Phenomenological Study of Pion Photoproduction with Polarized Photons*

G. T. HOFF
Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received December 6, 1960)

Expressions for the angular distributions and polarizations of pion photoproduction with polarized photon beams are derived from phenomenological production matrix. The experiments necessary for the complete determination of the multipole amplitudes are discussed in general, and in particular for the case in which only contributions up to p waves in the final state are important. Complete determination requires circularly polarized beams. But if only s and p waves contribute, experiments with linearly polarized beams completely determine the production matrix. The knowledge of these amplitudes would allow the determination of the unknown p-wave scattering phase shifts (α_{11}, α_{13}, and α_{31}) up to energies of about 300 Mev.

Invariance properties of the angular distributions and polarizations are found and tables are given.

TAKING into account invariance under reflection, gauge invariance, and the pseudoscalar nature of the π mesons, it is seen that the most general expression for the nonrelativistic production matrix in the reactions

$$\gamma+p\to p+\pi^0,\quad \gamma+n\to n+\pi^0,$$
$$\gamma+p\to n+\pi^+,\quad \gamma+n\to p+\pi^-$$

is given by

$$M=gI+i(\mathbf{h}\cdot\boldsymbol{\sigma}),\tag{1}$$

where $g=a(\hat{k}\times\hat{q}\cdot\boldsymbol{\varepsilon})=a(\hat{n}\cdot\boldsymbol{\varepsilon})\sin\theta$ and $\mathbf{h}=b\boldsymbol{\varepsilon}+c(\boldsymbol{\varepsilon}\cdot\hat{q})\hat{k}+d(\boldsymbol{\varepsilon}\cdot\hat{q})\hat{q}$.[1] $\boldsymbol{\varepsilon}$ is the polarization vector of the incident photon, $\hat{k}$ and $\hat{q}$ are unit vectors in the directions of the momenta of the incident photon and the emerging pion, $\hat{n}$ is the unit vector along the normal to the production plane $\hat{k}\times\hat{q}/|\hat{k}\times\hat{q}|$, and a, b, c, d are complex functions of the energy and the cosine of the angle θ between $\hat{k}$ and $\hat{q}$. All the quantities are taken in the center-of-mass system.

The angular dependence may be made explicit through a multipole expansion involving Legendre polynomials and its derivatives ($x=\cos\theta$).[1]

$$\begin{aligned}
a&=-\sum_{l=1}^{\infty}[(l+1)M_{l+}+lM_{l-}]P_l'(x),\\
b&=\sum_{l=1}^{\infty}\{[M_{l+}-M_{l-}][l(l+1)P_l(x)-xP_l'(x)]\\
&\qquad+[E_{(l-1)+}+E_{(l+1)-}]P_l'(x)\},\\
c&=\sum_{l=1}^{\infty}\{-[M_{l+}-M_{l-}][P_l'(x)+xP_l''(x)]\\
&\qquad+[E_{(l-1)+}+E_{(l+1)-}]P_l''(x)\}\\
d&=\sum_{l=1}^{\infty}[M_{l+}-M_{l-}-E_{l+}-E_{l-}]P_l''(x).
\end{aligned}\tag{2}$$

Here the notation is as shown in Table I. As a, b, c, d are complex functions, we need seven independent equations linking them in order to determine their absolute values and relative phases. Our purpose now is to discuss the experiments that are necessary for their complete determination.

TABLE I. Orbital angular momenta, total angular momentum, and parity corresponding to the different multipole amplitudes.

	l_π	l_γ	J	ω
M_{l+}	l	l	$l+\frac{1}{2}$	$(-1)^{l+1}$
M_{l-}	l	l	$l-\frac{1}{2}$	$(-1)^{l+1}$
E_{l+}	l	$l+1$	$l+\frac{1}{2}$	$(-1)^{l+1}$
E_{l-}	l	$l-1$	$l-\frac{1}{2}$	$(-1)^{l+1}$

