Angular Distributions of Secondary Particles Produced in High-Energy Nuclear Collisions and the Two-Center Model of Multiple Meson Production*

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The angular distributions of shower particles from 54 nuclear interactions of protons and neutrons with energies $>10^{12}$ ev in a stack of nuclear emulsions are analyzed. The method consists essentially in normalizing the angular distributions of all events in the $x=\log\tan\theta$ scale to the same dispersion σ. One finds a very significant deviation from the normal distribution predicted by hydrodynamical models. The deviation goes in the direction indicated by the two-center model (two maxima in the plot over the x coordinate). The correlation between the separation of the two emitting centers and σ is also in qualitative agreement with the model. The angular distribution in the rest system of the emitting centers is found to be roughly isotropic. The two-center model also offers an explanation for certain characteristic features observed for the angular distribution of events with a small number of shower particles ($n_s \leqq 5$). On the basis of this model a coefficient of inelasticity of ≈ 0.2 is obtained for these events.

Interactions characterized by small evaporation ($N_h \leqq 5$) and small numbers of shower particles ($n_s \leqq 20$) show the characteristic two-maximum shape. The same shape is found for presumably central collisions with heavy nuclei in the emulsion ($N_h > 8$, $n_s > 40$). However, the group of collisions with $N_h \leqq 5$, $n_s > 20$ does not show the two maxima. The last two observations cannot be explained by the present simple form of the two-center model.

The results of this paper are in good agreement with a similar analysis carried out by Gierula, Miȩsowicz, and Zielinski.

1. INTRODUCTION

RECENTLY Gierula, Miesowicz, and Zielinski,[1] starting from the two-center model of multiple meson production,[2–4] introduced a new method of statistical analysis for angular distributions of secondary particles produced in high-energy nuclear collisions. They have applied this method of analysis to 65 jets produced by singly charged and neutral primaries having energy higher than 10^{12} ev and have shown that the observed distributions are in rather good agreement with the predictions of the two-center model. Because this method of analysis seems to be more efficient in finding some new properties of the angular distributions than standard methods used before, we have applied it in this investigation of angular distributions of high-energy nuclear interactions found recently in this laboratory.

2. EXPERIMENTAL MATERIAL AND PROCEDURE

The experimental material which is used in this paper is taken from an investigation of high-energy nuclear interactions carried out in this laboratory.[5] A detailed description of the experimental procedure is contained in reference 5, therefore only a short summary will be given here.

In a 22-liter stack of nuclear emulsion exposed over Texas at an altitude above 110 000 feet, high-energy nuclear interactions were located by tracing showers of parallel minimum tracks back to the origin. Fifty-four interactions produced by singly charged or neutral particles with $n_s \geqq 6$ and with dip angles $<17°$ were found. The method used for finding these events imposes a lower limit of about 10^{12} ev on the primary energy. The average primary energy of the group is estimated to be 3.5×10^{12} ev. The angles θ_i of all the shower particles with respect to the line of flight of the primary particle were measured. The results, in the $\log_{10}\tan\theta_i$ coordinate, are presented in the paper of Barkow *et al.*[5]

We shall now evaluate this experimental material by a method quite similar to the one used by GMZ.[1] From the angular distributions represented in the coordinate $x_i=\log_{10}\tan\theta_i$, three parameters are calculated for each event:

1. The mean value of x, the first moment of the distribution, which is connected with an estimate of the primary energy by the well-known relation[6]

$$-\log_{10}\gamma_c = \langle \log_{10}\tan\theta_i \rangle. \tag{1}$$

2. The dispersion of the distribution or the second moment, defined by

$$\sigma = [\sum (x_i - \langle x \rangle)^2/(n_s-1)]^{\frac{1}{2}}, \tag{2}$$

where n_s is the number of shower particles.

3. The value D which is defined by

$$D=(N_e-N_i)/n_s, \tag{3}$$

where N_i is the number of tracks inside the interval $(\langle x\rangle - 0.67\sigma,\ \langle x\rangle + 0.67\sigma)$ and $N_e = n_s - N_i$.

$D>0$ indicates a deviation of the given distribution from the normal distribution towards the one predicted by the two-center model. For events with small numbers

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[1] J. Gierula, M. Miȩsowicz, and P. Zielinski, Acta Phys. Polon. **19**, 119 (1960); Nuovo cimento **18**, 102 (1960). Hereafter referred to as GMZ.

[2] P. Ciok, T. Coghen, J. Gierula, R. Hołyński, A. Jurak, M. Miesowicz, T. Saniewska, O. Stanisz, and J. Pernegr, Nuovo cimento **8**, 166 (1958).

[3] G. Cocconi, Phys. Rev. **111**, 1699 (1958).

[4] K. Niu, Nuovo cimento **10**, 994 (1958).

[5] A. G. Barkow, B. Chamany, D. M. Haskin, P. L. Jain, E. Lohrmann, M. W. Teucher, and M. Schein, preceding paper [Phys. Rev. **122**, 617 (1961)].

[6] C. Castagnoli, G. Cortini, D. Moreno, C. Franzinetti, and A. Manfredini, Nuovo cimento **10**, 1539 (1953).

