 - \bar{N}_3\gamma_\mu N_3]A_\mu + \tfrac{1}{2}ie[\bar{K}\tau_3\partial_\mu K - \partial_\mu\bar{K}\tau_3 K]A_\mu, \quad (5)$$
$$H_c = \tfrac{1}{2}ie[\bar{N}\gamma_\mu\tau_3 N_1 + \bar{N}_2\gamma_\mu\tau_3 N_2 + \bar{N}_3\gamma_\mu\tau_3 N_3 + \bar{N}_4\gamma_\mu\tau_3 N_4]A_\mu + ie\bar{\pi}T_3\partial_\mu\pi A_\mu.$$

Here, K is a component isotopic spinor for the K meson[9] and π is an isotopic vector for the π meson. The reason for writing H_{em} in three parts is that the three parts transform differently under the transformations considered below. The quantity

$$\Gamma_\mu(\Lambda) = T\langle\bar{\Lambda}(x)\Lambda(y)A_\mu(z)\rangle_0 \quad 6)$$

is related to the electromagnetic vertex operator for Λ, where T is the time-ordering operator and $\langle\ \rangle_0$ designates the expectation value in the physical vacuum.[3] Since $\Gamma_\mu(\Lambda)$ depends linearly on A_μ, one can consider the transformation of the three parts of H_{em} separately so long as H_{em} remains invariant under the transformation.

In order to obtain relations among the Γ_μ, let us first consider the conventional isospin rotation. The transformation is not the isospin rotation for the doublet representation.

$$\begin{array}{llll}
N_1 \to e^{i\frac{1}{2}\pi T_2}N_1 & \text{or} & p\to n, & n\to -p, \\
N_2 \to -e^{i\frac{1}{2}\pi T_2}N_3 & \text{or} & \Sigma^+\to -\Sigma^-, & Y^0\to Z^0, \\
N_3 \to e^{i\frac{1}{2}\pi T_2}N_2 & \text{or} & Z^0\to Y^0, & \Sigma\to -\Sigma^+, \\
N_4 \to e^{i\frac{1}{2}\pi T_2}N_4 & \text{or} & \Xi^0\to\Xi^-, & \Xi^-\to -\Xi^0, \\
\pi \to e^{i\pi T_2}\pi & \text{or} & \pi^+\to -\pi^-, & \pi^-\to -\pi^+, \\
 & & & \pi^0\to -\pi^0, \\
K \to e^{i\frac{1}{2}\pi T_2}K & \text{or} & K^+\to K^0, & K^0\to -K^+.
\end{array} \quad (7)$$

The strong interactions given by Eqs. (1) and (2) are invariant under transformation (7) since they are charge independent. The H_a part of H_{em} is also invariant under (7) provided $A_\mu\to A_\mu$. One thus obtains relations among the contributions of H_a. The H_b and H_c parts of H_{em} are invariant under transformation (7) provided $A_\mu\to -A_\mu$. This yields relations among the contributions of H_b and H_c. The contribution of H_a to $\Gamma_\mu(\Lambda)$, for instance, will be denoted by $T\langle\bar{\Lambda}\Lambda A_\mu\rangle_0{}^a = \langle\Lambda\rangle_a$. Then we have

$$\Gamma_\mu(\Lambda) = \langle\Lambda\rangle_a + \langle\Lambda\rangle_b + \langle\Lambda\rangle_c. \quad (8)$$

Let U be the unitary transformation that generates the transformation (7) and that leaves the vacuum invariant. Then, for instance,

$$\langle\Sigma^+\rangle_a = T\langle\bar{\Sigma}^+\Sigma^+A_\mu\rangle_0{}^a = T\langle U\bar{\Sigma}^+\Sigma^+A_\mu U^\dagger\rangle_0{}^a = T\langle\bar{\Sigma}^-\Sigma^-A_\mu\rangle_0{}^a = \langle\Sigma^-\rangle_a.$$

In a similar manner, transformation (7) leads to

$$\begin{aligned}
\Gamma(\Sigma^+) &= \langle\Sigma^+\rangle_a + \langle\Sigma^+\rangle_b + \langle\Sigma^+\rangle_c, \\
\Gamma(\Sigma^-) &= \langle\Sigma^+\rangle_a - \langle\Sigma^+\rangle_b - \langle\Sigma^+\rangle_c, \\
\Gamma(\Sigma^0) &= \langle\Sigma^0\rangle_a, \\
\Gamma(\Lambda) &= \langle\Lambda\rangle_a.
\end{aligned} \quad (9)$$

The H_b and H_c do not contribute to $\Gamma(\Sigma^0)$ and $\Gamma(\Lambda)$ because of cancellations.

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[6] R. E. Marshak, S. Okubo, and E. C. G. Sudarshan, Phys. Rev. **106**, 599 (1957).

