Nuclear Interactions and Mean Free Paths of Protons, Neutrons, and Alpha Particles at Energies Around 250 Bev/Nucleon*

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Nuclear interactions of protons, neutrons, α particles, and heavier nuclei of average energy 250 Bev/nuc were studied in nuclear emulsion. The source of these particles were fragmentations of heavy primary nuclei of the cosmic radiation. Their energy was determined from multiple scattering measurements. The interaction mean free path for protons is 41 ± 10 cm, for α particles 27 ± 7 cm. The mean free path shows no significant change compared with measurements at lower energies. The mean number of shower particles $\langle n_s \rangle$ depends appreciably on the mass of the target nucleus. Our best estimate for nucleon-nucleon collisions at 250 Bev is $\langle n_s \rangle = 8.8\pm1.9$. A detailed comparison of the estimate of the primary energy obtained from the angular distribution of shower particles with the true primary energy is carried out. The angular distribution of the shower particles will, in an individual case, give a quite unreliable value for the primary energy. In the average, the angular distribution method will overestimate the true primary energy by a factor of 1.3 for interactions with a number of heavy prongs $N_h \leq 5$. If $N_h > 5$, the angular distribution will underestimate the true energy in the average by a factor of 1.8. The angular distributions can be transformed into a system in which they are symmetric. This is even true for collisions with heavy target nuclei ($N_h > 5$). The results for alpha particle and heavy nucleus collisions are quite similar.

The inelasticity for the proton and neutron interactions shows large fluctuations for individual events. It depends weakly on the number of shower particles and on the mass of the target nucleus. Its mean value is 50%. The mean value for the alpha-particle collisions is 22%.

1. INTRODUCTION

It is, at present, impossible to investigate the interactions and the mean free paths of protons and neutrons in the 250-Bev range using artificially accelerated particles. Some knowledge of these phenomena seems to be highly desirable for theoretical as well as for experimental reasons, such as the proposed construction of new high-energy accelerators and the interpretation of high-energy cosmic-ray data. In this paper an attempt will be made to obtain such information in nuclear emulsions using protons and neutrons emerging from fragmentations of heavy nuclei in the primary cosmic radiation. This method does not depend on energy estimates based merely upon angular distributions of "jets." In all events used in this paper the energy is determined by measurements of the relative multiple Coulomb scattering between particles which is, in general, much more reliable than other methods. The fragmentations of energetic heavy nuclei have been widely investigated during the past decade.[1–3]

It is generally considered to be an evaporation process in the coordinate system of the moving primary heavy nucleus. In such a process the energies of the emitted protons, neutrons, deuterons, tritons, alpha particles, and heavier fragments ($Z>2$) are of the order of 10 Mev/nucleon only. If the energy of the heavy nucleus in the laboratory system is sufficiently high, say >10 Bev/nucleon, the evaporating particles are emitted in this system under very small angles and have approximately the same energy/nucleon. In this paper all these particles will be called "fragments" of the primary nucleus. In many cases several alpha particles or heavier fragments are emitted. Kaplon *et al.*[1] and later Jain *et al.*[4] used such events for their investigations of the energy spectrum of the heavy nuclei in the primary cosmic radiation at high energies (>7 Bev/nucleon). They measured the relative multiple Coulomb scattering between alpha particles and fragments as well as the angular distribution of the alpha particles and fragments. In addition, Jain *et al.*[5] studied the interactions, the meson production, and the mean free path of alpha particles ($\langle E \rangle = 40$ Bev/nucleon) emerging from such fragmentations. Thus, they were able to compare the energy estimates obtained from the angular distribution of the mesons produced in the interactions of the alpha particles with the energies determined by relative scattering measurements.

In this paper such a comparison is extended to energies of the order of 250 Bev/nucleon. In addition, protons and neutrons emerging from fragmentations are included.

2. EXPERIMENTAL PROCEDURE

Most of the events included in this paper were found in a stack of Ilford $G5$ emulsion consisting of 200 pellicles 30×60 cm, 600 μ thick, flown for 13 hr at an altitude of 116 000 ft over Texas. Each pellicle of this stack has been scanned around the edges and along certain grid lines across the pellicle for electromagnetic cascades and for "parallel" alpha particles and heavy

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[4] P. L. Jain, E. Lohrmann, and M. Teucher, Phys. Rev. **115**, 654 (1959).

[5] P. L. Jain, E. Lohrmann, and M. Teucher, Phys. Rev. **115**, 643 (1959).

TABLE I. Primary fragmentations.

Number of event	Charge of primary nucleus	Energy Bev/nucleon	N_h	n_s	N_α	Charge of fragments	Number of interactions by protons	Number of interactions by neutrons	Number of protons scattered
Guam	12	140_{-40}^{+60}	0	13	0	7	3	0	3
3	15	240 ± 60	3	50	2	3	2	4	5
15	22	180_{-60}^{+140}	10	50	3	8	2	3	a
17	22	80 ± 25	5	28	2	7; 5	3	0	3
28	6	160 ± 50	0	8	2	...	0	0	2
30	7	320 ± 80	0	7	2	...	0	0	2(+1)[b]
46	10	360 ± 100	17	89	3	...	0	0	2
48	6	70 ± 20	0	5	2	...	2	0	3(+1)[b]
50	7	165 ± 50	7	7	2	...	0	0	1
51	20	76 ± 16	0	16	6	3	2	1	6
52	7	330_{-80}^{+150}	0	45	1	...	1	0	6
54	23	125 ± 40	1	15	1	16	6	6	a
58	12	100 ± 25	0	12	4	...	1	0	3
69	15	180 ± 50	0	36	2	...	4	3	a
74	14	800 ± 300	5	77	2	3	4	2	a
76	6	90 ± 20	6	10	2	...	2	0	2

[a] In these cases following of individual tracks of minimum ionization was impossible. Therefore, only the volume scan for secondary interactions was done.
[b] The "proton" listed in brackets had to be rejected by the scattering measurements.

fragments ($Z>2$). All cases in which the angular divergence between the "parallel" alpha particles or fragments was $<2\times 10^{-3}$ rad were traced backwards to the primary fragmentation, i.e., to the first nuclear interaction of a heavy nucleus entering the stack. All heavy fragments and alpha particles emerging from such a primary event were then followed along the track until they produced secondary interactions or until they left the stack. In cases where the primary heavy fragments caused secondary interactions, all emerging alpha particles and heavy fragments were treated the same way. In addition, all singly charged particles emerging from primary or subsequent fragmentations were followed along the track, provided their angle θ_i against the line of flight of the heavy nucleus was $\leq 2\theta_{\max}$ in the laboratory system. $\theta_{\max}$ is the largest angle of any alpha particle or heavy fragment emerging from the fragmentation under consideration. This scan was carried out very carefully in order to obtain all interactions including those with zero heavy prongs and only very few particles of minimum ionization.

In a number of cases of favorable geometry and of suitably high energy, the total volume of a cone of half-opening-angle $2\theta_{\max}$ around the line of flight of the heavy nucleus was scanned for secondary nuclear interactions. In cases of charged primaries it was usually very easy to determine if the interaction belonged to the event or not. In cases of neutral primaries five or more minimum tracks were required to form a "jet" parallel to the line of flight of the heavy nucleus. By this procedure it is quite likely that neutrally induced secondary interactions of low n_s and N_h might have been overlooked or excluded by the selection criteria.

The energy of all events was then determined by relative multiple Coulomb scattering measurements. These are described in full detail in Sec. 4. We introduced a low-energy cutoff at 70 Bev/nucleon and rejected all heavy nuclei below that energy. The angular distribution of the shower particles in all interactions has been measured as carefully as possible. We did not attempt any comparison between the scattering measurements and the energy estimates obtained from the angular distribution of alpha particles and heavy fragments in the fragmentations. Our selection criteria at the "pickup" favor events where fragments are emitted under very small angles. Our sample of fragmentations, therefore, cannot be considered unbiased for a study of the angular distribution of the fragments. The same, of course, is true for several other aspects of the fragmentation process. In our sample it is not meaningful to discuss average values of N_h, n_s, N_{alpha}, etc. The details of our selected primary fragmentations are given in Table I.

3. IDENTITY OF THE SINGLY CHARGED PARTICLES

The main purpose of this paper is to obtain some information about the behavior of protons at energies above 100 Bev. Certainly not all minimum-ionization tracks emerging from fragmentations within $2\theta_{\max}$ are protons. We have to consider the possibility of mesons produced in the fragmentation. At a primary energy E (in Bev) the median angle of the produced mesons is of the order of $\theta_{\frac{1}{2}}\approx 1.4\times E^{-\frac{1}{2}}$, whereas the most probable value of the protons emerging from the fragmentation of a heavy nucleus of energy E (in Bev/nucleon) is of the order of $\langle\theta^2\rangle^{\frac{1}{2}}\approx 0.1\times E^{-1}$. Therefore, the number of mesons in the specified angular interval $2\theta_{\max}\approx 2\langle\theta^2\rangle^{\frac{1}{2}}$ can be expected to be small. In order to be safe, we measured the energy of all the minimum ionization tracks within $2\theta_{\max}$ by relative Coulomb scattering measurements against alpha particles and heavy fragments. This resulted in the rejection of 2 tracks out of 40.