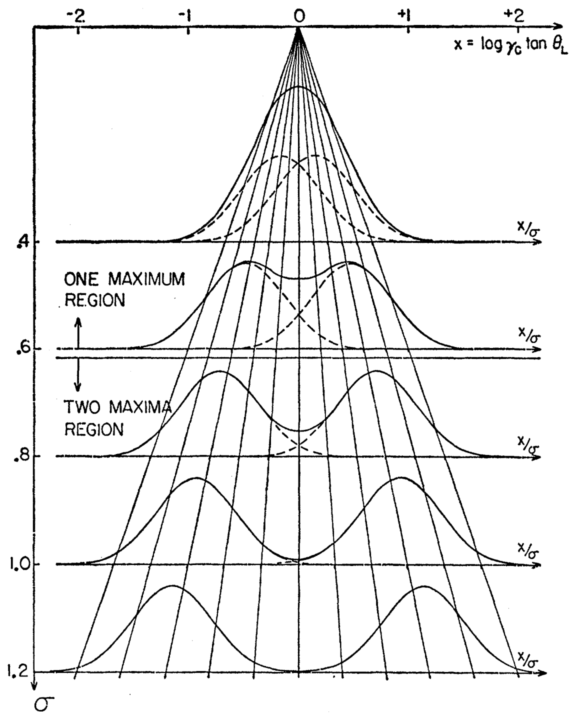

FIG. 1. Shapes of angular distributions vs their dispersion σ expected by the two-center model. Dashed curves correspond to the isotropic emission from individual centers. The family of divergent lines shows how the individual shapes become more similar to each other if they are presented in the $(x-\langle x\rangle)/\sigma$ coordinate.

of tracks, the same correction in calculating D values is introduced as in GMZ.

Each distribution is then normalized to a common dispersion $\sigma=1$ and to the same mean by dividing $x-\langle x\rangle$ by its own dispersion and a composite distribution $(x-\langle x\rangle)/\sigma$ of all events normalized in this way is constructed. Figure 1 shows some examples of the shapes of the angular distributions predicted by the two-center model as a function of the dispersion σ. As seen from this figure, the procedure of adding the angular distributions in the coordinate $(x-\langle x\rangle)/\sigma$ should be more efficient in visualizing the double-maximum shape of the angular distributions than the adding in the $x-\langle x\rangle$ coordinate normally used, especially if applied to jets having a dispersion $\sigma\gtrsim 0.6$.

Table I shows the whole collection of 54 jets divided into groups according to the number of evaporation tracks N_h and multiplicity n_s. We have not used any restriction on γ_c as was done by GMZ which accepted only events with $\gamma_c>23$, as defined by Eq. (1). Jets in their collection were only partly found by tracing back cascades. Another part was found by area scanning. In contrast to this, all our jets were found using unique scanning criteria. Owing to this, it is worthwhile to compare the distributions of γ_c and σ in both collections of jets (Fig. 2). A marked difference in both distributions is easily seen, suggesting that tracing back cascades favors higher energy events. In the special case of the analysis carried out here, this difference is not of great importance.

3. RESULTS OF THE ANALYSIS

(a) Events with $n_s\geqq 6$

First, we have constructed the composite histogram of the whole collection of the 54 jets in the $(x-\langle x\rangle)/\sigma$ coordinate. For easier comparison of the distributions with the normal one having the same dispersion ($\sigma=1$ by definition), the intervals on the $(x-\langle x\rangle)/\sigma$ axis were chosen in such a way that for the normal distribution a constant number of tracks was expected in each interval (Fig. 3). The deviation from the normal distribution towards the distribution expected by the two-center model is very significant, more than 3 standard deviations, as follows from a χ^2 test. A normal distribution would be predicted by the hydrodynamical model.[7] In order to be sure that the observed shape of the histogram is not produced by the applied procedure, we checked this result by a Monte Carlo experiment. It was tentatively assumed that all the observed distributions can be obtained by random sampling from the composite angular distributions of all jets taken together in the $x-\langle x\rangle=\log_{10}(\gamma_c\tan\theta_i)$ coordinate. This distribution is presented in reference 5. From this experimental distribution "Monte Carlo showers" were constructed by choosing $x-\langle x\rangle=\log_{10}(\gamma_c\tan\theta_i)$ values by a random method. A sample of 54 such showers was obtained, which had the same distribution of n_s as the experimental events. The Monte Carlo events were then treated in exactly the same way as the real events. The histogram obtained this way together with the experimental histogram is presented in Fig. 4. The deviation of the experimental histogram from the Monte Carlo one is clearly seen. The direction of this deviation shows that the effect of the double-maximum shape is stronger in the experimental jets than in the Monte Carlo samples. The χ^2 test[8] indicates a difference of about 2 standard deviations. This number gives some idea about the increase of efficiency in visualizing the shape of the angular distributions gained by applying the procedure of GMZ instead of using the composite distributions in the $x-\langle x\rangle$ coordinate. The discrepancy between the Monte Carlo showers and the real experimental data suggests that the observed angular distributions are not just the result of statistical fluctuations around a single shape of the angular distribution. In describing the angular distribution of individual events, one more parameter must be introduced, for example σ. As a matter of fact, the spread of σ values in the Monte Carlo showers is smaller than the one for the real events. The number of events deviating from the mean by more than one

[7] G. A. Milekhin, Zhur. Eksp. i Teoret. Fiz. **35**, 1185 (1958) [translation: Soviet Phys.—JETP **35**, 829 (1959)].

[8] For the application of the χ^2 test to compare two experimental distributions see A. Hald, *Statistical Theory with Engineering Applications* (John Wiley & Sons, Inc., New York, 1952), Chap. 23, Sec. 4.

TABLE I. List of interactions used in this analysis.