The angular distribution is given by

$$\begin{aligned}
d\sigma/d\Omega&=\tfrac{1}{2}\,\mathrm{Tr}M^\dagger M=gg^*+\mathbf{h}\cdot\mathbf{h}^*\\
&=aa^*\sin^2\theta\,|(\hat{n}\cdot\boldsymbol{\varepsilon})|^2+(bb^*+cc^*+bd^*\\
&\quad+b^*d)\,|(\boldsymbol{\varepsilon}\cdot\hat{q})|^2+(cd^*+c^*d)\,|(\boldsymbol{\varepsilon}\cdot\hat{q})|^2(\hat{k}\cdot\hat{q}),
\end{aligned}\tag{3}$$

Defining the angle φ as in Fig. 1, we have for linearly polarized photons

$$\begin{aligned}
d\sigma/d\Omega&=|a|^2\sin^2\theta+|b|^2+[|c|^2+|d|^2+2\,\mathrm{Re}(bd^*)\\
&\quad+2\,\mathrm{Re}(cd^*)\cos\theta-|a|^2]\sin^2\theta\sin^2\varphi.
\end{aligned}\tag{4}$$

Since this equation has one part independent of φ and another dependent on φ, measurements of the angular distributions will give two independent equations for the determination of the four complex functions. We could measure, for example, $d\sigma/d\Omega$ for $\varphi=0°$ and $\varphi=90°$. Unpolarized photons will give no new information since the angular distribution in that case is the average of the angular distributions for $\varphi=0°$ and $\varphi=90°$.

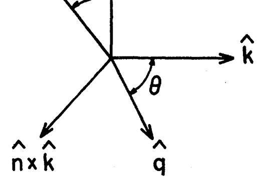

FIG. 1. Definition of angles.

* This work supported by the U. S. Atomic Energy Commission.

[1] R. F. Peierls, thesis, Cornell University (unpublished); G. F. Chew, *Encyclopedia of Physics* (to be published), Vol. 43. We have written our formulas in a slightly different form more convenient for calculations.

The polarization $\mathbf{P}$ is given by

$$\mathbf{P}d\sigma/d\Omega=\tfrac{1}{2}\,\mathrm{Tr}(M^{\dagger}\sigma M)=2\,\mathrm{Im}(g\mathbf{h}^*)+i\mathbf{h}\times\mathbf{h}^*=2\,\mathrm{Im}\{\sin\theta(\hat{n}\cdot\boldsymbol{\varepsilon})[ab^*(\boldsymbol{\varepsilon}^*\cdot\hat{n})\hat{n}+ab^*(\boldsymbol{\varepsilon}^*\cdot\hat{n}\times\hat{k})(\hat{n}\times\hat{k})$$
$$+ac^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})\hat{k}+ad^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\hat{q}\cdot\hat{k})\hat{k}+ad^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\hat{q}\cdot\hat{n}\times\hat{k})(\hat{n}\times\hat{k})]+bc^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\boldsymbol{\varepsilon}\cdot\hat{n}\times\hat{k})\hat{n}$$
$$-bc^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\boldsymbol{\varepsilon}\cdot\hat{n})(\hat{n}\times\hat{k})+bd^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\boldsymbol{\varepsilon}\cdot\hat{n})(\hat{q}\cdot\hat{n}\times\hat{k})\hat{k}+bd^*(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\boldsymbol{\varepsilon}\cdot\hat{n}\times\hat{k})(\hat{q}\cdot\hat{k})\hat{n}$$
$$-bd^*(\boldsymbol{\varepsilon}\cdot\hat{n})(\hat{q}\cdot\hat{k})(\boldsymbol{\varepsilon}^*\cdot\hat{q})(\hat{n}\times\hat{k})-cd^*|(\boldsymbol{\varepsilon}\cdot\hat{q})|^2\sin\theta\hat{n}\}. \quad (5)$$

For linearly polarized photons, we get the following expressions for the components of the polarization along $\hat{k}$, $\hat{n}$, and $\hat{n}\times\hat{k}$ in terms of the angles θ and φ:

$$\begin{aligned}P(\hat{k})d\sigma/d\Omega&=-\sin2\varphi\sin^2\theta\,\mathrm{Im}(ac^*+ad^*\cos\theta+bd^*),\\P(\hat{n})d\sigma/d\Omega&=2\sin\theta\,\mathrm{Im}[ab^*\cos^2\varphi+(bc^*+bd^*\cos\theta-cd^*\sin^2\theta)\sin^2\varphi],\\P(\hat{n}\times\hat{k})d\sigma/d\Omega&=-\sin2\varphi\sin\theta\,\mathrm{Im}(ab^*+ad^*\sin^2\theta-bc^*-bd^*\cos\theta).\end{aligned}\quad (6)$$