More difficult is the problem of deuterons and tritons among the minimum ionization tracks. It is well known that at low energies these particles constitute a very appreciable fraction of the singly charged particles emitted in nuclear evaporation processes. No detailed studies have been carried out for high-energy fragmentation processes. We tried to obtain at least some very rough estimate of this fraction by the observation of the neutral secondary interactions in the "volume scan" described above. For this purpose we used events Nos. 15, 54, 69, 74, Guam, 3, 52. Within the same volume we observed 22 secondary interactions caused by singly charged primaries and 18 caused by neutral primaries. From this we conclude that the fraction of deuterons and tritons among the interactions produced by singly charged particles is probably not more than 30%. Much better statistics would be necessary to obtain more reliable results.

4. ENERGY MEASUREMENTS OF THE PRIMARY FRAGMENTS

Most of the interactions of the heavy primary nuclei considered by us have two or more fragments. Their energy/nucleon agrees with the energy/nucleon of the primary heavy nucleus within about 10%. It is therefore possible to determine the energy of the primary nucleus by making relative multiple scattering measurements between the tracks of the fragments. Because this is a standard procedure, only some special questions connected with this work shall be discussed.

The high energies which have to be measured require the use of a large cell length. The smallest cell length used in this requirement was 1 cm. In most cases longer cells are required, the longest being 8 cm. The total track length which is available in the stack is in general not sufficient to permit a statistically meaningful measurement on a single pair of particles. However, in all events more than two fragments were available (if protons and deuterons are included, which will be discussed later). Then the scattering between combinations of fragments can be measured. The results of all statistically independent combinations can be combined, because the energy spread between the individual fragments is smaller than the error of the scattering measurement.

The ordinary noise which is encountered in conventional scattering measurements with shorter cells can be neglected compared to the following three kinds of noise which existed in this experiment: (1) noise produced by small random fluctuations of the cell length; (2) effects of large scale distortions; and (3) spurious scattering.

The influence of (1) is small compared to (2) and (3), because the angle between the fragments is very small. (2) and (3) are mainly produced by the large separation between the tracks. Their influence was reduced as much as possible by avoiding combinations of tracks with large relative separations. The largest separation which was allowed was about 100 μ. In addition, no measurements were made closer than 2 cm from the edge of the stack.

Numerical values for the noise vary greatly, depending on the position in the stack and on the geometry of the event.

Two independent methods were applied for measuring and eliminating the noise. The first one is the standard method of using the different cell length dependence of noise and of multiple Coulomb scattering. The three contributions to noise mentioned above are expected to be independent of cell length. This is true for contribution (1) and (2) if the measuring errors of the cell length and the effects of distortion in two different plates are not correlated. Contrary to ordinary scattering measurements, (3) is also independent of cell length, because it depends on the average separation between the tracks. This average separation is cell length independent, because the same total track length is always used for the measurements.

Experimentally, measurements of the events having the highest energies with short (1-cm) cells showed that the noise depended only very slightly on the cell length. It was therefore possible to eliminate the noise by the standard method, using two times and four times the basic cell length.

The second method of noise elimination can be used if more than two fragments are available. Then measurements on pairs of fragments with different separations can be compared. The contribution due to Coulomb scattering is independent of the separation. The contribution to the noise due to (1) and (3) is proportional to the separation. (2) is expected to increase also at least proportional to the separation, particularly with the separation in the direction normal to the plane of the emulsion.

By comparing the results of measurements as a function of the separation, a reliable estimate of the Coulomb scattering is possible by extrapolating the measurements to zero separation.

In many cases a check can be made by applying both methods of noise elimination. One gets then the same values for the primary energy.

Special considerations are required for the singly charged particles, which are emitted under similar angles as the fragments. Most of these particles are expected to be protons or deuterons from the primary heavy nucleus. This is substantiated by our measurements. Relative scattering measurements were made on all of those singly charged particles which lay within the very small angle specified in Sec. 3 and which could be followed along the track. In some cases only a lower limit for the energy could be obtained. Forty-two singly charged particles were measured. This number of particles is larger than the one following from Table I, because it includes particles originating under larger angles and coming from interactions of the fragments. On two tracks only a lower limit of about 30% of the primary energy could be established. These two tracks were left off.

Because the contamination of π mesons is so small, as is expected, scattering measurements made on singly charged particles can be used in the same way as the heavier fragments to obtain a better estimate of the primary energy. Only tracks emitted under very small angles ($\leqq$the angles of the heavier fragments) were used for this purpose. In particular, this method was used for the two events (Guam and No. 52) in which only one heavier fragment was available. In evaluating the scattering data to find E_0, the singly charged particles were assumed to be protons having the same energy/nucleon as the primary nucleus. A deuteron contamination would lead to a systematic overestimate of the primary energy. Our data indicate that about 20% of singly charged fragments are deuterons. This would result in an overestimate of the average energy of our events by approximately 6%. Because this effect is small and the deuteron contamination is somewhat uncertain, this correction was not applied.

Scattering Measurements on Shower Particles

Scattering measurements were made on the shower particles emerging from the secondary proton and neutron collisions, if the geometry of the event was favorable. This was the case for 8 events. As the energy of these events is <1000 Bev, mainly particles in the backward cone could be measured. Noise elimination was carried out by measuring the scattering of tracks of known high energy very close to the interaction. Under the conditions of this experiment such tracks can always be found. The total amount of noise (including spurious scattering and effects of distortion) is found in this way. For all energy measurements a ratio signal/noise>2 was required, otherwise only a lower limit for the energy of the track was claimed. The results of these measurements will be discussed in another section.

5. MEAN FREE PATHS

A total of 38 tracks of minimum ionization which fulfilled the angular criterion and were not rejected by scattering measurements were followed through the stack. 16 nuclear interactions were found. The observed mean free path is

$$\mathrm{MFP}_p = (41 \pm 10)\ \mathrm{cm},$$

in nuclear emulsion. The average energy of the particles followed was 170 Bev. Our value is in agreement with results obtained with artificially accelerated protons at 6.2 Bev.[6]

There seems to be no indication that the mean free path for nuclear interactions of protons decreases at higher energies. One has always to keep in mind that our method yields only a lower limit for the mean free path of protons as long as the fraction of deuterons and tritons among the singly charged particles is not known with sufficient accuracy. As we have shown in Sec. 3, this fraction is probably not higher than 30%.

The sample of protons used in this work is unbiased for the measurement of the interaction mean free path, because in the selection of events at the pickup no minimum tracks between the alpha particles or fragments were required.

Alpha Particles

All alpha particles (or more accurately, all tracks of particles) of four times minimum ionization have been followed from primary or higher order fragmentations through the stack. For the calculation of the mean free path only the track length "below the pickup" of the event could be used because only this track length leads to an unbiased estimate of the mean free path. In this way, 427 cm of "acceptable" track length were followed which yielded 16 interactions and a mean free path of

$$\mathrm{MFP}_{\mathrm{alpha}} = (27 \pm 7)\ \mathrm{cm},$$

in nuclear emulsion. This again is in agreement with results obtained at lower energies.[2] Reference 2 contains also a bibliography of other measurements.

There is no indication of a drastic change of the mean free path with energy. The average energy of the alpha particles reported in this paper was 166 Bev/nucleon.

At present, we are not in a position to estimate the fraction of so-called alpha particles which are not He^4. But this fraction is generally considered to be small.

6. INTERACTIONS PRODUCED BY PROTONS AND NEUTRONS

General Remarks

In this section 52 interactions produced by singly charged and neutral particles will be described, which lie within the angle specified in Sec. 3. Most of the singly charged particles are protons with an admixture of about 20% deuterons and tritons. The correction of our data for the deuteron contamination will be discussed further below. In what follows, the singly charged particles will be denoted briefly as "protons."

By arguments very similar to those used for the singly charged particles, it is clear that most of the neutral interactions were produced by neutrons of approximately the same energy as the protons.

Among the 32 proton interactions, 16 were found in an unbiased way, by following the protons along the track. The rest of the events were found by a volume scan. A comparison of the two groups of events will be given below. These are no significant differences. Therefore, no serious effects due to scanning bias should exist. This is furthermore substantiated by the fact that three

[6] W. Winzeler, B. Klaiber, W. Koch, M. Nikolic, and M. Schneeberger, Nuovo cimento **17**, 8 (1960).

TABLE II. Characteristics of proton and neutron collisions.