[7] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[8] A. Pais, Phys. Rev. **112**, 624 (1958).

[9] It is convenient to use the shorthand notation $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ for the singlets K^+ and K^0 in the following equations.

The second transformation which leaves H_π and H_K invariant is given as

$$\begin{array}{llll} N_1 \to e^{i\frac{1}{2}\pi T_2}N_1 & \text{or} & p \to n, & n \to -p, \\ N_2 \to e^{i\frac{1}{2}\pi T_2}N_2 & \text{or} & \Sigma^+ \to Y^0, & Y^0 \to -\Sigma^+, \\ N_3 \to e^{i\frac{1}{2}\pi T_2}N_3 & \text{or} & Z^0 \to \Sigma^-, & \Sigma^- \to -Z^0, \\ N_4 \to e^{i\frac{1}{2}\pi T_2}N_4 & \text{or} & \Xi^0 \to \Xi^-, & \Xi^- \to -\Xi^0, \\ \pi \to e^{i\pi T_2}\pi & \text{or} & \pi^+ \to -\pi^-, & \pi^- \to -\pi^+, \\ & & & \pi^0 \to -\pi^0, \\ K \to K & \text{or} & K^+ \to K^+, & K^0 \to K^0. \end{array} \quad (10)$$

This is essentially the product of the transformation (7) and the $N_2 \to N_3$ symmetry discussed by Pais.[8,10] Moreover H_a and H_b are invariant under (10), provided $A_\mu \to A_\mu$. The H_c is invariant under (10) provided $A_\mu \to -A_\mu$. Transformation (10) leads to

$$\langle \Sigma^0 \rangle_a = \tfrac{1}{2}(\langle \Sigma^+ \rangle_a + \langle \Sigma^- \rangle_a) = \langle \Lambda \rangle_a, \quad (11)$$

with the aid of conservation of charge.

We obtain from Eqs. (9) and (11) the relation

$$\Gamma(\Sigma^+) + \Gamma(\Sigma^-) = 2\Gamma(\Sigma^0) = 2\Gamma(\Lambda), \quad (12)$$

and, in particular,

$$\mu_+ + \mu_- = 2\mu_0 = 2\mu_\Lambda. \quad (13)$$

As stressed previously, this relation is rigorously true to all orders in the strong interactions.[1,6] It has been proved under the assumptions of charge independence of the strong π and K interactions and symmetry between the Σ and Λ hyperons. The latter assumes that the (Σ,Λ) parity is even and that the (Σ,Λ) mass difference, the smallest such difference in the baryon system, can be neglected. In the following calculation of μ_Λ, the Λ hyperon is regarded as an isospin singlet $(I=0)$.

3. METHOD

The linear interaction of the Λ hyperon with the electromagnetic field can be expressed in terms of two real scalar functions $F(q^2)$ and $G(q^2)$, where q^2 is the square of the energy-momentum four vector. The charge form factor $F(q^2)$ and the magnetic form factor $G(q^2)$ are normalized such that

$$F(0)=0, \quad G(0)=\mu_\Lambda. \quad (14)$$

The magnetic form factor $G(q^2)$ can be represented by an expression such as

$$G(q^2) = \frac{1}{\pi}\int_{(3\mu_\pi)^2}^{\infty} dm^2 \frac{g(m^2)}{m^2+q^2-i\epsilon}, \quad (15)$$

where the variable m^2 represents the square of the mass of the various intermediate states through which the photon-Λ interaction is effected, and the limits of integration will be explained later. The charge form factor $F(q^2)$ can also be written in a form similar to Eq. (15), but it will not be considered further.

The weight function $g(m^2)$ in Eq. (15) is calculated from[4]

$$I_\mu = \bar{u}(p')[i\gamma_\mu \,\mathrm{Im}F(q^2) + \sigma_{\mu\nu}q_\nu \,\mathrm{Im}G(q^2)]u(p)$$

$$= \pi \sum_s \int d^3k\, \delta(q+k)\langle p' | f(0) | sk \rangle \langle sk | j_\mu(0) | 0 \rangle u(p), \quad (16)$$

where

$$\bar{f}(x) = (-\gamma_\mu{}^T \partial/\partial x_\mu + M)\bar{\psi}(x), \quad \mathrm{Im}G(q^2) = g(-q^2).$$

The initial and final Λ hyperon momenta are designated by p and p', respectively, and $q = p - p'$.

The states that enter into sum in Eq. (16) must have nucleon number zero, zero strangeness, and zero charge, and thus must consist of π mesons, an even number of K mesons, and baryon pairs of zero strangeness. First let us examine the states with π mesons. From G invariance one knows that the isotopic scalar part of j_μ contributes to $\langle s | j_\mu | 0 \rangle$ for states having an odd number of π mesons, whereas the isotopic vector of j_μ involves states with an even number. Since the Λ hyperon has $I=0$, only states with an odd number of mesons contribute to $\langle s | j_\mu | 0 \rangle$. The state with one π meson does not contribute because of invariance under charge conjugation.