No.	Designation of the event	Type	γ_c	σ_{n-1}	D	Remarks
			Jets with $N_h \leqq 5$ and $n_s \leqq 20$			
1	116	$0+\ 7p$	10.7	0.49	+0.43	
2	63	$2+17p$	135	0.65	+0.18	
3	117	$0+17p$	28.2	0.67	−0.29	
4	56	$4+\ 6n$	58.9	0.69	−0.33	
5	70	$4+16p$	31.6	0.71	+0.37	
6	23	$3+18n$	53.7	0.71	+0.11	
7	83	$2+20p$	35.5	0.75	+0.20	
8	9	$1+11p$	18.1	0.76	+0.27	
9	18	$1+10p$	85.1	0.77	0	
10	49	$0+\ 9p$	30.2	0.81	+0.11	
11	51	$2+14n$	68.4	0.85	+0.28	
12	88	$1+14p$	38.1	0.85	+0.14	
13	91	$0+\ 7p$	115	0.87	−0.14	
14	47	$2+13p$	79.4	0.92	+0.38	
15	4	$2+16p$	55.0	0.93	+0.37	
16	102	$5+18p$	93.3	0.95	+0.11	Taken for analysis of individual cones.
17	107	$1+12p$	38.1	1.03	+0.50	
18	40	$0+13p$	295	1.11	+0.08	
			Jets with $N_h \leqq 5$ and $n_s > 20$			
19	68	$4+27p$	41.7	0.35	−0.18	
20	45	$2+41p$	72.5	0.58	−0.17	
21	69	$3+32p$	31.6	0.63	−0.19	
22	72	$1+26p$	91.2	0.65	−0.31	
23	20	$2+27p$	17.8	0.66	+0.11	
24	15	$4+27p$	16.2	0.69	+0.04	
25	118	$0+21p$	89.1	0.69	+0.24	
26	13	$3+33p$	36.3	0.76	+0.15	
27	62	$0+22n$	25.1	0.78	+0.27	
28	25	$5+34p$	79.4	0.80	−0.18	
29	64	$4+58p$	110	0.82	+0.14	
30	7	$5+33p$	46.8	0.84	+0.09	
31	89	$0+22p$	39.8	0.94	+0.27	Taken for analysis of individual cones.
32	82	$5+33p$	18.2	0.97	−0.15	
33	28	$1+23p$	39.8	0.98	+0.22	Taken for analysis of individual cones.
34	12	$3+23p$	72.5	1.03	+0.22	
35	120	$3+23p$	57.6	1.11	−0.13	
			Jets with $5 < N_h \leqq 8$			
36	119	$6+19p$	9.1	0.66	+0.16	
37	108	$6+13p$	28.2	0.87	+0.38	
38	1	$6+16p$	58.9	1.11	−0.12	
			Jets with $N_h > 8$			
39	76	$18+32p$	15.5	0.41	+0.12	
40	84	$14+11p$	36.3	0.46	+0.27	
41	95	$25+37p$	11.2	0.58	+0.03	
42	11	$15+67p$	49.0	0.64	+0.07	
43	31	$11+18p$	85.2	0.64	0	
44	50	$10+23p$	87.1	0.70	+0.43	
45	86	$11+33p$	49.0	0.73	+0.03	
46	3	$11+13p$	24.6	0.76	+0.39	
47	73	$15+17p$	93.3	0.78	+0.18	
48	34	$22+26p$	19.1	0.82	+0.08	
49	14	$17+34p$	74.2	0.82	0	
50	80	$23+60p$	38.0	0.83	−0.03	
51	103	$29+31p$	14.1	0.88	+0.22	
52	35	$23+34p$	53.7	0.89	+0.29	
53	99	$11+29p$	26.9	1.02	−0.03	
54	97	$11+21p$	47.9	1.22	+0.33	
	94	$18+64p$	38	0.70	+0.09	
		$16+63p$	31	0.76	+0.05	Events used in the last step of the analysis only (Sec. 4).
		$14+52n$	28	0.81	+0.08	
	38	$22+51p$	23	0.90	+0.14	

(a)

(b)

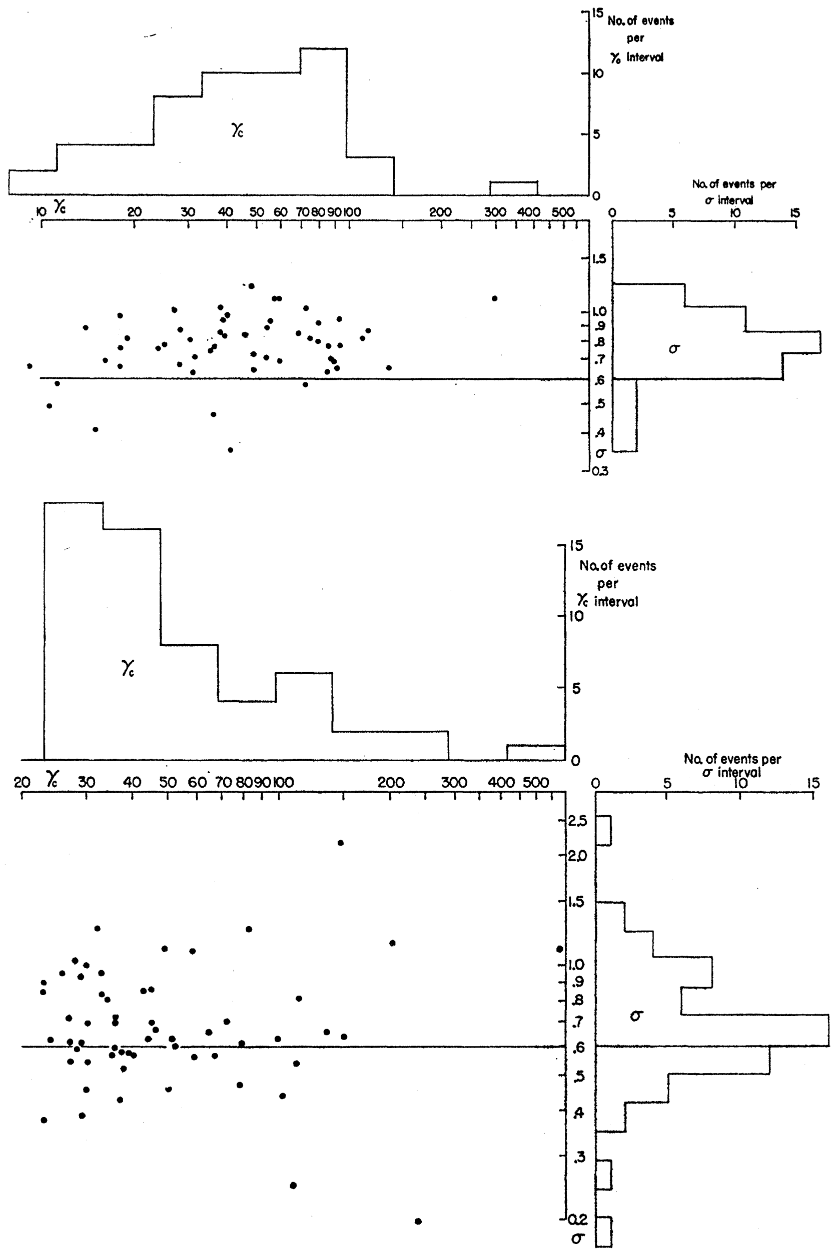

FIG. 2. γ_c vs σ: (a) for events analyzed in the present paper, (b) for events from GMZ paper.

standard deviation is 14 for the Monte Carlo showers, and 23 for the real events. All this can probably be explained by the fact that the events have a rather wide range of primary energies and are produced by collisions with different kinds of target nuclei.