	Protons $N_h \leq 5$	Protons $N_h > 5$	All p	Along the track	Neutrons $N_h \leq 5$	Neutrons $N_h > 5$	All n
No. of events	18	14	32	16	10	10	20
$\langle n_s \rangle$	9.0 ±2.1	17.1 ±4.6	12.5 ±2.2	12.0 ±3.0	11.6 ±3.7	15.6 ±4.9	13.6 ±3.0
$\langle N_h \rangle$	2.7 ±0.6	13.7 ±3.6	7.6 ±1.3	7.0 ±1.7	3.0 ±1.7	17.0 ±5.4	10.0 ±2.2
$\langle N_m \rangle$ [a]	7.1 ±2.0	14.5 ±4.4	10.3 ±2.1	9.9 ±2.9	10.6 ±3.7	14.6 ±4.9	12.6 ±3.0
$\langle E_0 \rangle$			240 Bev				275 Bev
$\langle \eta \rangle$ [b]	0.49±0.12	0.47±0.14	0.48±0.09	0.55±0.15	0.48±0.15	0.50±0.16	0.49±0.11

[a] N_m: number of charged particles produced.
[b] η: inelasticity.

of the events found in the area scan have no heavy prongs.

The following abbreviations will be used: N_h: number of grey and black prongs (ionization>1.4 minimum ionization); n_s: number of shower particles (ionization <1.4 minimum ionization); θ_i: angle of the ith shower particle with respect to the line of flight of the primary particle in the laboratory system.

7. MULTIPLICITIES OF THE EVENTS

All information regarding N_h and n_s is contained in Fig. 1. Each point represents one event. Both proton- and neutron-induced interactions are included. The distribution functions of N_h and n_s can easily be obtained from Fig. 1. For N_h and $n_s<20$ the integral track number distributions are approximately exponential. The positive correlation between N_h and n_s is evident. It suggests a rather strong influence of the target nucleus on the multiplicity.

For further analysis the interactions were divided into two groups according to N_h. The first group with $N_h \leq 5$ includes the interactions with free protons, and with the light nuclei (C,N,O) in emulsion. Depending on the elementary nucleon-nucleon cross section, an estimated 30%–50% of the interactions of this group are produced on a heavy target nucleus. The second group ($N_h>5$) contains predominantly interactions with the heavy elements (Ag and Br). The average multiplicities $\langle n_s \rangle$ and $\langle N_h \rangle$ are given in Table II. $\langle n_s \rangle$ is seen to depend strongly on N_h. The exact dependence of $\langle n_s \rangle$ on the mass of the target nucleus cannot easily be obtained from our data, since in the group with $N_h \leq 5$ no clear-cut separation between the interactions on light and on heavy target nuclei seems possible.

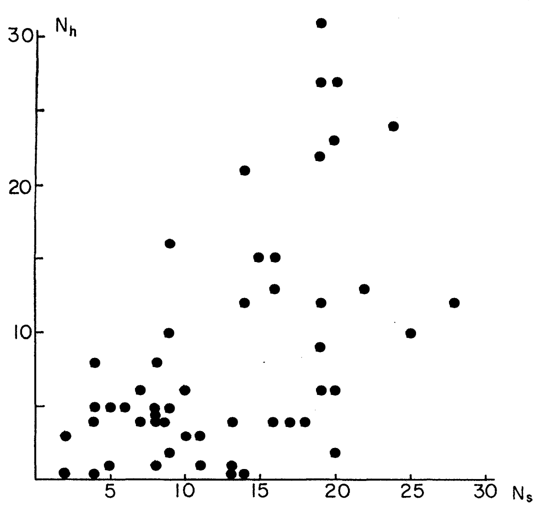

FIG. 1. Distribution of N_h and n_s for proton and neutron induced events.

As will be pointed out in more detail, also the events with $N_h \leq 5$ show in the average a significantly different behavior from what can be expected for nucleon-nucleon collisions. Thus, the average multiplicity $\langle n_s \rangle$ of this group is larger than for a nucleon-nucleon collision. In order to estimate the number of charged mesons produced, the contribution to the number of shower particles due to nucleons was subtracted. This can only be done in an approximate way. In the average two tracks were subtracted for the proton collisions and one track for the neutron collisions. The average number of charged particles created $\langle N_m \rangle$ is shown in Table II.

The dependence of the average multiplicities $\langle N_h \rangle$ and $\langle n_s \rangle$ on the primary energy E_0 can be established by comparison with work done in nuclear emulsion at different energies.[6–12] These figures have been compiled in Table III. The average number of heavy prongs $\langle N_h \rangle$ does not depend on the primary energy between 6 Bev and 4000 Bev within statistics.

The dependence of $\langle n_s \rangle$ on energy is shown in Fig. 2. Within the energy interval shown, an approximate fit to the experimental data of the form,

$$\langle n_s \rangle \propto E^{0.37}, \tag{1}$$

can be obtained.

[7] L. F. Hansen and W. B. Fretter, Phys. Rev. **118**, 812 (1960).
[8] R. M. Kalbach, J. J. Lord, and C. H. Tsao, Phys. Rev. **113**, 325 (1959).
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[12] N. A. Dobrotin and S. A. Slavatinsky, Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

TABLE III. Average multiplicities in emulsion as a function of energy.

$\langle n_s\rangle$ \ Primary energy (Bev)	6.2	9[a]	100[b]	250[c]	1000[b]	3500[d]
p-p collisions	2.8 $\pm$0.4[e, f]	3.24$\pm$0.08	4.6$\pm$0.2	8.8$\pm$1.9 8 $\pm$1[g]	6.3$\pm$0.9	15 $\pm$5
Emulsion:						
$N_h\leq5$		3.0 $\pm$0.2		10.0$\pm$1.9		20.0$\pm$3.5
$N_h>5$		3.5 $\pm$0.3		16.5$\pm$3.4		28.0$\pm$6.5
$N_h\geq0$	2.65$\pm$0.1[f, h]	3.2 $\pm$0.2		12.9$\pm$1.8		22.5$\pm$3.0
$\langle N_h\rangle$	8.9 $\pm$0.4[f, h]	7.8 $\pm$0.7		8.5$\pm$1.2		6.5$\pm$0.9

[a] See reference 10. [b] See reference 7, calculated by us. [c] This work. [d] See reference 11.
[e] See reference 8. [f] See reference 6. [g] See reference 12. [h] See reference 9.

The best estimates available for the average number of shower particles $\langle n_s\rangle$ in nucleon-nucleon collisions are also included.

Our value at 250 Bev was obtained by selecting from interactions with $N_h\leq5$ only those whose angular distribution was compatible with a nucleon-nucleon collision (for details see Sec. 11).

Our value of $\langle n_s\rangle$ is somewhat higher than the one obtained by Hansen and Fretter[7] in a cloud-chamber measurement. However, it is in good agreement with recent cloud-chamber measurements in LiH by Dobrotin and Slavatinsky.[12]

The energy dependence of $\langle n_s\rangle$ for nucleon-nucleon collisions is weaker than given by Eq. (1). It is in agreement with an assumed $E^{\frac{1}{4}}$ dependence. It appears that the dependence of $\langle n_s\rangle$ on the mass of the target nucleus is stronger at high energies than at low energies.

A model to describe nucleon-nucleus collisions, which can be compared with these results, was proposed by Landau and Belenki[13] and by Milekhin.[14] They assume that the collision takes place with a cylindrical section of the target nucleus, and that all the nucleons in this column act coherently to form a single compound system ("nuclear fluid model"). This model predicts a dependence of the multiplicity of the form

$$\langle n_s\rangle \propto A^{0.19}E_0^{0.25}. \qquad (2)$$

It cannot therefore explain all the data of Fig. 2 because it predicts the same energy dependence for all values of A. The discrepancy at lower energies (6.2 Bev and 9 Bev) is not surprising, since the model is expected to be valid only at extremely high energies. The model is, however, not in contradiction with the experimental data in Fig. 2 at energies >100 Bev. The average multiplicity for collisions with emulsion nuclei can be calculated from Eq. (2), assuming a geometrical cross section. It is higher than the multiplicity of a nucleon-nucleon collision by a factor of 1.6. This is consistent with our present data.

[13] S. S. Belenki and L. D. Landau, Suppl. Nuovo cimento **3**, 15 (1956).

[14] G. A. Milekhin, Zhur. Eksp. i Teoret. Fiz. **35**, 1185 (1958) [translation: Soviet Phys.-JETP **35**(8), 829 (1959)].

8. TRANSVERSE MOMENTUM OF THE SHOWER PARTICLES

The transverse momentum p_t of a shower particle is defined by

$$p_t=p_i \sin\theta_i, \qquad (3)$$

where p_i is the momentum of the particle in the laboratory system.

Transverse momenta of shower particles were determined for 8 geometrically favorable interactions by scattering measurements. These measurements have been described in detail above. Even when measuring favorable events, only lower limits of the energy can be obtained for some of the shower particles. If the proportion of such particles is large, the value of the information obtained will be greatly reduced. Therefore, only tracks were considered which satisfied the following criterion:

$$l \sin\theta_i > 1000\mu. \qquad (4)$$

Here, l is the track length available for scattering measurements.