The less massive states that contribute to μ_Λ are the states with an odd number of π mesons (up to seven), and next comes the state with two K mesons. Under the present assumptions the π-meson states have been found to give a contribution of $-0.06e/2M$ to μ_Λ.[11] This contribution will be added to the K-meson contribution of the μ_Λ. The contribution from these states vanishes when $M_\Xi = M_N$ with suitable equalities among the coupling constants.[3] Let us now examine the contribution from the $|2K\rangle$ state, since this computation will offer a direct comparison with previous calculations of μ_Λ.

Before going on to the determination of the weight function, let us discuss the limitation due to unitarity in the range $m^2 > 4M^2$ of the dispersion integral (15). This can be studied according to the FGT method, suitably generalized to the present case.

Let β_S and β_D be the S-matrix elements for production of a p-wave K-meson pair by a hyperon pair in the ${}^3S_1$ and ${}^3D_1$ states, respectively. For the region $m^2 \geqslant 4M^2$, the unitarity condition yields $|\beta_S|$, $|\beta_D| \leqslant 1$. One can formulate the quantity $g(m^2)$ of Eq. (15) in terms of β_S and β_D, put $\beta_S=1$, $\beta_D=0$, double the result, and obtain an upper limit on the contribution coming from $m^2 \geqslant 4M^2$ when evaluating Eq. (15). We then find that for pseudoscalar K mesons and $g_{\Xi K}{}^2 = g_{NK}{}^2 = 10$, the calculated magnetic moments of Λ and Σ receive

[10] In the doublet representation, transformation (10) is the isospin rotation $\exp[i\pi T_2{}^{\mathrm{total}}]$.

[11] G. Feinberg (to be published).

too large a contribution for $m^2 \gtrsim 4M^2$. This indicates that the limit placed on $|\beta_S|$ and $|\beta_D|$ by considerations of unitarity is violated by a factor of 1.5 to 2. This factor is less than that involved in the calculation of the magnetic moment of the nucleon.[12] For scalar K mesons and $g_{\Xi K}^2 = g_{NK}^2 = 10$, the unitarity limit is found to be violated by a factor of about 4.

If one uses a smaller K-meson coupling constant, the violation will appear to be still less significant but the maximum contribution from the region $m^2 \gtrsim 4M^2$ is probably overestimated so that the violation of unitarity will persist. In any event, because of the possible smallness of the K coupling constant compared to the π coupling constant, the violation of unitarity is less significant for the present case than in that of the magnetic moment of the nucleon.

The weight function $g_{2K}(m^2)$ of the $|2K\rangle$ state, adapted to the present case, is given as[13]

$$g_{2K}(m^2) = |F_K(m^2)|^2 \frac{1}{\pi} \int_{-\infty}^{a} \frac{d\sigma^2 \, \mathrm{Im} J(\sigma^2)}{(\sigma^2 + m^2 - i\epsilon) F_K(\sigma^2)},$$

where $a = 4\mu^2[1 - (\mu^2/4M^2)]$ and F_K is the K-meson form factor. Here $J(\sigma^2)$ is an appropriate projection of the amplitude for the process $\langle \Lambda\bar{\Lambda} | K\bar{K} \rangle$. We shall make the assumption of a point K meson, in which case $g_{2K}(m^2) \to J(m^2)$ may be obtained from Eq. (16).

The evaluation of the $|2K\rangle$ state is very similar to that of the $|2\pi\rangle$ state for the nucleon magnetic moment. One finds from Eq. (16) for the $|2K\rangle$ state,

$$I_\mu(2K) = -\tfrac{1}{2}\pi \sum_{jk} (2\pi)^{-3} \int d^3q_1 d^3q_2 \, \delta(q_1 + q_2 + q) \times \langle p' | \bar{f}(0) | q_1 j, q_2 k \rangle \langle q_1 j, q_2 k | j_\mu(0) | 0 \rangle u(p). \quad (17)$$