In the next step of this analysis we have divided our collection of jets into two groups, with $N_h \leqq 5$ and with $N_h > 8$. The first group was then subdivided into 2 groups with $n_s \leqq 20$, and $n_s > 20$. The composite histograms of these groups are shown in Figs. 5 and 6. In testing the deviations from the normal distribution special care was taken for the group with $N_h \leqq 5$ and $n_s \leqq 20$. The histogram was composed, not with the straight line corresponding to the normal distribution but with the so-called modified t distribution, which arises if we apply our procedure to small random samples taken from the normal distribution.[9] As can be seen from the figures, all deviations are towards the shape expected from the two-center model. The deviations obtained from the χ^2 test are given in Table II. No subdivision into events with $\sigma < 0.6$ and $\sigma > 0.6$ was done because the total number of events with $\sigma < 0.6$ was very small (6 events).

[9] We would like to thank Dr. R. G. Glasser for the discussion concerning this question and for preparing curves for the modified t distribution.

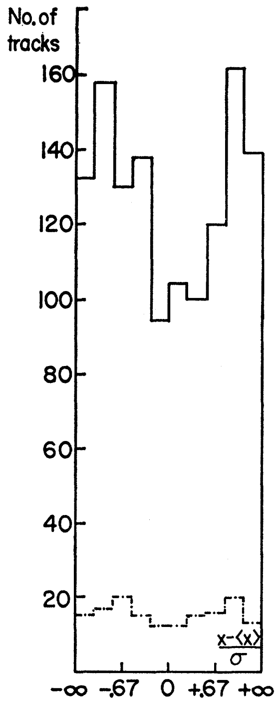

FIG. 3. Histograms of composite angular distributions in the $(x-\langle x\rangle)/\sigma$ coordinate divided into intervals corresponding to equal areas of the normal (Gaussian) curve. Full line represents 54 events presented in Table I; dot-dashed line represents events having $\sigma<0.6$ (6 events).

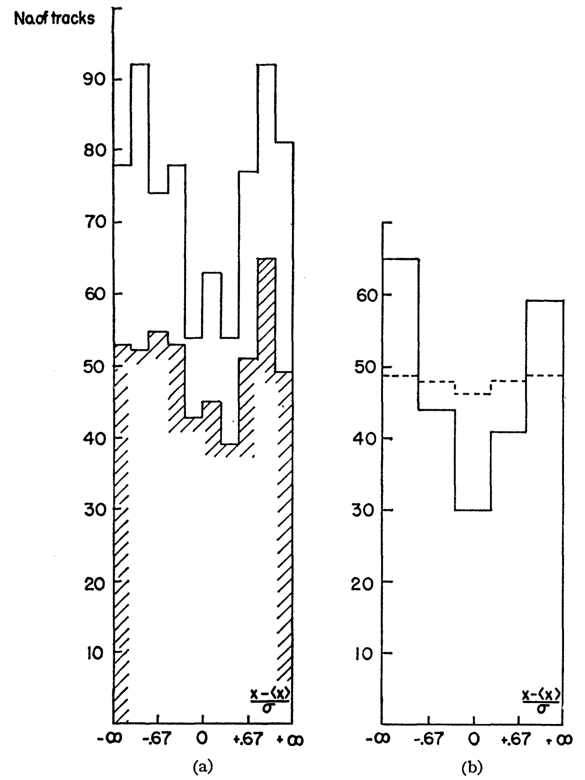

FIG. 5. Histograms of composite angular distributions for events characterized by $N_h\leq 5$: (a) all events (35) (upper line); lower line, those having $n_s>20$ (17 events); (b) 18 events having $n_s\leq 20$ (full line) and the histogram corresponding to the modified t distribution (dashed line) used for evaluation of the deviation of the experimental histogram from the normal shape.

The distribution of D values is presented in Fig. 7, both as a function of the dispersion σ and of multiplicity n_s. For comparison, the distribution of D values of our "Monte Carlo showers" is also presented as a function of σ. The striking asymmetry in the number of positive and negative D values for the experimental jets is clearly seen. For 38 events D is positive; for 3 events $D=0$, and for 13 events it is negative. On the contrary, for "Monte Carlo showers" this distribution is symmetric, 21 positive D values, 10 values $D=0$, and 23

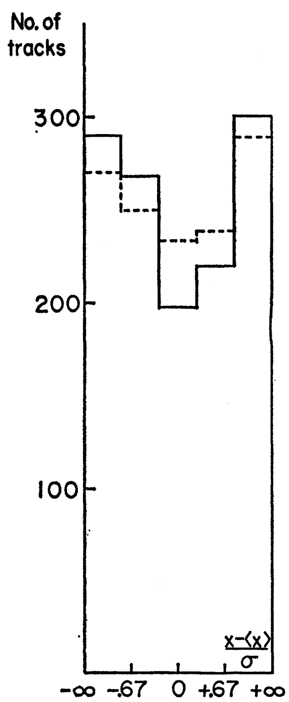

FIG. 4. Histogram from Fig. 3 (full line) in comparison with the histogram of "Monte Carlo events" (dashed line).