For three particles only lower limits for p_t could be obtained. To estimate the influence of these particles, the mean value of $p_t(\langle p_t\rangle)$ was plotted for several values of the acceptance parameter $l \sin\theta_i$ against the relative number of tracks for which only lower limits were obtained, assuming the lower limits to represent the true energy. Extrapolation of this plot leads to the following

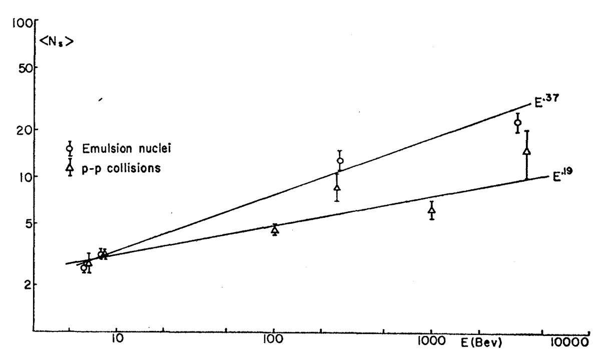

FIG. 2. Dependence of the average number of shower particles $\langle n_s\rangle$ on primary energy.

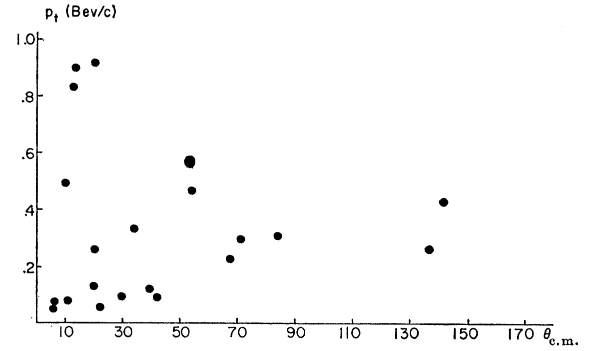

FIG. 3. Correlation between transverse momentum p_t and angle of emission $\theta_{c.m.}$ for shower particles emitted from proton and neutron collisions.

best estimate for $\langle p_t \rangle$:

$$\langle p_t \rangle = 370 \pm 85 \text{ Mev}/c.$$

This result is in good agreement with several other measurements at primary energies ranging from 9 Bev to several 1000 Bev.[7,12,15] This shows that $\langle p_t \rangle$ does not depend on the primary energy within the accuracy of the measurements.

The acceptance criterion Eq. (4) will influence these results in a systematic way, if there is a correlation between p_t and θ_i. Figure 3 shows p_t plotted against $\theta_{c.m.}$. $\theta_{c.m.}$ is the angle in the center-of-mass system of a nucleon-nucleon collision.

The distribution of p_t can be easily obtained from Fig. 3. It is in good agreement with other published results.

Within the rather limited statistics no strong correlation between p_t and either θ_i or $\theta_{c.m.}$ can be seen. Also, no dependence of $\langle p_t \rangle$ on the number of black and grey tracks associated with the interaction, i.e., on the target nucleus, could be found. This is further substantiated by the agreement of other measurements on interactions in emulsion, carbon, and LiH.[7,12,15]

9. INELASTICITY

In a collision between a high-energy nucleon and a nucleus, only a fraction of the energy of the incident nucleon is used for the production of mesons and other particles. This fraction is called inelasticity. In the laboratory system it is defined by

$$\eta = E_t / E_0. \tag{5}$$

E_t is the sum of the energies in the laboratory system of all particles created in the collision. E_0 is the kinetic energy of the primary particle also in the laboratory system. Frequently, the inelasticity is also discussed in the center-of-mass system. For the high energies considered here, the difference in the numerical values of η as defined in the two different systems is about 10%. The definition, Eq. (5), was chosen here because in composite collisions the question of the center-of-mass system requires further consideration.

If the transverse momentum p_t is independent of θ_i, the total energy of the created particles can be calculated from the angular distribution of the shower particles:

$$E_{\text{tot}} = 1.5 \sum \left[m_\pi^2 + (\langle p_t \rangle / \sin\theta_i)^2 \right]^{\frac{1}{2}}. \tag{6}$$

Measurements made on high-energy interactions, and also on the interactions described above, are, at present, not in contradiction with this assumption. Moreover, as was pointed out above, $\langle p_t \rangle$ seems not to depend on the primary energy or the target nucleus within the statistical accuracy of present experiments. Therefore, the same value of $\langle p_t \rangle$ can be used for all events. Combining the results of all published measurements[7,12,15] of $\langle p_t \rangle$, a better value of $\langle p_t \rangle = 330$ Mev/c was adopted for the calculations. In Eq. (6) it is furthermore assumed that all shower particles are π mesons. It neglects the influence of nucleons and strange particles among the shower particles. Measurements at this and at higher energies show that the proportion of these particles is only about 20%. Their influence on Eq. (6) was therefore neglected.

The inelasticity η was calculated for all events using Eqs. (5) and (6). The primary energy E_0 was taken from the scattering measurements. This is a considerable advantage compared to other methods for finding E_0, because our method is quite accurate and independent of the angular distribution and other characteristics of the interaction.

Two corrections were applied to η. First, the influence of nucleons among the shower particles has to be taken into account. In collisions with a small value of η, a large part of the primary energy is retained by a single particle, which continues under a very small angle because of its high energy. It is reasonable to assume that this is the primary nucleon. This track has consequently to be left off in the sum equation (6). Assuming a 50% probability for a charge exchange of the incident nucleon in the collision, the continuing nucleon is expected to be a proton in $\frac{1}{2}$ of the events. This agrees with our observation, that about 50% of the events have one track under a particularly small angle. Therefore, the track with the smallest angle θ_i was left off when calculating E_{tot} in 50% of the events, which were selected from the group having one track under a very small angle. The procedure of always omitting the track with the smallest angle might lead to a slight underestimate of η. This will be counterbalanced by the presence of more nucleons among the shower particles, for which no correction was applied.

A second correction to η has to be applied, because some of the interactions may have been produced by deuterons and not by protons. Our data indicate that

[15] M. Schein, D. M. Haskin, E. Lohrmann, and M. W. Teucher, Phys. Rev. 116, 1238 (1959); this paper contains also more references.

the percentage of deuterons among the singly charged primary particles is probably smaller than 30%. For the correction 20% was assumed. As will be shown below from a comparison of interactions of protons and alpha particles having the same energy/nucleon, the energy dissipation for the alpha particles is only increased by a factor of 2, i.e., it increases more slowly than the mass number. It was therefore assumed that the energy dissipation by a deuteron is increased by a factor of 1.5. This results in an 8% correction to η. Furthermore, a corresponding number of additional tracks under very small angles were left off from the sum equation (6) to allow for the increased number of nucleons contained in the primary particle.

The results corrected for these two effects are shown in Table II. They should be compared with the work of Dobrotin and Slavatinsky,[12] who obtained $\langle\eta\rangle=0.30$ for collisions of 300-Bev protons and neutrons with LiH. This indicates a very slow increase of $\langle\eta\rangle$ with the mass of the target nucleus, especially if Dobrotin's value is compared with our value for the collisions with heavy nuclei ($N_h>5$).

Our data show that $\langle\eta\rangle$ is independent of N_h whereas a small increase with N_h would be expected. Apart from the possibility of statistical fluctuations, this could be explained, if the transverse momentum for events with $N_h\leq 5$ and $N_h>5$ would be slightly different.

The distribution of η obtained from the individual collisions is shown in Fig. 4. The events with $N_h\leq 5$ are cross-hatched. They appear to have the same distribution. It should be noted that events with very small inelasticity do occur, a fact which cannot be well explained by most models of multiple meson production. The events with $\eta>1$ can be attributed to the presence of deuterons among the primary particles and to statistical fluctuations. Figure 5 shows the correlation between η and n_s. Apart from the existence of a group of events with very small $n_s(\leq 5)$ and small η, no strong correlation between the inelasticity η and the multiplicity n_s is found.

10. DISCUSSION OF ENERGY ESTIMATES FROM THE ANGULAR DISTRIBUTION

In very many investigations of high-energy nuclear interactions, the angular distribution of the emitted shower particles provides the only information about the primary energy. Several of the assumptions underlying the use of the angular distribution to determine the primary energy are, however, highly questionable. A direct check of this method appeared therefore desirable. Such a check is possible, because in this work the primary energy is known from the scattering measurements.

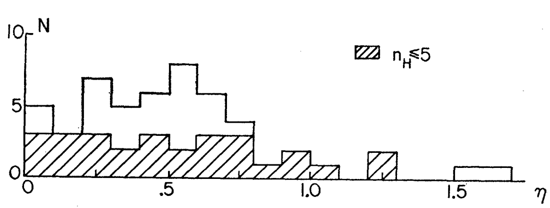

FIG. 4. Frequency distribution of the inelasticity η for p and n collisions.