An isospin treatment similar to that for charged π mesons is used for the K mesons. Substitution of

$$\langle q_1 j, q_2 k | j_\mu(0) | 0 \rangle = -ie(q_1 - q_2)_\mu (\delta_{j1}\delta_{k2} - \delta_{j2}\delta_{k1}) / (4w_1 w_2)^{\frac{1}{2}}, \quad (18)$$

in which a K-meson form factor has been replaced by unity, into Eq. (17) yields

$$I_\mu(2K) = \tfrac{1}{2} e \int d^3q_1 d^3q_2 \, \delta(q_1 + q_2 + q)(q_1 - q_2)_\mu \times [\langle p', -q_2 K^- | T | p, q_1 K^- \rangle - \langle p', -q_2 K^+ | T | p, q_1 K^+ \rangle] / (4w_1 w_2)^{\frac{1}{2}}, \quad (19)$$

where $\langle p' | \bar{f}(0) | q_1 j, q_2 k \rangle u(p) = \langle p' | \bar{f}(0) | q_1 K_j, q_2 K_k \rangle u(p)$ has been used. The first and second amplitudes on the right-hand side of Eq. (19) are related to the (K^-, Λ) and (K^+, Λ) scattering amplitudes, respectively. By writing

$$\langle p', -q_2 K^\pm | T | p, q_1 K^\pm \rangle = (-A_\pm + i\gamma_\nu Q_\nu B_\pm) / (4w_1 w_2)^{\frac{1}{2}}, \quad (20)$$

and

$$-q = q_1 + q_2, \quad Q = \tfrac{1}{2}(q_1 - q_2), \quad P = \tfrac{1}{2}(p + p'),$$

one gets

$$I_\mu(2K) = -(e/4\pi^2) \int d^3q_1 d^3q_2 \, \delta(q_1 + q_2 + q)(4w_1 w_2)^{-1} \times Q_\mu \{ (A_- - A_+) - i\gamma_\nu Q_\nu (B_- - B_+) \}. \quad (21)$$

The dispersion relations for (K^-, Λ) and (K^+, Λ) scattering can be constructed in a similar way to (K, N) scattering. The (K^-, Λ) scattering has a bound-state contribution at the mass of the negatively charged cascade hyperon M_Ξ and (K^+, Λ) scattering has a bound-state contribution at the mass of the proton M_N. We shall neglect the contributions that come from the additional states and confine ourselves to the bound-state contributions.

The dispersion relations obtained in a standard manner for pseudoscalar interaction [meaning that the $(\Lambda\Xi K)$ and $(\Lambda N K)$ couplings are pseudoscalar] are

$$A_- - A_+ = A_\Xi - A_N, \quad B_- - B_+ = B_\Xi - B_N, \quad (22)$$

where

$$A_H = \tfrac{1}{2} F_H^2 (M - M_H) \left[\frac{1}{(P+Q)^2 + M_H^2} - \frac{1}{(P-Q)^2 + M_H^2} \right], \quad (23)$$

$$B_H = \tfrac{1}{2} F_H^2 \left[\frac{1}{(P+Q)^2 + M_H^2} + \frac{1}{(P-Q)^2 + M_H^2} \right].$$

The remaining steps are to substitute Eqs. (22) and (23) into Eq. (21) and obtain $g_{2K}(m^2)$ which in turn is put into Eq. (15) to find μ_Λ. Since the relevant contribution is from $(2\mu)^2$ to $(2M)^2$, the integration is carried out to a finite upper limit L^2. Since the algebra is similar to that of Chew *et al.*,[4] it will not be repeated here. The result is

$$\mu_\Lambda(H,l) = \frac{e}{2M} \frac{g_{HK}^2}{8\pi} \{ (1 - 3n + 3\kappa - 4\kappa^{\frac{1}{2}})(l - n/l)^{\frac{1}{2}} + (1 + \kappa - n)(l^2 - ln)^{\frac{1}{2}}/(l-1) + 2[(\kappa - n)(\kappa - n - \kappa^{\frac{1}{2}}) - n][\ln(n/2) - \ln((l^2 - ln)^{\frac{1}{2}} + l - (n/2))] + [(-\tfrac{1}{2} - \kappa - n - 3\kappa n + \tfrac{3}{2}\kappa^2 + \tfrac{3}{2}n^2 + 2\kappa^{\frac{1}{2}}(1 - \kappa + n))((l-1)/l)^{\frac{1}{2}} + (\tfrac{1}{2} + \kappa - 3n - 5\kappa n + \tfrac{5}{2}\kappa^2 + \tfrac{5}{2}n^2 - 2\kappa^{\frac{1}{2}}(1 + \kappa - n))(l/(l-1))^{\frac{1}{2}} + (-\tfrac{1}{2} - \kappa + n + \kappa n - \tfrac{1}{2}\kappa^2 - \tfrac{1}{2}n^2)(l^2 - l)^{\frac{1}{2}}/(l^2 - 1)] \ln[(l - \tfrac{1}{2}(1 - \kappa + n) + ((l-1)(l-n))^{\frac{1}{2}})/(l\kappa - n + \tfrac{1}{4}(1 - \kappa + n)^2)^{\frac{1}{2}}] + 4[\kappa(1-\kappa)(\kappa^{\frac{1}{2}} - \kappa) + n(-1 + \kappa^{\frac{1}{2}} - \kappa + 2\kappa^{\frac{3}{2}} - 3\kappa^2) + n^2(2 - \kappa^{\frac{1}{2}} + 3\kappa) - n^3](4n - (1 - \kappa + n)^2)^{-\frac{1}{2}} \cos^{-1}[l(n - 1 + \kappa)^2/n(4l\kappa + n^2 - 2n(1+\kappa) + (1-\kappa)^2)]^{\frac{1}{2}} \}. \quad (24)$$

[12] The author would like to thank Dr. P. Federbush for a communication on the FGT method.
[13] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959).