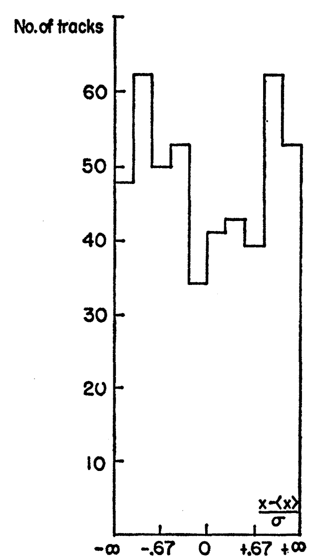

FIG. 6. Histogram of the composite angular distributions in the coordinate $(x-\langle x\rangle)/\sigma$ for 16 events characterized by $N_h>8$. Intervals on the $(x-\langle x\rangle)/\sigma$ axis correspond to equal areas of the normal curve.

TABLE II. Comparison of angular distributions with a normal distribution for several groups of interactions.

Group	Fig.	Number of jets	Number of tracks	Level of significance of the deviation from the normal distr.	Remarks
All events	3	54	1277	more than 3 stand. dev.	
"Monte Carlo jets"	4, dashed line	54	1277	$\sim$2 stand. dev.	This is the deviation between experimental jets and "Monte Carlo jets." In evaluation of the χ^2 it was taken into account that both distributions are fluctuating.
Events with $N_h \leqq 5$	5(a)	35	744	more than 2 stand. dev.	
Events with $N_h \leqq 5$ and $n_s > 20$	5(a)	17	505	less than 1 stand. dev.	
Events with $N_h \leqq 5$ and $n_s \leqq 20$	5(b)	18	239	more than 2 stand. dev.	In comparison to the "modified t distribution."
Events with $N_h > 8$	6	16	485	2 stand. dev.	

negative D values. The continuous line on the diagram D versus σ corresponds to the dependence predicted by the two-center model for nucleon-nucleon collisions. Since any asymmetry in the number of charged particles emitted from both centers diminishes the D value, the observed distribution can be considered as being in rather good agreement with the prediction of the two-center model, especially for events having a small number of evaporation tracks, $N_h \leqq 5$, and a small number of shower tracks, $n_s \leqq 20$—full dots in Fig. 7(a). The increase of D with decreasing multiplicity n_s is seen in Fig. 7(c). The mean D value for events having $N_h \leqq 5$ and $n_s > 20$ is $+0.03$, while for events with $N_h \leqq 5$ and $n_s \leqq 20$ it is $+0.15$.

Having in mind the last effect, it would be important to have some information about the angular distribution of events characterized by a very low number of shower particles ($n_s < 6$).

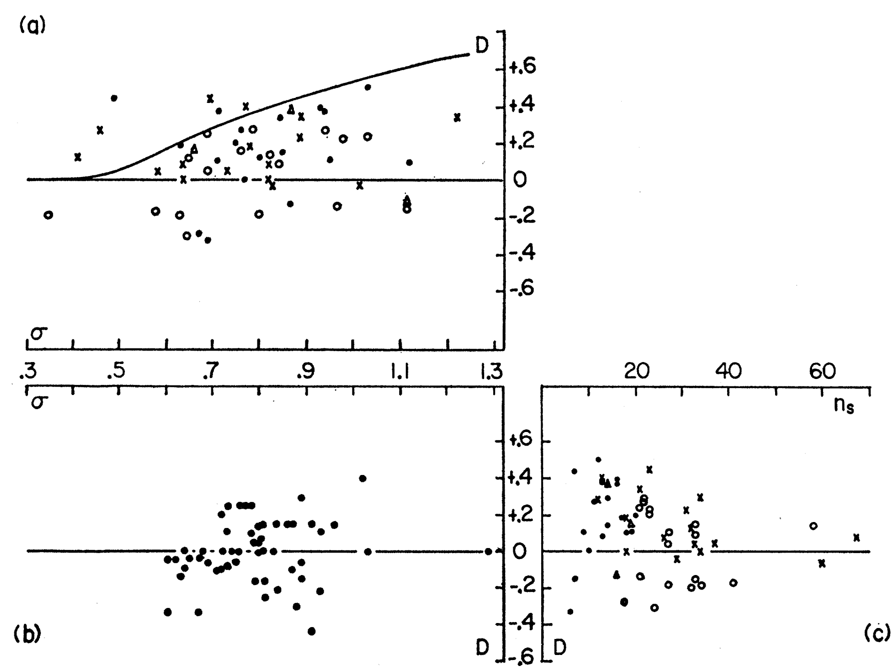

FIG. 7. (a) D vs σ distribution for experimental events. ●—events with $N_h \leqq 5$ and $n_s \leqq 20$. ○—events with $N_h \leqq 5$ and $n_s > 20$. △—events with $5 < N_h \leqq 8$. ×—events with $N_h > 8$. (b) D vs σ distribution for "Monte Carlo events." (c) D vs n_s distribution for experimental events.