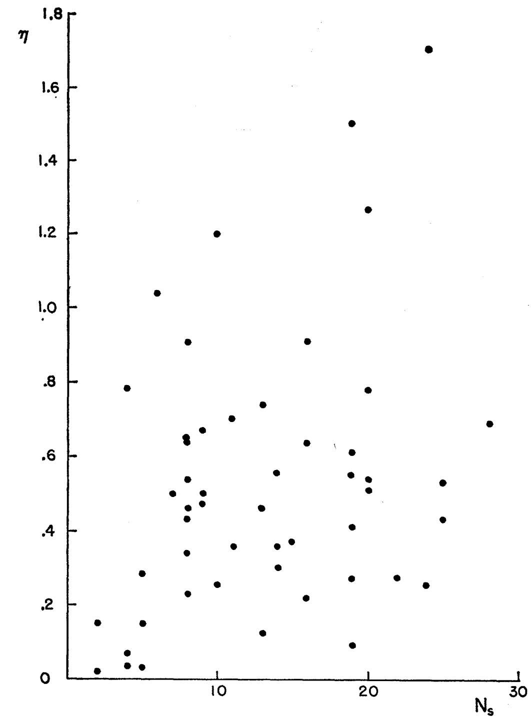

FIG. 5. Correlation between inelasticity η and n_s.

Among the many methods proposed to find the primary energy from the angular distribution, the one introduced by Castagnoli *et al.*[16] was chosen because this method was widely used in the past and is easy to apply. A detailed discussion of this and of other methods was given in an earlier paper.[5,15]

According to Castagnoli *et al.*,[16] the best estimate for the primary energy of an interaction is given by

$$\log_{10}\gamma = -\frac{1}{n}\sum_{i=1}^{n}\log_{10}\tan\theta_i, \tag{7}$$

where γ is connected with the primary energy E_c in the laboratory system by

$$E_c = Mc^2(2\gamma^2-1). \tag{8}$$

E_c will, in general, be different from the actual primary energy E_0. The value of E_c was found from the angular distribution of the shower particles of all events with $n_s>5$. For the comparison of E_c and E_0, the energy found from the scattering measurements can be substituted for E_0. This will not influence our conclusions

[16] C. Castagnoli, G. Cortini, D. Moreno, C. Franzinetti, and A. Manfredini, Nuovo cimento 10, 1539 (1953).

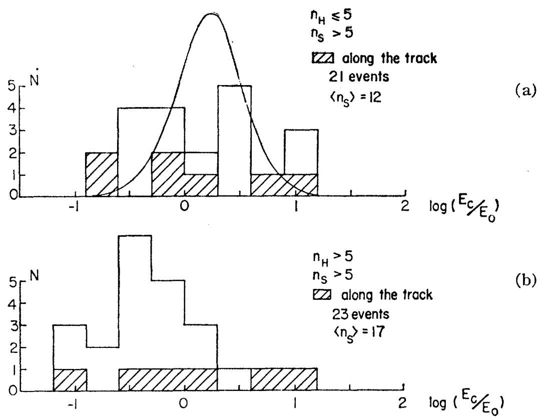

FIG. 6. Distribution of $\log_{10}(E_c/E_0)$. E_c: estimate of primary energy from angular distribution of shower particles according to Eqs. (7) and (8). E_0: true energy. (a) Events with $N_h \leq 5$; (b) events with $N_h > 5$. The smooth curve is result of Monte Carlo calculation

as the accuracy of the scattering measurements is much better than the accuracy of the method using the angular distribution.

In Figs. 6(a) and 6(b) the distribution of $\log_{10}(E_c/E_0)$ is plotted for the two groups of events with $N_h \leq 5$ and $N_h > 5$. Events found along the track are cross-hatched.

It is evident from Figs. 6(a) and 6(b) that for a single interaction the angular distribution of the shower particles gives a very unreliable value for the primary energy. It can be over as well as underestimated by more than a factor of 10.

The extreme fluctuations appearing in the determination of E_c can be attributed, apart from statistical fluctuations, to the breakdown of the following two assumptions which are necessary for deriving Eqs. (7) and (8): (i) The collision is a nucleon-nucleon collision. It has already been pointed out that this assumption is not justified for most of the collisions. (ii) $\beta_m = \beta$. β_m is the velocity of the mesons in the center-of-mass system and β is the velocity of the center-of-mass system. Data on multiple meson production show that this assumption is also violated. In this case, the primary energy will in the average be overestimated if one uses Eqs. (7) and (8) (compare reference 5).

It is important to know if the widespread of the distributions of Figs. 6(a) and 6(b) can be explained by statistical fluctuations and by the breakdown of assumptions (i) and (ii) only, or if still another physical process is involved. We have, therefore, calculated the distributions of $\log_{10}(E_c/E_0)$ to be expected in the case of a nucleon-nucleon collision, using a Monte Carlo method. In the limit of very many shower particles the distribution of $\log_{10}(E_c/E_0)$ should go towards a normal distribution.

A first calculation was carried out for the strongly simplified model of isotropic emission of the shower particles in the center-of-mass system and with assumption (ii). The distribution of $\log_{10}(E_c/E_0)$ was in good

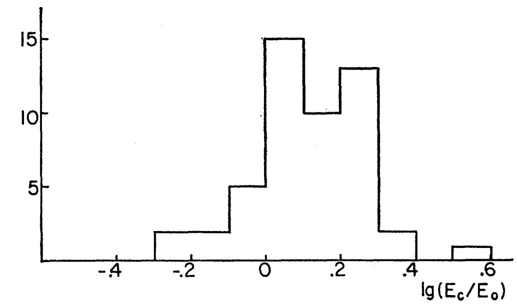

FIG. 7. Distribution of $\log_{10}(E_c/E_0)$ according to Monte Carlo calculation.

agreement with a normal distribution for $n_s = 8$ shower particles. The rms value of the distribution was

$$\langle[\log_{10}(E_c/E_0)]^2\rangle^{\frac{1}{2}} = 0.72/n_s^{\frac{1}{2}}. \qquad (9)$$

For the actual comparison with experiment, a second Monte Carlo calculation was carried out, using the experimental data about meson production. Meson showers were constructed in the center-of-mass system. The number of shower particles was chosen to be 12. The angle and the transverse momentum of each shower particle was chosen by a random process from the experimentally determined distributions. The angular distribution was taken to be symmetric in the center-of-mass sytem and somewhat more peaked towards 0^0 and $180°$ than a $1/\sin\theta$ distribution. This is the angular distribution derived from our events with $N_h \leq 5$. For the distribution of p_t a distribution averaged over various published data was used.

Values for the angle and for p_t can only be chosen by a random process, if they are independent. Actually, there exists a correlation between these parameters for different tracks because of energy and momentum balance in the center-of-mass system. A calculation taking into account this correlation accurately appears not to be feasible. Therefore, some simplifications were introduced. For the small value of n_s which was used, only momentum conservation has to be considered. The conservation of p_t can be taken into account by properly adjusting the azimuthal angles. This does not affect the results. Balance of the longitudinal momentum p_l was established in the following way: p_l was found from the randomly chosen values of the angle and p_t for 11 of the 12 tracks. For the 12th track p_l was chosen in such a way as to make

$$\sum p_l = 0. \qquad (10)$$

Equation (10) together with a randomly chosen value of p_t determines the angle of the last track. Since Eq. (7) is only applied to the charged particles, 35% of the tracks were omitted by random process, assuming them to be neutral particles. The average multiplicity of charged particles was, therefore, $n_s = 7.8$. These tracks were transformed into the laboratory system, using the exact transformation equations and assuming a primary energy of 200 Bev. Altogether, 50 events were calcu-

TABLE IV. Characteristics of the angular distribution of the shower particles.

	Protons and neutrons			Alpha particles			
	$N_h\leq 5$ $n_s<10$	$N_h\leq 5$ $n_s\geq 10$	$N_h>5$	$N_h\leq 5$	$N_h>5$	Primary fragmentations	Fragments
$\langle x\rangle$	−0.10	0.05	0.15	−0.03	0.09	−0.17	0.11
σ	0.67	0.60	0.59	0.56	0.53	0.62	0.51
E_c'/E_0	1.6	0.80	0.50	1.2	0.65	2.2	0.60
	$N_h\leq 5$		$N_h>5$				
No. of events	21		23	9	10	16	9
$\langle\log_{10}(E_c/E_0)\rangle$	0.125		−0.25	0.09	−0.17	0.30	−0.17
$10^{\langle\log_{10}(E_c/E_0)\rangle}$	1.34		0.56	1.2	0.7	2.0	0.7

lated. The Castagnoli energy E_c was determined for these events using Eqs. (7) and (8). The resulting distribution of $\log_{10}(E_c/E_0)$ is shown in Fig. 7. The average is

$$\langle\log_{10}(E_c/E_0)\rangle=0.22. \qquad (11)$$

This means that Eqs. (7) and (8) overestimate the true primary in the average by a factor of 1.7 in the case of a nucleon-nucleon collision.

The rms value around the average is

$$\langle(\log_{10}(E_c/E_0)-0.22)^2\rangle^{\frac{1}{2}}=0.86/n_s^{\frac{1}{2}}. \qquad (12)$$

This is quite similar to the value found for the simplified calculation [Eq. (9)]. It indicates that the details of the Monte Carlo calculation do not influence the rms value strongly.

The results of the calculation show that for a nucleon-nucleon collision Eq. (8) should be substituted by

$$E_c=Mc^2(2\gamma^2-1)/1.7. \qquad (13)$$

The expected fluctuations of the best energy estimate based on Eq. (13) around the true energy are then given by Eq. (12).