We see that the measurements of the polarization along the direction of the incident photon gives only one equation (any angle φ can be used but the best one is 45°). Measurements along the normal to the production plane give two equations (we could use $\varphi=0°$ and $\varphi=90°$), and measurements along the direction normal to these two give one more equation (again the best angle is 45°). Thus, we get only four equations.

In order to get one more relationship, we have to use circularly polarized photons. Evidently measurements of the angular distribution will not give us any new information since the angular distribution for circularly polarized photons is equal to that for unpolarized photons. The same happens with the component of the polarization along the normal to the production plane. But we still have the other two components of the polarization of the recoil nucleon at our disposal. For circularly polarized photons

$$\boldsymbol{\varepsilon}_{r,l}=[-\hat{n}\pm(\hat{n}\times\hat{k})i]/\sqrt{2},$$

where the upper sign refers to right-circularly polarized (r) and the lower sign to left-circularly polarized (l). Substitution in (5) gives

$$\begin{aligned}&[P(\hat{k})d\sigma/d\Omega]_{r,l}=\pm\sin^2\theta\,\mathrm{Re}(ac^*+ad^*\cos\theta+bd^*),\\&[P(\hat{n}\times\hat{k})d\sigma/d\Omega]_{r,l}\\&\qquad=\pm\sin\theta\,\mathrm{Re}(ab^*+ad^*\sin^2\theta-bc^*-bd^*\cos\theta).\end{aligned}\quad (7)$$

So, we get one more equation than we need.

Measurements of angular distributions and polarizations with linearly polarized photons are being carried out at the present time at Stanford, but we cannot expect at present experiments using circularly polarized photons.

Nevertheless, it is possible to get the needed information in the low-energy region where only s and p wave contributions in the final state are important. This is due to the fact that for $l<2$, $d=0$, so we have only three unknown parameters.

The equations for the angular distribution and polarization using linearly polarized photons then reduce to

$$\begin{aligned}d\sigma/d\Omega&=|a|^2\sin^2\theta+|b|^2\\&\quad+(|c|^2-|a|^2)\sin^2\theta\sin^2\varphi,\\P(\hat{k})d\sigma/d\Omega&=-\sin2\varphi\sin^2\theta\,\mathrm{Im}(ac^*),\\P(\hat{n})d\sigma/d\Omega&=2\sin\theta\,\mathrm{Im}(ab^*\cos^2\varphi+bc^*\sin^2\varphi),\\P(\hat{n}\times\hat{k})d\sigma/d\Omega&=-\sin2\varphi\sin\theta\,\mathrm{Im}(ab^*-bc^*).\end{aligned}\quad (8)$$

Thus we can obtain from experiment the value of the following combinations of the parameters as a function of the angle θ and the energy:

$$\begin{gathered}|a|^2\sin^2\theta+|b|^2,\\|c|^2-|a|^2,\\\mathrm{Im}(ac^*)=|a|\,|c|\,\sin(\alpha-\gamma),\\\mathrm{Im}(ab^*)=|a|\,|b|\,\sin(\alpha-\beta),\\\mathrm{Im}(bc^*)=|b|\,|c|\,\sin(\beta-\gamma),\\\mathrm{Im}(ab^*-bc^*).\end{gathered}\quad (9)$$

We notice that the sixth expression is a linear combination of the fourth and the fifth, so the measurement of $P(\hat{n}\times\hat{k})$ is not necessary. We have five equations for the determination of five unknowns, but there will be a certain ambiguity in the phase-differences due to the fact that there are only sines appearing in our formulas. We notice that if $(\alpha-\gamma)=A$, $(\alpha-\beta)=B$, and $(\beta-\gamma)=C$ satisfy the equations and the condition $(\alpha-\gamma)-(\alpha-\beta)-(\beta-\gamma)=0$, then also $180-A$, $180-B$, C and $180-A$, B, $180-C$ will satisfy them.