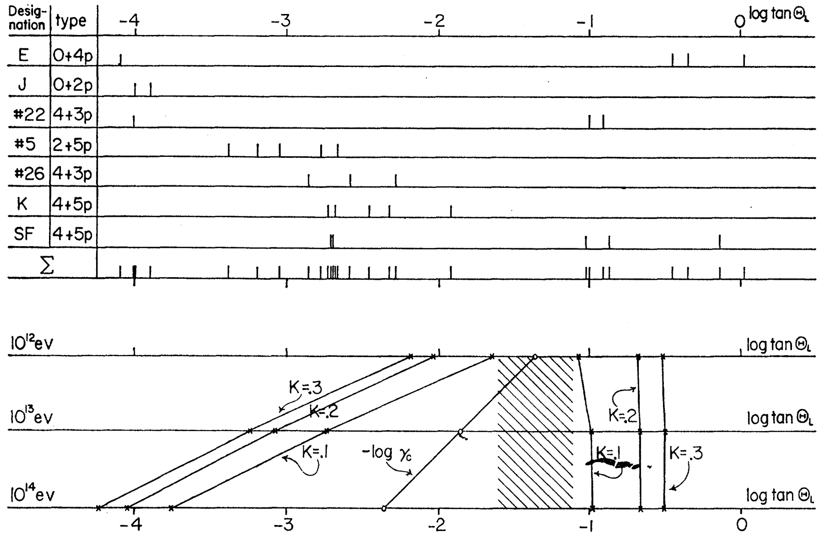

FIG. 8. Angular distributions in the x coordinate (laboratory system) for 7 events with low multiplicity ($n_s<6$) and the composite distribution in the laboratory system (Σ). The points marked by $\times$ on the lower part of the figure mark the positions of the centers of the two maxima, as predicted by the two-center model for the multiplicity $n_s=5$. Energies and inelasticities K are marked in the figure. The shadowed part should show the smallest density of tracks and may be compared with the composite distribution Σ.

(b) Low Multiplicity Events

All events having a multiplicity <6 had been rejected from the present analysis because of very large uncertainties in the parameters γ_c, σ, and D characterizing the event. However, it is possible to draw some conclusions from the angular distributions of these low-multiplicity events. This is possible because of very characteristic features of the angular distributions in the laboratory system predicted by the two-center model for such events. Let us write the principle of energy conservation applied to the two-center model [see Ciok *et al.*, Eq. (8)[10]]:

$$1.5n_sE_\pi\bar{\gamma}=2K\gamma_c, \tag{4}$$

where n_s=multiplicity, E_π=the mean total energy (in units of nucleon mass) of a particle in the rest system of the center from which it is emitted, $\bar{\gamma}$=the Lorentz factor of the center in the c.m. system, K=the inelasticity coefficient, and γ_c=the Lorentz factor of the c.m. system.

Starting from this formula and assuming $n_s=5$ and $E_\pi=0.5$ Bev (from the well-known value of 300 Mev/c for the transverse momentum), we can derive the relation γ_c versus $\bar{\gamma}$ for a given value of K taken as a parameter. We can also easily find the relation γ_c versus δ, where 2δ is the separation between two maxima of the distribution in the $x=\log_{10}\tan\theta$ coordinate [see Ciok *et al.*, Eq. (16)[10]].

In the lower part of Fig. 8 we see the results of this calculation shown for three values of the inelasticity coefficient. The two very characteristic features of this picture are the energy independence of the position of the diffuse cone in the laboratory system, and a minimum of the density of tracks in the region of x from about -2 to -1. The position of this region is also energy independent for energies $\gtrsim 10^{12}$ ev.

Looking for the experimental evidence for this, we have collected all low-multiplicity ($n_s<6$) and low-evaporation ($N_h\leqq 5$) events having high energy. Four such events were found in this laboratory by tracing back high-energy electromagnetic cascades and another three were taken from the published materials of the Bristol laboratory.[11] The angular distributions for individual events as well as the sum of these distributions in the laboratory system are given in the upper part of Fig. 8. The agreement with the predictions of the two-center model is evident from the figure. The striking asymmetry in the number of particles may be caused by the method of finding these events by scanning for cascades. An unbiased sample of such collisions can be obtained from the work of Lohrmann, Teucher, and Schein[12] on nuclear interactions of protons and neutrons of average energy about 250 Bev. These events were obtained by a systematic scanning in the core of high-energy fragmentations of heavy nuclei.

This sample of jets has two advantages. The scanning is unbiased, along the track scanning for the p interactions, and the primary energy is known from scattering measurements. So we can add the individual angular distributions in the c.m. system, assuming that the collision was a nucleon-nucleon collision. The individual angular distributions in the c.m. system of 7 such events as well as the composite distribution of all the tracks are shown in Fig. 9. It is worthwhile to stress the following features of the composite distribution:

[10] P. Ciok, T. Coghen, J. Gierula, R. Hołyński, A. Jurak, M. Mięsowicz, T. Saniewska, and J. Pernegr, Nuovo cimento **10**, 741, (1958).

[11] C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of Elementary Particles by the Photographic Method* (Pergamon Press, New York, 1959), p. 550, Fig. 15–12.

[12] E. Lohrmann, M. Teucher, and M. Schein, this issue [Phys. Rev. **122**, 672 (1961)].

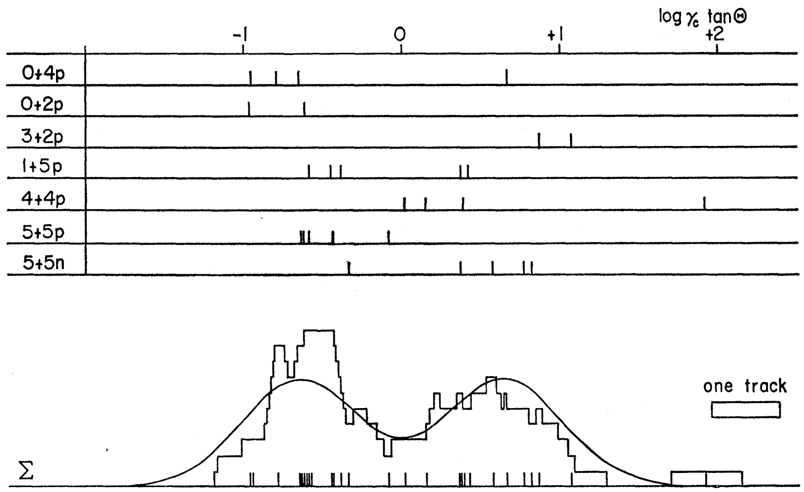

FIG. 9. Angular distributions in the c.m. system of low-multiplicity events ($n_s<6$) in the energy region around 300 Bev and the corresponding composite distribution (Σ). The "constant area" histogram of the composite distribution is drawn for comparison with the distribution expected by the two-center model (continuous line).

1. There is symmetry in the number of particles in both cones (14 and 13 tracks in the narrow and diffuse cone, respectively) suggesting that the explanation of the asymmetry in the high-energy events by scanning bias was right.