The comparison of the Monte Carlo calculation with our experiments is given in Fig. 6(a) and Table IV. Table IV shows the mean value $\langle\log_{10}(E_c/E_0)\rangle$ as found experimentally for the collisions with $N_h\leq 5$ and $N_h>5$. It is smaller than the theoretical value. This can be attributed to the influence of secondary collisions inside the same nucleus. For the events with $N_h\leq 5$ the true energy is in the average overestimated by a factor of 1.3 by Eqs. (7) and (8), to be compared with the theoretical value of 1.7.

For the events with $N_h>5$, the effect resulting from hitting a heavy target nucleus is much more pronounced. In the average, for this group of events Eqs. (7) and (8) will underestimate the true energy by a factor of 1.8.

The result of the Monte Carlo calculation has been included in Fig. 6(a). In order to get a valid comparison with experiment, also the uncertainties of the scattering measurements and the energy spread of the beam of primary protons and neutrons (30% each) were folded into the distribution. The experimental distribution is much wider than the theoretical one. This shows again the influence of the heavier target nuclei. For practical purposes, Eq. (12) underestimates, therefore, the limits of error; they have to be estimated from the experimental distributions.

There is a tail in the experimental distribution at small values of E_c/E_0. It suggests, that also for the events with $N_h\leq 5$ secondary collisions can strongly influence the characteristics of the interactions. There appears to be also a slight excess of events with large values of E_c/E_0 compared to theory. This effect cannot be produced by secondary interactions. It will be necessary to improve the statistical accuracy of this result, in order to establish this effect with certainty. It would be explained by assuming that the collision of the primary nucleon (core) occurred with a virtual pion from the target nucleus. An indication of such an effect has also been reported by Friedlander[17] for 9-Bev proton collisions.

Recently, Dobrotin and Slavatinski[12] have observed large deviations of the angular distribution of 300-Bev p and n collisions with LiH from a symmetrical shape, which they have also interpreted in terms of collisions of a nucleon core with a virtual pion.

11. ANGULAR DISTRIBUTION OF SHOWER PARTICLES

The differential angular distribution of the shower particles in the laboratory system is given in Figs. 8(a), (b) and Fig. 9. It is shown for 3 groups of collisions separately according to the number N_h and n_s of heavily and lightly ionizing tracks of the primary event. The angular distribution is plotted in terms of the variable,

$$x=\log_{10}\tan\theta_i+\tfrac{1}{2}\log_{10}E_0/(2Mc^2). \qquad (14)$$

The second term is added to normalize all events to the same primary energy.

Within the statistical limits of error, all three distributions are symmetric. Under assumption (ii) this means that a system can be found, in which the angular distribution is symmetric with respect to a plane perpendicular to the line of flight ("center-of-momentum

[17] E. M. Friedlander, Nuovo cimento 14, 796 (1959).

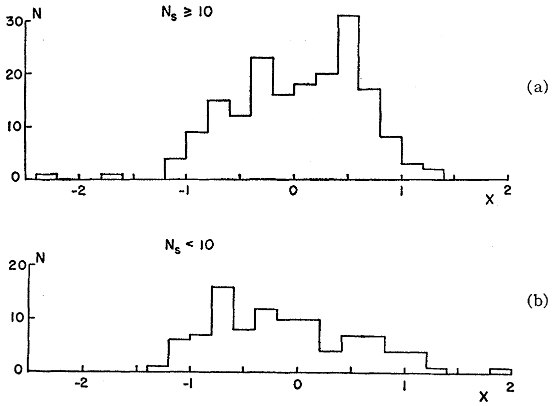

FIG. 8. Angular distribution of shower particles normalized to the same primary energy, as given by Eq. (14). (a) p and n interactions with $N_h \leq 5$, $N_s \geq 10$; (b) p and n interactions with $N_h \leq 5$, $N_s < 10$.

system," c.m. system). As was already pointed out, assumption (ii) is not completely satisfied.

A correction for this was introduced in an approximate and statistical way. The transverse momentum of all tracks was assumed to be the same. The best value of p_t was obtained by averaging over the distribution of p_t with a weight function p_t^{-2}, since the biggest influence comes from the particles which have a small value of p_t. With this procedure, all angular distributions were transformed into a system in which the distribution should be symmetric (c.m. system; angle $\theta_{\text{c.m.}}$). The Lorentz factor of this system is given by Eq. (13). A χ^2 test[18] for symmetry was performed in the $\log_{10} \tan\theta_{\text{c.m.}}/2$ and $\theta_{\text{c.m.}}$ coordinate. Table V shows the result. Given is the probability of obtaining the observed deviations by a statistical fluctuation. In Figs. 10(a), (b); 11(a), (b); 12(a), (b); distributions are presented in the $\theta_{\text{c.m.}}$ coordinate. The results for the alpha-particle and heavy nucleus interactions are also included. The distributions are all compatible with symmetry, even those from collisions with heavy target nuclei ($N_h > 5$).

The distributions in the c.m. system can be approximated by

$$N(\theta_{\text{c.m.}})d\Omega_{\text{c.m.}} \propto \sin^{-1}\theta_{\text{c.m.}} d\Omega_{\text{c.m.}}, \qquad (15)$$

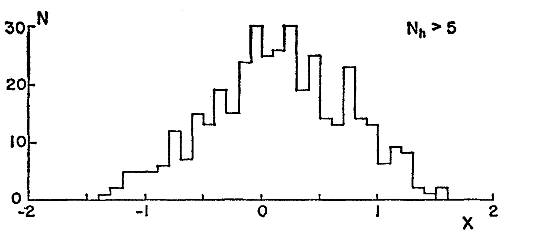

FIG. 9. Angular distribution of shower particles normalized to the same primary energy, as given by Eq. (14). p and n interactions with $N_h > 5$.

[18] A. Hald, *Statistical Theory with Engineering Applications* (John Wiley & Sons, Inc., New York, 1952), Chap. 23, Sec. 4.

for the events with $N_h \leq 5$. Events with $N_h > 5$ have a more isotropic distribution.

The symmetry of the angular distribution in the case of the events with $N_h > 5$ is interesting, since they represent collisions with heavy nuclei. The same effect was found at higher energies.[11,19] This shows that arguments of symmetry are not sufficient to prove that an individual collision can be considered as a nucleon-nucleon collision.

Since we are dealing with composite collisions, there is no *a priori* reason why the angular distribution should by symmetric.

Belenki, Landau, and Milekhin[14] have proposed a specific model of collision between a nucleon and an extended nucleus in which it is assumed that the nucleons in the target act coherently ("nuclear fluid" model). This has been discussed above. Such a model would explain the symmetric angular distribution observed by us. It would also be consistent with other observations as has been discussed and as will be described below. It is, however, not known at present if this model should be used at our relatively low primary energies of 250 Bev.

TABLE V. χ^2 test for symmetry of angular distributions.

		P	No. of tracks
p and n interactions	$N_h \leq 5$	60%	275
	$N_h > 5$	40%	390
Alpha interactions	$N_h \leq 5$	30%	155
	$N_h > 5$	12%	289
Heavy-nucleus interactions	Primary	50%	274
	Fragments	≈1%	392

The mean value $\langle x\rangle = \langle \log_{10}\tan\theta_i\rangle + \frac{1}{2}\langle \log_{10} E_0/(2Mc^2)\rangle$ of the distributions Figs. 8(a), (b), and Fig. 9, is listed in Table IV. Each track is given equal weight. For Table IV there is also listed a quantity E_c'/E_0 defined by $\log_{10}(E_c'/E_0) = -2\langle x\rangle$. E_c' is the value derived for the primary energy from Eqs. (7) and (8) for a group of interactions which have all the same true primary energy E_0, giving the same weight to each track.

The value for (E_c'/E_0) found for the events with $N_h > 5$ is quite similar to the one given in the previous section. The total factor by which the energy is underestimated by the angular distribution is 3.4, if one compares with the theoretical value for a nucleon-nucleon collision. If one accepts the hypothesis that the nucleons of the target nucleus act coherently in the part which is hit in the collision, as was discussed in connection with the symmetry of the angular distributions, the average number of the nucleons taking part in the collision is given by

$$1.7E_c'/E_0 = 3.4. \qquad (16)$$

[19] J. Bartke, P. Ciok, J. Gierula, R. Hołynski, M. Miesowicz, and T. Saniewska, Nuovo cimento **15**, 18 (1960).

This is in agreement with what is to be expected for the collisions with Ag and Br nuclei.

The value of $\langle x\rangle$ for the group of events with $N_h \leq 5$ and $n_s < 10$ agrees quite well with the calculation for a nucleon-nucleon collision. However, the group of collisions with $N_h \leq 5$ and $n_s \geq 10$ shows very markedly the effects of a more complex collision. If one again calculates the mean number of nucleons which would be coherently involved in the collision in the target nucleus, one gets

$$1.7E_c'/E_0 = 2.1. \qquad (17)$$

This is again in reasonable agreement with the number expected for collisions with the light nuclei in emulsion.