If we were able to make measurements using circularly polarized photons, we would get

$$\begin{aligned}\mathrm{Re}(ac^*)&=|a|\,|c|\,\cos(\alpha-\gamma),\\\mathrm{Re}(ab^*-bc^*)&=|a|\,|b|\,\cos(\alpha-\beta)\\&\qquad-|b|\,|c|\,\cos(\beta-\gamma),\end{aligned}\quad (10)$$

and that ambiguity would disappear.[2] Nevertheless, we will see presently that there is another way of getting rid of it.

Making the expansions only up to p waves in the

[2] We note that the first equation still leaves an ambiguity, so we need the second one specifically.

final state, we have

$$\begin{aligned} a&=-2M_{1+}-M_{1-},\\ b&=E_{0+}+(M_{1+}-M_{1-}+3E_{1+})\cos\theta, \\ c&=-M_{1+}+M_{1-}+3E_{1+}. \end{aligned} \qquad (11)$$

Thus knowing a, b, and c as functions of $\cos\theta$ for a certain energy, we can determine the four multipole amplitudes E_{0+}, E_{1+}, M_{1+}, and M_{1-} for that particular energy.

The determination of these amplitudes for both reactions $\gamma+P\rightarrow\pi^{+}+n$, $\gamma+p\rightarrow\pi^{0}+p$ would make it possible to find the three p-wave scattering phase-shifts that are not well determined (α_{13}, α_{31}, and α_{11}) since the phase shifts and the multipole amplitudes are related.[3] On the other hand, the previous knowledge of the two s-wave phase shifts α_1, α_3, and the p-wave phase shift α_{33} will allow us to discriminate among the three possible sets of solutions for the multipole amplitudes and to fix their absolute phases.[4]

At energies above 300 Mev, d-wave contributions become important and we can no longer set $d=0$.[5] We need then to perform experiments using circularly polarized photons for the determination of the multipole amplitudes that contribute. The system of equations involved becomes more complicated.

If we redefine the multipole amplitudes $R_m=|R_m|e^{i\delta m}$ in such a way that we have for the total cross section for unpolarized photons the expression

$$\sigma=(\pi/2k^2)\Sigma(2J+1)|R_m|^2,$$

we can express the angular distributions and polarizations in the following way:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\varphi=0^\circ}=\frac{1}{4k^2}\{\sum_m|R_m|^2M_{mm}{}^{\mathrm{I}}+2\sum_{mn,m<n}|R_m||R_n|\cos(\delta_m-\delta_n)M_{mn}{}^{\mathrm{I}}\},$$

$$\left[P(\hat{n})\frac{d\sigma}{d\Omega}\right]_{\varphi=0^\circ}=\frac{1}{4k^2}2\sum_{mn,m>n}|R_m||R_n|\sin(\delta_m-\delta_n)M_{mn}{}^{\mathrm{I}}\sin\theta,$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\varphi=90^\circ}=\frac{1}{4k^2}\{\sum_m|R_m|^2M_{mn}{}^{\mathrm{II}}+2\sum_{mn,m<n}|R_m||R_n|\cos(\delta_m-\delta_n)M_{mn}{}^{\mathrm{II}}\},$$

$$\left[P(\hat{n})\frac{d\sigma}{d\Omega}\right]_{\varphi=90^\circ}=\frac{1}{4k^2}2\sum_{mn,m>n}|R_m||R_n|\sin(\delta_m-\delta_n)M_{mn}{}^{\mathrm{II}}\sin\theta,$$

$$\left[P(\hat{k})\frac{d\sigma}{d\Omega}\right]_{cp}=\pm\frac{1}{4k^2}\sum_{mn,m\leq n}|R_m||R_n|\cos(\delta_m-\delta_n)M_{mn}{}^{\mathrm{III}}\sin^2\theta,$$

$$\left[P(\hat{k})\frac{d\sigma}{d\Omega}\right]_{lp}=-\frac{1}{4k^2}\sum_{mn,m>n}|R_m||R_n|\sin(\delta_m-\delta_n)M_{mn}{}^{\mathrm{III}}\sin^2\theta\sin2\varphi,$$