2. The two-maximum shape of the distribution is in rather good agreement with the shape predicted by the two-center model for the same dispersion σ. Another more quantitative formulation of this fact would be the following. The measured dispersion of the composite distribution is $\bar{\sigma}=0.73$; the corresponding D value predicted by the two-center model is $D=+0.30$, while the measured $D=+0.26$.

3. The 7 low-multiplicity events correspond to the high-σ tail of the σ distribution of all the 250-Bev interactions found in the systematic scan. The mean σ value for the whole sample is 0.57.[12]

4. The inelasticity coefficient found from Eq. (4) assuming $\bar{n}_s=4$, $E_\pi=0.5$ Bev, and a value of $\bar{\gamma}$ corresponding to $\sigma=0.73$ is $K\approx0.3$, which should be compared with the mean value $K\approx0.19$, which can be obtained for these 8 events from Fig. 5 in the work of Lohrmann, Teucher, and Schein.[12]

Concluding, we can say that there is a rather good agreement between the predictions of the two-center model and the experimental material for low-multiplicity collisions in the primary energy region of 250 Bev as well as over 1 Tev.

(c) Isotropy in Individual Cones

The experimental material concerning the evidence of the isotropic emission of particles from individual centers collected up to now is rather small The largest material was presented by Ciok *et al.*[10] and it includes 7 events with a total number of ~70 tracks As can be seen from Fig. 1 only the events with $\sigma\gtrsim0.9$ can be used for this investigation In these events, the separation of both cones is large enough that we can be rather sure that there is no, or only very small, mixing of tracks belonging to different cones.

In choosing the events for this investigation we have introduced the additional criterion $D>0$. Of course, we have not used jets with $N_h>8$ to eliminate the collisions with heavy nuclei for which the two-center model does not give any clearcut predictions as yet. We have 8 jets satisfying these criteria. They are marked in the Table I. The total number of tracks in these events is 140. The composite integral angular distributions in individual cones for these events are presented in Fig. 10 for both cones separately in Duller-Walker[13] coordinates. For comparison, the straight lines corresponding to the isotropic distributions are also drawn. There is a good agreement with isotropy. This agreement can also be

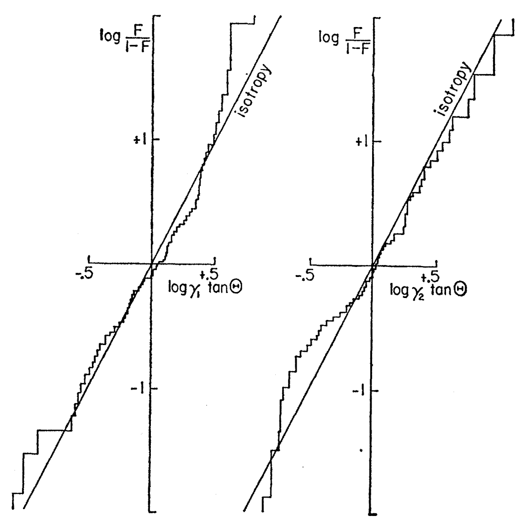

FIG. 10. Integral angular distributions in individual cones for events having $N_h\leq5$, $\sigma>0.9$, and $D>0$: (a) forward cones, (b) backward cones. Straight lines correspond to isotropic distribution and are given for comparison.

[13] N. M. Duller and W. D. Walker, Phys. Rev. 93, 215 (1954).

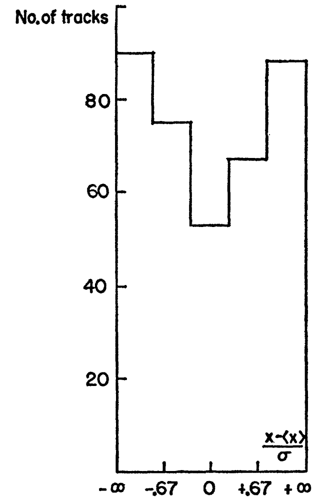

FIG. 11. Histograms of composite angular distribution for 24 events having $N_h \leqq 1$. The total number of tracks is 373. (Concerning coordinates, see caption for Fig. 3.)

seen when comparing the dispersions of both distributions with the dispersion for isotropic distribution: σ (isotropic) $=0.39$, σ (narrow cone) $=0.40$, and σ (diffuse cone) $=0.43$.

4. COMPARISON AND COMPILATION OF RESULTS

There is good agreement of the results reached in this investigation with the ones in the paper of GMZ. This is best seen when comparing the Tables II of both papers, including the levels of significance for the deviations from the normal distribution in several groups of jets. Very significant and comparable deviations towards the two-maximum shape of the angular distribution have been observed for the whole sample of jets and for the groups with $N_h \leqq 5$, $n_s \leqq 20$, $\sigma > 0.6$. No significant deviations or deviations not in the direction predicted by the two-center model were observed in the group having $N_h \leqq 5$, $n_s > 20$, $\sigma > 0.6$. Also both the distributions in the groups of jets with $N_h > 8$ show a very significant two-maximum structure, but this evidence seems to be very much stronger in the GMZ collection than in ours. In view of this general agreement of the results reached on independent samples of events, it seems justified to combine both groups of experimental materials.

Two important questions arise in connection with the observations of angular distributions having two maxima.

1. What is the experimental evidence that the two-maximum shape of the angular distribution is attached to nucleon-nucleon collisions?
2. How can the two-maximum structure of angular distributions in collisions with heavy nuclei be explained?

Concerning the first question, we can say that there are no very efficient criteria for deciding if a high-energy collision is a collision with single nucleon or not. We can only obtain a sample of events enriched in nucleon-nucleon collisions rejecting, for instance, all events with $N_h > 1$. In the joint experimental materials of GMZ and the present work, there are 24 jets which satisfy this condition and have $\sigma > 0.6$. The composite angular distribution of this group of events is presented in Fig. 11. It has a typical two-maximum shape which deviates from the normal distribution by more than 2 standard deviations.