The result of Eq. (17) shows that even the interactions with $N_h \leq 5$ will, in general, not represent nucleon-nucleon collisions in a good approximation, in particular if n_s is high, i.e., there is a correlation between n_s and $\log_{10}(E_c/E_0)$. This was used to get a better estimate of the multiplicity for nucleon-nucleon collisions, by omitting from the group with $N_h \leq 5$ all events for which $\log_{10}(E_c/E_0) < -0.70$. These events lie outside the limits given by the Monte Carlo calculations and are the ones most likely not to be nucleon-nucleon collisions. The result has already been discussed in Sec. 7 (Table III).

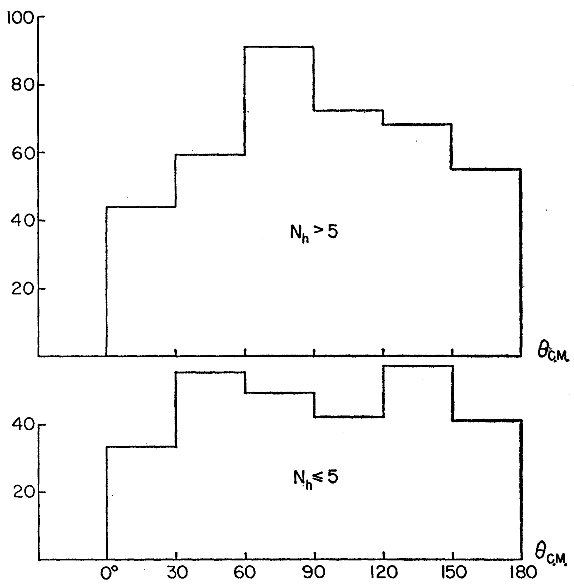

FIG. 10. Angular distribution in center-of-momentum (c.m.) system vs angle $\theta_{\text{c.m.}}$ for p and n interactions. (a) $N_h \leq 5$; (b) $N_h > 5$.

The angular distributions show no indication of a two-maximum structure characteristic of the two-center model. However, due to the relatively low primary energy, no such structure should be expected. The only exceptions are events with very low multiplicity ($n_s \leq 5$). They will be discussed elsewhere.[20]

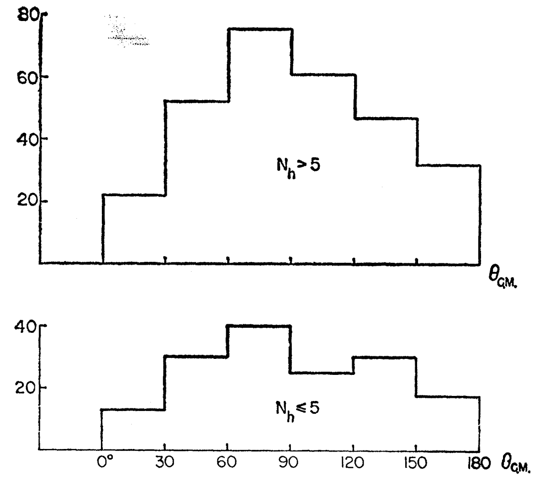

FIG. 11. Angular distribution in c.m. system vs $\theta_{\text{c.m.}}$ for alpha-particle interactions. (a) $N_h \leq 5$; (b) $N_h > 5$.

The angular distributions can also be discussed in terms of the rms value of x, defined in Eq. (14):

$$\sigma = \langle (x - \langle x\rangle)^2\rangle^{\frac{1}{2}}. \qquad (18)$$

An isotropic distribution in the symmetry system corresponds to $\sigma = 0.39$; a distribution of the form of Eq. (15) corresponds to $\sigma = 0.67$. The values of σ following from the experimental data, Figs. 8(a), 8(b), and 9, are given in Table IV.

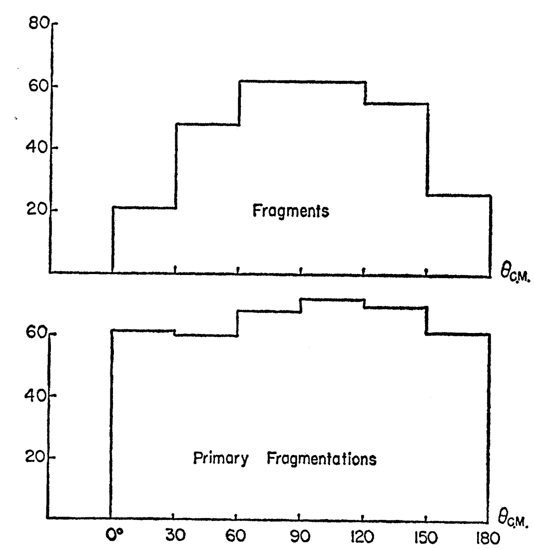

FIG. 12. Angular distribution in c.m. system vs $\theta_{\text{c.m.}}$ for interactions of heavy nuclei. (a) Primary fragmentations; (b) fragments.

[20] J. Gierula, D. M. Haskin, and E. Lohrmann, this issue [Phys. Rev. 122, 626 (1961)].

TABLE VI. Characteristics of interactions of alpha particles and heavy nuclei.

	Alpha particle $N_h \leq 5$	Alpha particle $N_h > 5$	All interactions	Primary fragmentations	Fragments
$\langle n_s \rangle$	14.7±4.2	31 ±10	22.2 ±4.7	29.4±7	35.0±12
$\langle N_h \rangle$	1.7±0.5	16.2± 5	8.3 ±1.8	3.4±0.8	9.1± 3.3
$\langle E_0 \rangle$			165 Bev	210 Bev	210 Bev
η			0.22±0.05		
No. of events	12	10	22	16	9
$\langle Z \rangle$			2.0	12.8	6.9

12. INTERACTIONS OF ALPHA PARTICLES

The alpha particles originating from the fragmentations of the heavy nuclei were traced through the stack. In this way 22 interactions were found in a systematic way. They were analyzed in the same way as the p and n interactions. The multiplicities, as given in Fig. 13, show again the correlation between N_h and n_s. Average values of N_h and n_s are listed in Table VI. The comparison of $\langle n_s \rangle$ for the two groups with $N_h \leq 5$ and $N_h > 5$ shows again the strong influence of the target nucleus on the multiplicity. The average number of charged mesons $\langle N_m \rangle$ has been estimated by subtracting from $\langle n_s \rangle$ two particles which correspond to the incident protons. Since one has to expect a contribution of knock-on nucleons, the average number of charged mesons is expected to be somewhat smaller than $\langle N_m \rangle$.

$\langle N_m \rangle$ for the alpha-particle collisions is higher than the corresponding value of the p and n collisions by about a factor of two. It will be shown that the angular distribution for the two classes of events looks the same. Furthermore, measurements of the transverse momentum p_t of an alpha-particle interaction of very high energy gave the same result as measurements on proton-induced interactions in the same energy region.[21] Therefore, the inelasticity η for the alpha-particle interactions can be derived from the calculations done for the p and n interactions [Eqs. (5) and (6)]. The average value $\langle \eta \rangle$ is smaller by roughly a factor of two for the alpha-particle interactions as compared to the p and n interactions. The accurate value of $\langle \eta \rangle$ is listed in Table VI.

An estimate E_c for the primary energy E_0 per nucleon was obtained from the angular distribution of the shower particles using Eqs. (7) and (8). For these equations only the mesons among the shower particles should be included. Therefore, the two tracks having the smallest angles θ_i were omitted from each interaction. It has been pointed out[9] that these tracks most probably are due to the nucleons contained in the primary alpha particle, and continue with very small angular deflection. The same conclusion is supported by our own data, because of the small value of the average inelasticity $\langle \eta \rangle$. This means that a large fraction of the primary energy must be carried away by the nucleons originally forming the alpha particle. The strong effect of these tracks on the estimate of E_c has been discussed in a previous paper.[5] The energy estimate E_c obtained from the angular distribution was compared with E_0, the energy per nucleon as determined from the scattering measurements. Figure 14 shows the distribution of $\log_{10}(E_c/E_0)$. The events with $N_h > 5$ are cross-hatched. They are correlated with small values of E_c/E_0. The theoretical distribution function for a nucleon-nucleon collision, as calculated with the Monte Carlo method, is drawn in for comparison. As in Fig. 6(a), the additional effects of the error in the scattering measurements and the energy spread of the alpha particles were included. The theoretical curve is not in disagreement with experiment for large values of E_c/E_0. For small values, there exists a tail in the experimental distribution. This can be explained by collisions with a heavy target nucleus, as in the case of the p and n collisions.

The average values $\langle \log_{10}(E_c/E_0) \rangle$ are shown in Table IV. In the average, the primary energy is underestimated by a factor of 1.5 for the events with $N_h > 5$ and overestimated by a factor of 1.2 for the events with

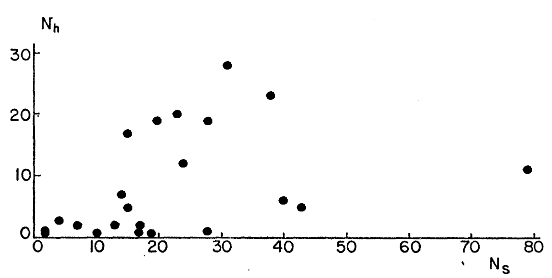

FIG. 13. Distribution of N_h and n_s for alpha-particle interactions.