The processes that have an anomalous threshold are not treated differently but Eq. (24) is correct for all processes. The correct branch is chosen as in reference 1. Where we have put

$$g_{HK}^2=F_H^2/4\pi,\quad l=L^2/4M^2,\quad n=\mu^2/M^2,\quad \text{and}\quad \kappa=M_H^2/M^2.$$

The notation $\mu_\Lambda(H,l)$ represents the H-baryon contribution to μ_Λ, with the upper limit l. From Eqs. (14), (15), (16), (21), and (22), $\mu_\Lambda(l)$ is expressible as

$$\mu_\Lambda(l)=\mu_\Lambda(\Xi,l)-\mu_\Lambda(N,l). \tag{25}$$

Equation (24) may be applied even when the intermediate baryon has a different mass from the initial one and when the upper limit has any arbitrary value l. We have obtained μ_Λ for the pseudoscalar coupling, but μ_Λ for the scalar coupling can be obtained by the replacement $\kappa^{\frac{1}{2}}\to-\kappa^{\frac{1}{2}}$ in Eq. (24). Equation (24) can also be used to find the rescattering terms in an effective-range approach, although these terms are neglected here.

4. EVALUATION

In order to compare our results with those of perturbation theory, we take the limit $l\to\infty$ in Eq. (24) and get[14]

$$\mu_\Lambda(H,\infty)=\frac{e}{2M}\frac{g_{HK}^2}{4\pi}\{1+2(\kappa-n-\kappa^{\frac{1}{2}})-[(\kappa-n)(\kappa-n-\kappa^{\frac{1}{2}})-n]\ln(\kappa/n)+2[\kappa(1-\kappa)(\kappa^{\frac{1}{2}}-\kappa)$$
$$+n(-1+\kappa^{\frac{1}{2}}-\kappa+2\kappa^{\frac{3}{2}}-3\kappa^2)+n^2(2-\kappa^{\frac{1}{2}}+3\kappa)-n^3][4n-(1-\kappa+n)^2]^{-\frac{1}{2}}\cos^{-1}[(n-1+\kappa)^2/4n\kappa]^{\frac{1}{2}}\}. \tag{26}$$

It has been shown by a rigorous argument based on unitarity that when the coupling constant is sufficiently large the perturbation theory is wrong for L^2 larger than $4M^2$ or for $l\geqslant 1$. The integration of $\mu_\Lambda(H,l)$ should therefore be cut off at $l=1$ so that

$$\mu_\Lambda(H,1)=(e/2M)(g_{HK}^2/4\pi)\{4(1-n)^{\frac{1}{2}}[\tfrac{1}{3}+\kappa-\kappa^{\frac{1}{2}}-n-\tfrac{1}{3}\kappa(1+\kappa-n)^{-1}]+[(\kappa-n)(\kappa-n-\kappa^{\frac{1}{2}})-n]$$
$$\times[\ln n-\ln[2(1-n)^{\frac{1}{2}}+2-n]]+2[\kappa(1-\kappa)(\kappa^{\frac{1}{2}}-\kappa)+n(-1+\kappa^{\frac{1}{2}}-\kappa+2\kappa^{\frac{3}{2}}-3\kappa^2)+n^2(2-\kappa^{\frac{1}{2}}+3\kappa)-n^3]$$
$$\times[4n-(n+1-\kappa)^2]^{-\frac{1}{2}}\cos^{-1}[(n-1+\kappa)/(1+\kappa-n)n^{\frac{1}{2}}]\}. \tag{27}$$

The numerical results are obtained by neglecting the mass differences within the baryon multiplets and using the observed mass values $M_N=939$, $M=1115$, $M_\Xi=1315$, and $\mu=496$ in units of Mev. The numerical values of μ_Λ for the case $l=\infty$ which corresponds to that of standard perturbation theory and the case $l=1$, together with that of static cutoff perturbation theory,[2] are shown in Table I. Although there is no compelling reason, the rather large value $g_{\Xi K}^2=g_{NK}^2=10$ is used.

When we add the π-meson contribution of $-0.06e/2M_N$ to the third and fourth rows of Table I, we obtain for $\mu_{\Lambda\,\mathrm{ps}}$ and $\mu_{\Lambda\,\mathrm{s}}$ the values -0.083 and 0.54 nuclear magneton, respectively. The following discussion on μ_Λ is based on the values given in Table I.