$$\left[P(\hat{n}\times\hat{k})\frac{d\sigma}{d\Omega}\right]_{cp}=\pm\frac{1}{4k^2}\sum_{mn,m\leq n}|R_m||R_n|\cos(\delta_m-\delta_n)M_{mn}{}^{\mathrm{IV}}\sin\theta,$$

$$\left[P(\hat{n}\times\hat{k})\frac{d\sigma}{d\Omega}\right]_{lp}=-\frac{1}{4k^2}\sum_{mn,m>n}|R_m||R_n|\sin(\delta_m-\delta_n)M_{mn}{}^{\mathrm{IV}}\sin\theta\sin2\varphi.$$

The M_{mn}'s are functions of $\cos\theta$ and are given in Tables II, III, IV, and V for dipole and quadrupole contributions.[6]

We notice that certain invariance properties hold in the parameters a, b, c, d.

a: $(l+1)M_{l+}\leftrightarrow lM_{l-}$

b and c: $M_{l+}\leftrightarrow -M_{l-}$, $E_{(l-1)+}\leftrightarrow E_{(l+1)-}$

d: We can exchange any two among M_{l+}, $-M_{l-}$, $-E_{l+}$, $-E_{l-}$.

[3] See M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1954), Vol. 4, p. 219, for formulas relating multipole amplitudes with phase shifts.

[4] There will still be an ambiguity of π in the absolute phases. We can absorb the sign into the real amplitudes which have then positive or negative values. See S. Hayakawa, M. Kawaguchi, and S. Minami, Suppl. Progr. Theoret. Phys. (Kyoto) **1**, 41 (1958).

[5] The retardation term gives higher l states than p at lower energies but we know its contribution.

[6] We do not give tables for the angular distributions and polarizations using unpolarized photons since they have been given previously by other authors. See Hayakawa *et al.*, reference 4, and R. F. Peierls, Phys. Rev. **118**, 325 (1960). There are several incorrect terms though in the latter, and only contributions up to $J=\frac{3}{2}$ are given in the former. The reader interested in knowing our results can easily take the average for $\varphi=0^\circ$ and $\varphi=90^\circ$.

TABLE II. Angular distributions and polarizations along the direction normal to the production plane for $\varphi=0°(M_{mn}^{\mathrm{I}})$.

$R_m \backslash R_n$	E_{0+}	M_{1-}	M_{1+}	E_{2-}	E_{1+}	M_{2-}	M_{2+}	E_{3-}
E_{0+}	1	$-x$	x	1	$\sqrt{3}x$	$\sqrt{3}(1-2x^2)$	$\sqrt{3}(-1+2x^2)$	$\sqrt{3}x$
M_{1-}	-1	1	$2-3x^2$	$-x$	$-\sqrt{3}x^2$	$\sqrt{3}x$	$\sqrt{3}(4x-5x^3)$	$-\sqrt{3}x^2$
M_{1+}	-2	$3x$	$4-3x^2$	x	$\sqrt{3}x^2$	$\sqrt{3}(5x-6x^3)$	$\sqrt{3}(5x-4x^3)$	$\sqrt{3}x^2$
E_{2-}	0	1	2	1	$\sqrt{3}x$	$\sqrt{3}(1-2x^2)$	$\sqrt{3}(-1+2x^2)$	$\sqrt{3}x$
E_{1+}	0	$\sqrt{3}x$	$2\sqrt{3}x$	0	$3x^2$	$3(x-2x^3)$	$3(-x+2x^3)$	$3x^2$
M_{2-}	$-2\sqrt{3}x$	$\sqrt{3}$	$2\sqrt{3}(1-3x^2)$	$-2\sqrt{3}x$	$-6x^2$	3	$3(-1+10x^2-10x^4)$	$3(+x-2x^3)$
M_{2+}	$-3\sqrt{3}x$	$\sqrt{3}(1+5x^2)$	$\sqrt{3}(-2+x^2)$	$-3\sqrt{3}x$	$-9x^2$	$15(-x+2x^3)$	$3(1+5x^2-5x^4)$	$3(-x+2x^3)$
E_{3-}	0	$\sqrt{3}x$	$2\sqrt{3}x$	0	0	$6x^2$	$9x^2$	$3x^2$

TABLE III. Angular distributions and polarizations along the direction normal to the production plane for $\varphi=90°(M_{mn}^{\mathrm{II}})$.