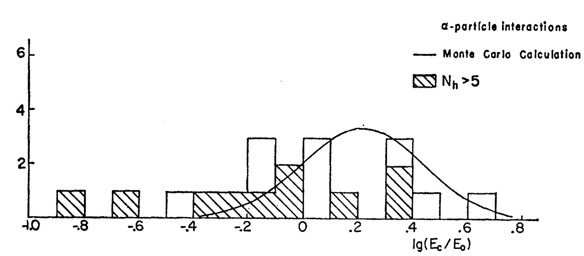

FIG. 14. Comparison of primary energy E_c, as estimated from the angular distribution of shower particles, with the true primary energy E_0 for alpha-particle interactions. Cross-hatched area: events with $N_h > 5$. Smooth curve: Monte Carlo calculation.

[21] E. Lohrmann and M. Teucher (unpublished).

$N_h \leq 5$. These factors agree with those found for the p and n interactions within the accuracy of the experiment.

The differential angular distributions in the laboratory system are shown in Fig. 15(a) and Fig. 15(b) for the two groups of events with $N_h > 5$ and $N_h \leq 5$. The tracks, which were left off for calculating E_c, and which most probably are nucleons, are shown cross-hatched.

The distributions were plotted over the variable x defined by Eq. (14) to normalize all events to the same primary energy E_0.

The questions of symmetry of these distributions and the transformation into the c.m. system have already been given in Sec. 11.

The values $\langle x \rangle$, σ [Eq. (18)], and E_c'/E_0 are given in Table IV. The cross-hatched tracks were omitted when calculating $\langle x \rangle$, σ, and E_c'/E_0.

13. INTERACTIONS PRODUCED BY HEAVY NUCLEI

The characteristics of the 16 primary heavy nuclei fragmentations are shown in Table I and Table VI.

They are not an unbiased sample, because their detection was based on the presence of at least two fragments (except for two events). One of the consequences is the small value of $\langle N_h \rangle$ for these interactions (see reference 2).

The primary energy of the interactions was again estimated from the angular distribution of the shower particles using Eqs. (7) and (8). The influence of shower particles, which are not mesons but fragments of charge one coming from the primary nucleus, is still larger than for the alpha-particle interactions. If they are included in the sum of Eq. (7), the energy estimate E_c would in some cases be very much higher than the true primary energy E_0. Therefore, all particles which are most likely to be protons or deuterons were not considered in Eq. (7). The expected number N_p of such particles is given by

$$N_p = Z_0 - \sum Z_f. \tag{19}$$

Z_0 is the charge of the primary nucleus; $\sum Z_f$ is the sum

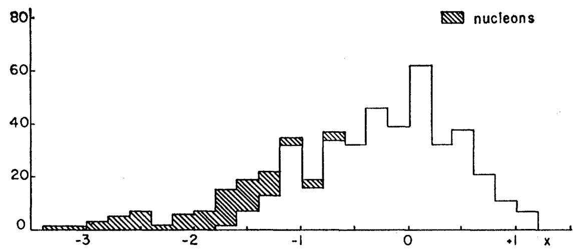

FIG. 16. Angular distribution of shower particles from the primary fragmentations. Nucleons are cross-hatched.

of the charges Z_f of all fragments with $Z_f > 1$. The number N_p of the shower particles having the smallest angles θ_i were left off. This procedure is the same as used in a previous paper.[5] The angular distribution of these tracks is represented by the shaded area in Fig. 16.

The average value $\langle \log_{10}(E_c/E_0) \rangle$, as shown in Table IV, should be compared with the p, n, and alpha interactions having $N_h \leq 5$, because $\langle N_h \rangle$ is also small for the primary fragmentations. The true primary energy is overestimated in the average by a factor of 2.0 by Eqs. (7) and (8).

The fragments originating in the primary fragmentations were traced through the stack. Nine interactions of nuclei with $Z_f > 2$ were found. These interactions can be regarded as an unbiased sample of interactions of heavy nuclei. The characteristics of these interactions are shown in Table VI. The energy estimate E_c was carried out in exactly the same way as for the primary fragmentations.

For Fig. 17 the distribution of $\log_{10}(E_c/E_0)$ for the primary fragmentations and for the fragments is shown. The theoretical distribution is also included. The interactions of the fragments are cross-hatched. Again, the experimental distribution is wider than the result of the Monte Carlo calculations. The tail at low values of E_c/E_0 can be explained by the influence of heavy target nuclei. However, there is also a group of interactions for which the angular distribution gives a much too high value for the primary energy. This might be partly due to uncertainties connected with the cutoff pro-

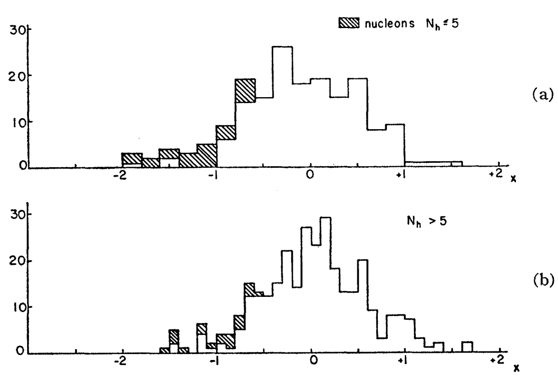

FIG. 15. Angular distribution of shower particles from alpha particle interactions. See also Fig. 8. Nucleons are cross-hatched. (a) $N_h \leq 5$; (b) $N_h > 5$.

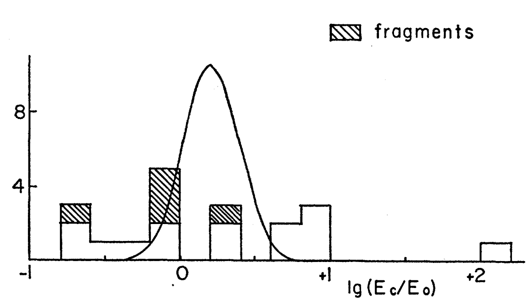

FIG. 17. Comparison of primary energy E_c, as estimated from the angular distribution of shower particles, with the true primary energy E_0, for the primary fragmentations and interactions by heavy fragments (cross-hatched).

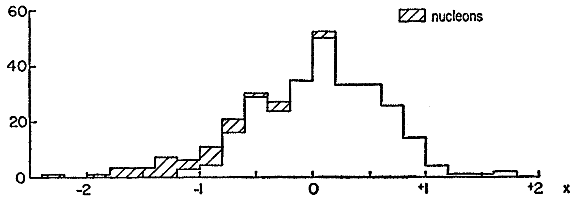

FIG. 18. Angular distribution of shower particles from interactions of fragments.

cedure for those shower particles, which are supposed to be protons or deuterons (errors in charge measurement). It appears, however, unlikely that the anomaly can entirely be explained by this and by statistical fluctuations. An explanation for a similar indication in the case of the p and n interactions has already been discussed. An effect of this kind is also to be expected, if one takes the "nuclear fluid" model seriously. In this model, part of the incident heavy nucleus (N_i nucleons) act coherently in colliding with a lump (N_t nucleons) in the target nucleus. If mesons are emitted symmetrically in the resulting center-of-momentum system, the conventional Eqs. (7) and (8) will overestimate the energy/nucleon in the laboratory system by a factor of (N_i/N_t).

Figures 16 and 18 show the differential angular distribution of the shower particles in the laboratory system for the primary fragmentations and the fragments. Again, all events have been normalized to the same primary energy by plotting over the variable x defined by Eq. (14). The tracks considered to be protons or deuterons according to Eq. (19) have been cross-hatched. They were left off in the further considerations of the angular distributions. Figures 12(a) and 12(b) show the angular distribution transformed into the c.m. system. The values of $\langle x\rangle$, σ, and E_c'/E_0 are shown in Table IV.

14. REMARKS ABOUT THE "NUCLEAR FLUID" MODEL

Our results about the angular distributions can be formally interpreted in a simple way on the basis of the hydrodynamical model proposed by Belenki and Landau,[13] and Milekhin,[14] which was described in Sec. 7 and Sec. 11. The following observations find a natural explanation by this model.

The angular distribution can be transformed into a system in which it is symmetric. This is even true for the collisions with heavy nuclei. The Lorentz-factor of this system is given by considering the collision as taking place between the incoming nucleon and the mass of a cylindrical column punched out of the target nucleus. Our measurements show that this mass is equal to 3.4 nucleons in the case of collisions with heavy nuclei and 2.1 nucleons for collisions with $N_h \leq 5$ and $N_s > 10$. These values are in agreement with a rough geometrical model.

It has to be pointed out, however, that there are discrepancies with more detailed predictions of this hydrodynamical model and observations at higher energy.[11,20,22] Also, it is not quite clear whether the model applies at our relatively small energy.

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[22] J. Gierula, M. Miesowicz, and P. Zielinski, Acta Phys. Polon. **19**, 119 (1960); Nuovo cimento **18**, 102 (1960).