The value of μ_Λ is due to the differences in the M_N and M_Ξ masses, since we assumed that $g_{\Xi K}=g_{NK}$. It is therefore expected that the magnetic moment would be small compared to that of the nucleon. The values of $\mu_\Lambda(\Xi,\infty)$ and $\mu_\Lambda(N,\infty)$ are in agreement with standard perturbation theory.[1]

For the pseudoscalar coupling, the $\mu_\Lambda(\Xi,1)$ and $\mu_\Lambda(N,1)$ contributions are only about a third or a quarter of $\mu_\Lambda(\Xi,\infty)$ and $\mu_\Lambda(N,\infty)$, respectively, but $\mu_\Lambda(1)$ is larger than $\mu_\Lambda(\infty)$ and both are negligible even for $g_{\Xi K}^2=g_{NK}^2=10$.

There is a remarkable discrepancy in the values of μ_Λ obtained by the present theory and by the static cutoff theory. This can be understood in the following way. In the static cutoff method, the same cutoff value (essentially the mass of the Λ particle) was used for both contributions. The μ_Λ is the difference between the K-current contributions in the virtual processes $\Lambda\to K^++\Xi^-$ and $\Lambda\to K^-+p$. In the process $\Lambda\to K^++\Xi^-$, the reduced mass of (K,Ξ) is larger than that of (K,Λ). The latter reduced mass corresponds to the case in which the intermediate baryon had the same mass as the Λ hyperon. Thus the range of interaction is reduced so that a higher momentum cutoff[15] is required in the contribution $\mu_\Lambda(\Xi,l)$. By a similar argument, a smaller momentum cutoff should be taken for the process

TABLE I. Summary of values of μ_Λ in units of $e/2M_N$ for pseudoscalar (ps) and scalar (s) coupling with $g_{\Xi K}^2=g_{NK}^2=10$.

	K coupling	$\mu_\Lambda(\Xi,l)$	$\mu_\Lambda(N,l)$	$\mu_{\Lambda\,\mathrm{ps}}$	$\mu_{\Lambda\,\mathrm{s}}$
$4\mu^2\to\infty$ Pert. theory	pseudoscalar	0.272	0.279	−0.007	
	scalar	−0.834	−1.522		0.69
$4\mu^2\to 4M^2$	pseudoscalar	0.057	0.080	−0.023	
	scalar	−0.303	−0.901		0.60
Static cutoff theory	pseudoscalar	0.13	0.30	−0.20	

[14] This expression is in agreement with the value of $\mu_\Lambda(H,\infty)$ obtained in reference 1. It can also be obtained by the method which is the relativistic generalization of the cutoff meson theory discussed by S. Okubo, Nuovo cimento **4**, 452 (1957); and K. Tanaka, Phys. Rev. **109**, 578 (1958).

[15] As the range of interaction is reduced, the critical radius which defines the region of ignorance is likewise reduced. This critical radius corresponds to the momentum cutoff in momentum space.

TABLE II. Contribution to the anomalous magnetic moments of Σ hyperons in units of $e/2M_N$ from π and K mesons for $g_\pi^2=g_{\Xi K}^2=g_{NK}^2=10$.

Coupling	$\mu_\Sigma(\Sigma\pi,l)$ ps	$\mu_\Sigma(\Lambda\pi,l)$ ps	$\mu_\Sigma(\Xi,l)$ ps	$\mu_\Sigma(\Xi,l)$ s	$\mu_\Sigma(N,l)$ ps	$\mu_\Sigma(N,l)$ s
$4\mu^2\to\infty$ Pert. theory	0.467	0.531	0.266	-0.915	0.275	-1.876
$4\mu^2\to 4M_\Sigma^2$	0.259	0.326	0.067	-0.396	0.092	-1.280
Static cutoff theory	0.56	0.56	0.13		0.42	

$\Lambda\to K_-+p$ since the reduced mass of (K,N) is smaller than that of (K,Λ). This adjustment (in the cutoff) that is perhaps necessary for a static cutoff theory would increase the value for $\mu_\Lambda(\Xi,l)$ and decrease the value of $\mu_\Lambda(N,l)$ in Table I. As shown in reference 2, the respective values depend very sensitively on the cutoff value so that a dramatic cancellation is expected to bring about an agreement.

For the scalar coupling, one has values of μ_Λ that are measurable if the K-coupling constants are as large as the π-coupling constants.[16]

The most reliable expression for μ_Λ is given by Eqs. (25) and (27). The magnetic moments μ_+, μ_-, and μ_0 are expressed as a sum over such terms as Eq. (27), and their numerical values are obtained in Sec. 5.