$R_m \backslash R_n$	E_{0+}	M_{1-}	M_{1+}	E_{2-}	E_{1+}	M_{2-}	M_{2+}	E_{3-}
E_{0+}	1	$-x$	x	$-2+3x^2$	$\sqrt{3}x$	$-\sqrt{3}x^2$	$\sqrt{3}x^2$	$\sqrt{3}(-4x+5x^3)$
M_{1-}	-1	1	-1	$-x$	$\sqrt{3}(1-2x^2)$	$\sqrt{3}x$	$-\sqrt{3}x$	$\sqrt{3}(1-2x^2)$
M_{1+}	1	0	1	x	$\sqrt{3}(-1+2x^2)$	$-\sqrt{3}x$	$\sqrt{3}x$	$\sqrt{3}(-1+2x^2)$
E_{2-}	$3x$	-2	2	$4-3x^2$	$\sqrt{3}(-5x+6x^3)$	$-\sqrt{3}x^2$	$\sqrt{3}x^2$	$\sqrt{3}(5x-4x^3)$
E_{1+}	$-\sqrt{3}$	$2\sqrt{3}x$	$-2\sqrt{3}x$	$2\sqrt{3}(1-3x^2)$	3	$3(x-2x^3)$	$3(-x+2x^3)$	$3(1-10x^2+10x^4)$
M_{2-}	$-\sqrt{3}x$	0	0	$4\sqrt{3}x$	$-6x^2$	$3x^2$	$-3x^2$	$3(x-2x^3)$
M_{2+}	$\sqrt{3}x$	0	0	$-4\sqrt{3}x$	$6x^2$	0	$3x^2$	$3(-x+2x^3)$
E_{3-}	$\sqrt{3}(-1+5x^2)$	$5\sqrt{3}x$	$-5\sqrt{3}x$	$\sqrt{3}(2-x^2)$	$15(-x+2x^3)$	$-9x^2$	$9x^2$	$3(1+5x^2-5x^4)$

TABLE IV. Polarizations along the direction of the incoming photons for linearly and circularly polarized beams (M_{mn}^{III}).

$R_m \backslash R_n$	E_{0+}	M_{1-}	M_{1+}	E_{2-}	E_{1+}	M_{2-}	M_{2+}	E_{3-}
E_{0+}	0	0	0	-3	0	$-\sqrt{3}$	$\sqrt{3}$	$-5\sqrt{3}x$
M_{1-}	0	-1	-1	$6x$	$-\sqrt{3}$	$-2\sqrt{3}x$	$-3\sqrt{3}x$	$\sqrt{3}(-1+10x^2)$
M_{1+}	0	-3	2	$3x$	$-2\sqrt{3}$	$-\sqrt{3}x$	$6\sqrt{3}x$	$\sqrt{3}(-2+5x^2)$
E_{2-}	3	$-6x$	$-3x$	-3	$-3\sqrt{3}x$	$4\sqrt{3}(-1+3x^2)$	$\sqrt{3}(4+3x^2)$	$-8\sqrt{3}x$
E_{1+}	0	$\sqrt{3}$	$2\sqrt{3}$	$-3\sqrt{3}x$	0	$-9x$	$-6x$	$-15x^2$
M_{2-}	$\sqrt{3}$	$-2\sqrt{3}x$	$5\sqrt{3}x$	$2\sqrt{3}(-1+6x^2)$	$-3x$	-3	$3(2-5x^2)$	$12(-2x+5x^3)$
M_{2+}	$-\sqrt{3}$	$-3\sqrt{3}x$	0	$\sqrt{3}(2+3x^2)$	$-12x$	$-15x^2$	$3(-1+5x^2)$	$3(3x+5x^3)$
E_{3-}	$5\sqrt{3}x$	$\sqrt{3}(1-10x^2)$	$\sqrt{3}(2-5x^2)$	$2\sqrt{3}x$	$15x^2$	$6(3x-10x^3)$	$-3(x+5x^3)$	$-15x^2$

TABLE V. Polarizations along the $\hat{n}\times\hat{k}$ direction for linearly and circularly polarized photons (M_{mn}^{IV}).