5. MAGNETIC MOMENT OF THE Σ HYPERONS

The magnetic moments μ_+, μ_-, and μ_0 can be found in the same spirit as for $\mu_\Lambda(l)$. One finds from Eqs. (9) and (11) that

$$\mu_+=A+B,\quad \mu_-=A-B,\quad \mu_0=A, \tag{28}$$

where

$$A(l)=\mu_\Sigma(\Xi,l)-\mu_\Sigma(N,l). \tag{29}$$

In order to obtain $B(l)$, the third component of a vector in isotopic-spin space, one must carry out an expansion of the type in Eq. (16) for the $|\Sigma\rangle$ state. The $|2\pi\rangle$ state is the configuration with lowest mass that gives a contribution to $B(l)$. Then the $|2K\rangle$ state is reached if the states with many π mesons are neglected. The contributions from the $|2\pi\rangle$ and the $|2K\rangle$ states are related to (π,Σ) and (K,Σ) scattering, respectively. Again, considering the bound-state contributions, one can find that

$$B(l)=\mu_\Sigma(\Xi,l)+\mu_\Sigma(N,l)+\mu_\Sigma(\Sigma\pi,l)+\mu_\Sigma(\Lambda\pi,l), \tag{30}$$

where $\mu_\Sigma(\Sigma\pi,l)$ and $\mu_\Sigma(\Lambda\pi,l)$ express the contributions from the intermediate $|\Sigma\rangle$ state and $|\Lambda\rangle$ state in (π,Σ) scattering, respectively.

The four terms on the right-hand side of Eq. (30) can be computed from Eqs. (26) and (27). It is assumed that π mesons are pseudoscalar and the relative (Σ,Λ) parity is even, so that there is no scalar π-meson contribution. The numerical values are obtained using the observed values $M_\Sigma=1192$ and $\mu_\pi=139.6$ in units of Mev and are tabulated in Table II. For the π-meson contributions in Table II, μ denotes the mass of the π meson. The π-coupling constant $g_\pi^2=G^2/4\pi$ where G appears in Eq. (1). The respective contributions for $l=\infty$ are in agreement with standard perturbation theory.[1]

The magnetic moments obtained for the Σ hyperons by use of Table II, from Eqs. (28), (29), and (30), and from the static cutoff theory[2] are given in Table III. The values of the magnetic moments in Table III do not include the fourth-order meson contributions that were included in reference 2.

6. REMARKS

We have made an estimate of μ_Λ on the basis of the $|2K\rangle$ state. The possible contributions from the π-meson states have been found to be of the same order of magnitude as $\mu_{\Lambda ps}$ given in Table I.[11] The value of μ_Λ when this contribution is included is also given after Table I. In the analysis of the $|2K\rangle$ state, the structure of the K meson and rescattering terms were not considered. It is argued that any analysis of μ_Λ with these two items should be postponed until some experimental data on the K-meson structure and (K,Λ) scattering become available and a reliable method of computation with such data is found. Moreover, the main features of the present analysis of μ_Λ based on the bound-state contributions are not likely to be altered by the proper treatment of these items.[5] A reason is that μ_Λ is the difference of two terms so that such modifications would perhaps tend to cancel out.

The magnetic moments of the Λ and Σ hyperons have their normal contribution from absorptive processes that correspond to the threshold for the creation of real particles. When the Λ hyperon, for instance, is considered as a composite particle consisting of N and K, a certain mass inequality (indicating that the Λ hyperon is loosely bound) gives rise to a structure contribution.[17] The threshold of this contribution is

TABLE III. Magnetic moments of the Σ hyperons in units of $e/2M_N$ for $g_\pi^2=g_{\Xi K}^2=g_{NK}^2=10$.

K coupling	μ_+ ps	μ_+ s	μ_0 ps	μ_0 s	μ_- ps	μ_- s
$4\mu^2\to\infty$ Pert. theory	1.53	-0.83	-0.01	0.96	-1.55	2.75
$4\mu^2\to 4M_\Sigma^2$	0.72	-0.21	-0.025	0.88	-0.77	1.97
Static cutoff theory	1.37		0.29		-1.91	

[16] The Λ magnetic moment will be measured in the near future by an Argonne group (W. Kernan, T. Novey, S. Warshaw, and A. Wattenberg) and a Brookhaven group (D. O. Caldwell, R. L. Cool, D. Hill, R. O. Jenkins, T. F. Kycia, L. Marshall, and R. A. Schluter).

[17] R. Oehme, Nuovo cimento **13**, 778 (1959); R. Blankenbecler and Y. Nambu (to be published). These articles have references to previous articles. The author is indebted to Professor Nambu for a discussion.

below the onset of the absorptive process that gives rise to an anomalous threshold.

In the anomalous case, the contribution to the Λ form factor for zero momentum transfer, resulting from a scalar interaction $\Lambda \to N+K$ (abbreviated ΛNK), is given from perturbation and dispersion theory as

$$F(0,M)=\frac{\Gamma_0^2}{2\pi^2}\left[\int_{4M_K^2}^{4M^2} dy\frac{a(M,y)}{y} + \int_{S(M^2)}^{4M_K^2} dy\frac{2\pi}{y(4M^2y-y^2)^{\frac{1}{2}}}\right], \quad (31)$$

where

$$a(M,y)=\frac{2}{(4M^2y-y^2)^{\frac{1}{2}}}\tan^{-1}\left[\frac{[(4M^2-y)(y-4M_K^2)]^{\frac{1}{2}}}{y-2(M^2-M_N^2+M_K^2)}\right],$$

and

$$S(M^2)=-[M^2-(M_K+M_N)^2]\times[M^2-(M_K-M_N)^2]/M_N^2.$$

The first term on the right-hand side of Eq. (31) is related to the normal contribution due to absorptive processes and the second term is the structure contribution. In order to estimate the relative size of the structure contribution to that of the normal contribution, we integrate the first and second terms on the right-hand side of Eq. (31) to obtain

$$\frac{\Gamma_0^2}{2\pi^2M^2}\left[\frac{\pi(1-n)^{\frac{1}{2}}}{n^{\frac{1}{2}}}-\frac{1}{2}\ln\left(\frac{2(1-n)^{\frac{1}{2}}+2-n}{n}\right) - \frac{(1-n+\kappa)}{\{4n-(1+n-\kappa)^2\}^{\frac{1}{2}}}\cos^{-1}\left(\frac{1-\kappa-n}{(1+\kappa-n)n^{\frac{1}{2}}}\right)\right], \quad (32)$$

and

$$\frac{\Gamma_0^2}{2\pi^2M^2}\left[\frac{\pi(1+\kappa-n)}{\{4n-(1-\kappa+n)^2\}^{\frac{1}{2}}}-\frac{\pi(1-n)^{\frac{1}{2}}}{n^{\frac{1}{2}}}\right], \quad (33)$$

respectively.

By a suitable substitution, Eqs. (32) and (33) can also be used for the cases ΣNK and $\Sigma\Lambda\pi$. The numerical result shows that the structure contributions for the cases ΛNK, ΣNK, and $\Sigma\Lambda\pi$, are about 3%, 10%, and 32% of the corresponding normal contributions. It thus appears that the structure contributions to the magnetic moments of the Λ and Σ hyperons are not appreciable except for the case of $\Sigma\Lambda\pi$.

It should be mentioned that the fact that perturbation theory is wrong for $m^2>(2M)^2$ does not prove that cutting off the integral of Eq. (15) at $(2M)^2$ is the correct procedure. It is, however, the most reasonable procedure. Needless to say, it is not possible to estimate the errors involved in leaving out the possible contributions from an infinite number of states, but the error should be small for μ_Λ. Keeping these points in mind, let us examine the main features of our result from our analysis of the magnetic moments.

Measurement of the magnetic moments of hyperons would give insight into the symmetries of the strong interaction which is implied by invariance under Eqs. (7) and (10). In particular, a small observed value ($\mu_\Lambda\approx 0$ to 0.1 nm) would support charge independence of the interaction between Λ and π and between Λ and K mesons and a pseudoscalar K coupling, as indicated in Table I. The interesting point is that although $\mu_\Lambda(\Xi,1)$ and $\mu_\Lambda(N,1)$ are considerably smaller than standard perturbation results, $\mu_\Lambda(1)$ (the difference between these two quantities) is much larger than $\mu_\Lambda(\infty)$ that is obtained from the K-current part of perturbation theory.

One can obtain a $\mu_\Lambda(1)$ as large as a few tenths of a nuclear magneton for pseudoscalar K coupling, if the relative parity of (Λ,N) and that of (Ξ,N) are different. On the other hand, scalar K couplings can give rise to $\mu_\Lambda(1)$ of a few tenths of a nuclear magneton with $g_{HK}^2\approx 3$ to 15.

With the anticipated accuracy of about 0.2 nm[18] in an experimental determination of μ_Λ one cannot, however, provide information on the relative magnitudes of $g_{\Xi K}^2$ and g_{NK}^2 for pseudoscalar K coupling. But for scalar K coupling the $\mu_\Lambda(1)$ would give information on $g_{\Xi K}^2$ and g_{NK}^2 as can be seen from the value of $\mu_\Lambda(1)$ in Table I.

Table III shows that $\mu_\pm(1)$ is about 2 to 3 times $\mu_\pm(\infty)$. Further, there is a distinct difference between the values for pseudoscalar and scalar K couplings, so that information on the coupling as well as charge independence of the π and K interactions can be obtained from the experimental values of μ_+, μ_-, and μ_0.

[18] S. Warshaw (private communication).