$R_m \backslash R_n$	E_{0+}	M_{1-}	M_{1+}	E_{2-}	E_{1+}	M_{2-}	M_{2+}	E_{3-}
E_{0+}	0	-2	-1	$3x$	$-\sqrt{3}$	$-3\sqrt{3}x$	$-2\sqrt{3}x$	$\sqrt{3}(-1+5x^2)$
M_{1-}	0	$2x$	$-x$	$1-3x^2$	$-\sqrt{3}x$	$\sqrt{3}(-1+6x^2)$	$\sqrt{3}(1-x^2)$	$-2\sqrt{3}(2x+5x^3)$
M_{1+}	-3	$3x$	$-x$	$5-3x^2$	$-2\sqrt{3}x$	$\sqrt{3}(1-3x^2)$	$\sqrt{3}(1+2x^2)$	$\sqrt{3}(8x-5x^3)$
E_{2-}	$-3x$	$-3+3x^2$	$-3+3x^2$	$3x$	$\sqrt{3}(-1+3x^2)$	$6\sqrt{3}(x-2x^3)$	$\sqrt{3}(4x-3x^3)$	$\sqrt{3}(-1+8x^2)$
E_{1+}	$\sqrt{3}$	$-\sqrt{3}x$	$4\sqrt{3}x$	$\sqrt{3}(1+3x^2)$	$-3x$	$-3(1+x^2)$	$3(1-4x^2)$	$3(-2x+5x^3)$
M_{2-}	$-\sqrt{3}x$	$\sqrt{3}(-1+2x^2)$	$\sqrt{3}(1-5x^2)$	$4\sqrt{3}(2x-3x^2)$	$3(1+x^2)$	$3(-x+4x^3)$	$3(2x-3x^3)$	$3(-1+14x^2-20x^4)$
M_{2+}	$-4\sqrt{3}x$	$\sqrt{3}(1+3x^2)$	$-\sqrt{3}$	$\sqrt{3}(2x-3x^3)$	$3(1-6x^2)$	$15x^3$	$-3(x+x^3)$	$3(1+6x^2-5x^4)$
E_{3-}	$\sqrt{3}(1-5x^2)$	$2\sqrt{3}(-3x+5x^3)$	$\sqrt{3}(-6x+5x^3)$	$\sqrt{3}(1-2x^2)$	$-15x^3$	$3(1-16x^2+20x^4)$	$3(-1-4x^2+5x^4)$	$3(-x+5x^3)$

From this we see that the following invariance properties hold in the angular distributions and polarizations along $\hat{n}$ for $\varphi=0°$ and $\varphi=90°$. For $\varphi=0°$ we have an invariance under the transformation

$$E_{(l-1)+} \leftrightarrow E_{(l+1)-}.$$

$E_{(l-1)+}$ and $E_{(l+1)-}$ are both electric $2l$ pole amplitudes [parity $(-1)^l$] but the first one corresponds to a total angular momentum $l-\frac{1}{2}$ while the second corresponds to $J=l+\frac{1}{2}$.

For $\varphi=90°$ the invariance that holds is under the transformation

$$M_{l+} \leftrightarrow -M_{l-}$$

M_{l+} and M_{l-} correspond both to magnetic $2l$ pole radiation [parity $(-1)^{l+1}$] but the total angular momentum is $l+\frac{1}{2}$ for M_{l+} and $l-\frac{1}{2}$ for M_{l-}. These invariance properties can be checked in the tables.

We also notice that under the transformation

$$M_{l+} \leftrightarrow -E_{(l+1)-}, \quad M_{l-} \leftrightarrow E_{(l-1)+},$$

the angular distribution for $\varphi=0°$ goes over into the angular distribution for $\varphi=90°$ and vice versa. This does not occur with the polarization. As a consequence, the angular distribution, but not the polarization for unpolarized photons, is invariant under this transformation. This is the well-known Minami invariance.[7] The relevance of this ambiguity to high-energy photoproduction experiments has been emphasized by Sakurai[8] and Moravcsik.[9]

ACKNOWLEDGMENT

We wish to give thanks to Professor J. J. Sakurai for his guidance and criticism while working on this article.

[7] See, for instance, Hayakawa *et al.*, reference 4. The discrepancy between the sign of E_{l-} in this article and in ours is due to a different definition of these amplitudes.
[8] J. J. Sakurai, Phys. Rev. Letters **1**, 258 (1958).
[9] M. J. Moravcsik, Phys. Rev. Letters **2**, 171 (1959).

PHYSICAL REVIEW VOLUME 122, NUMBER 2 APRIL 15, 1961

Range of Proton-Antiproton Annihilation Near 1.0 Bev

OSAMU HARA*
School of Physics, University of Minnesota, Minneapolis, Minnesota
(Received December 9, 1960)

The range of the proton-antiproton annihilation was calculated for antiproton with energy near 1 Bev. The point is to get the range of pure annihilation interaction, separating the effect of pion production. It was found that the root mean square of this range is given by $(1.19\pm0.07)\times10^{-13}$ cm almost independently of the energy.

THE range of proton-antiproton annihilation is of special interest in connection with the problem of nucleon structure, since it gives in some sense direct information about the core size. Lévy estimated this to be 1.43×10^{-13} cm.[1] His result is, however, subject to an ambiguity, since in his calculation the effect of pion production is not separated. Recently, the pion production cross section in p-$\bar{p}$ collision was found to be (5 ± 1) mb at 940 Mev.[2] It seems possible to use this new information to eliminate to some extent the ambiguity in Lévy's calculation.

If we denote the phase shift due to pure annihilation as η_l, and the correction to η_l due to pion production as $\delta\eta_l$, the total inelastic cross section and the scattering amplitude are given by

$$\sigma_{\text{inel}}=\pi\lambda\!\!\!^{-2} \sum (2l+1)(1-e^{4i(\eta_l+\delta\eta_l)}), \tag{1}$$

and

$$f(\theta)=\tfrac{1}{2}i\lambda\!\!\!^{-} \sum (2l+1)(1-e^{2i(\eta_l+\delta\eta_l)})P_l(\cos\theta), \tag{2}$$

respectively, where $\lambda\!\!\!^{-}$ is the wavelength of the incident particle in the c.m. system, and the spins of both particles were neglected. η_l and $\delta\eta_l$ are assumed to be imaginary.

Equation (1) can be written as

$$\sigma_{\text{inel}}=\pi\lambda\!\!\!^{-2}\{ \sum (2l+1)(1-e^{4i\eta_l}) + \langle e^{4i\eta}\rangle \sum (2l+1)(1-e^{4i\delta\eta_l})\}, \tag{3}$$

where $\langle e^{4i\eta}\rangle$ is the average value of $e^{4i\eta_l}$ over the range of l in which $\delta\eta_l$ is appreciably different from zero. The first term on the right-hand side of (3) is the pure annihilation cross section. To relate the second term to observed quantities, we assume first that the phase shifts due to annihilation and pion production can be defined separately and are additive, and second that the mechanism of pion production is essentially the same for p-p and p-$\bar{p}$ collisions. Then $\delta\eta_l$ can be identified with the phase shift for p-p collision with the same energy, and (3) can be written as

$$\sigma_{\text{inel}}=\sigma_{\text{an}}+\langle e^{4i\eta}\rangle\sigma_{\text{pro}}(pp),$$

where $\sigma_{\text{pro}}(pp)$ is the pion production cross section for p-p collision with the same energy. The second term in the right-hand side of (3) is the pion production cross section in p-$\bar{p}$ collision, and from this we see that $\langle e^{4i\eta}\rangle$ can be expressed as

$$\langle e^{4i\eta}\rangle=\sigma_{\text{pro}}(p\bar{p})/\sigma_{\text{pro}}(pp), \tag{4}$$

where $\sigma_{\text{pro}}(p\bar{p})$ is the pion production cross section in p-$\bar{p}$ collision.

* On leave of absence from Nihon University, Tokyo, Japan.
[1] M. Lévy, Phys. Rev. Letters **5**, 377 (1960).
[2] O. Chamberlain, *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960* (Interscience Publishers, New York, 1960).