In GMZ and in the present investigation, the criterion $N_h \leqq 5$ and $n_s \leqq 20$ was used. The additional condition limiting n_s seems to be justified by the results reached by Lohrmann, Teucher, and Schein[12] in the investigation of collisions in the energy region of 250 Bev. The histogram showing the composite angular distribution for these events (having $\sigma > 0.6$) based on the

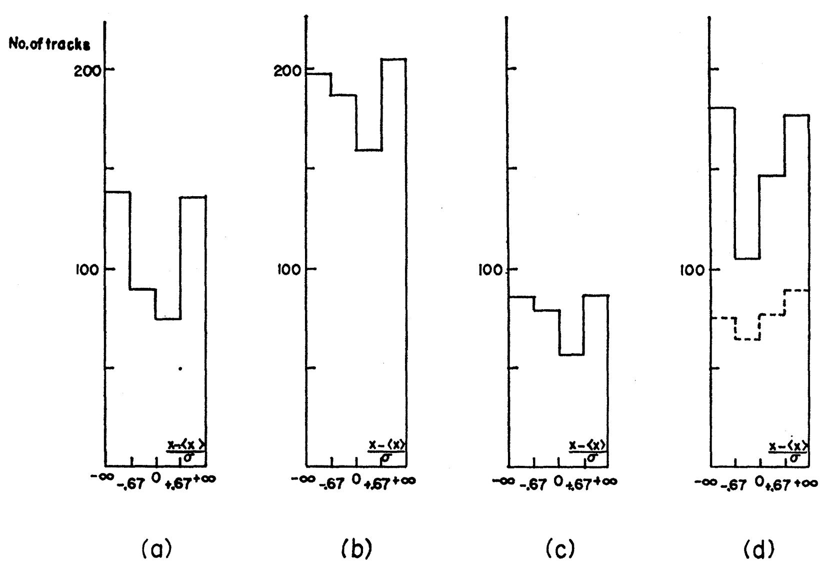

FIG. 12. Histograms of the combined angular distributions for events characterized by $\sigma > 0.6$ and $N_h \leqq 5$, $n_s \leqq 20$; (b) $N_h \leqq 5$, $n_s > 20$; (c) $N_h > 8$, n_h 40; (d) $N_h > 8$, $n_s > 40$. Dashed line corresponds to the subgroup of events with no visible double-maximum structure.

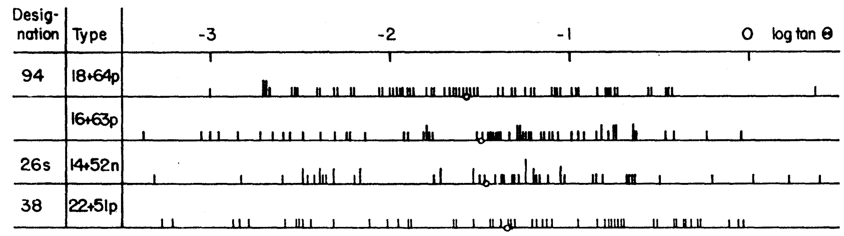

FIG. 13. Angular distribution of four additional events characterized by $N_h>8$, $n_s>40$.

joint material is presented in Fig. 12(a). It shows again the typical two-maximum shape deviating by more than 3 standard deviations from the normal distribution.

Both results seem to give a rather strong support to connecting the two-maximum shape of the angular distribution with nucleon-nucleon collisions.

In connection with the second question, we present 3 other histograms based on the joint materials. These are the angular distributions for groups of events obeying the following conditions: $N_h \leqq 5$ and $n_s>20$ [Fig. 12(b)], $N_h>8$ and $n_s<40$ [Fig. 12(c)], and $N_h>8$ and $n_s>40$ [Fig. 12(d)]. In addition, there is a common condition for all groups: $\sigma>0.6$. Four additional events from our laboratory were added to the last group to increase statistics. The characteristics of these events are given at the end of Table I and the distributions are presented in Fig. 13. Some parameters corresponding to the four groups of events presented in Fig. 12 are given in Table III.

The following conclusions can be drawn from Fig. 12 and the data included in Table III. In all groups the distributions show the deviation from the normal shape towards the two-maximum shape. The rather small deviation in the group with $N_h \leqq 5$, $n_s>20$ in comparison with the mean dispersion σ is in disagreement with the prediction of the simple two-center model for nucleon-nucleon collisions. There is a striking similarity in the shape of angular distributions between the group with $N_h \leqq 5$, $n_s \leqq 20$, and both groups corresponding to collisions with heavy nuclei ($N_h>8$).

Events in the last group have so high a number of tracks that some characteristics of the shape of the angular distribution can be easily seen in single events. By comparing these shapes, large differences between individual events have been found. To have some idea about the significance of these differences, all jets in this group were divided into two subgroups with clear two-maximum structure and without such structure. The dashed line in Fig. 12(d) corresponds to events without visible double structure. This distribution does not deviate from the normal one, while the distribution corresponding to the jets with visible two maxima deviates very strongly. Both distributions deviate from each other by more than 2 standard deviations. There is a correlation between the shape and the dispersion of the distribution. The mean dispersion in the group having the normal shape is 0.76, while in the group having two maxima, the dispersion is 1.0.

A similar correlation was observed by GMZ for collisions characterized by $N_h \leqq 5$ and $n_s \leqq 20$ for lower dispersions. On the basis of similar observations, Bartke *et al.*[14] and GMZ suggest that there may be some relationship in physical processes for nucleon-nucleon, and nucleon-nucleus collisions.

TABLE III. Combined results of this paper and reference 1.

N_h	$\leqq 5$		>8			
n_s	$\leqq 20$	>20	<40	>40		
No. of events	34	22	14	11	5	6
No. of tracks	440	749	309	610	300	310
$\bar{N}_h$	1.74 ± 0.23	2.90 ± 0.36	13.2 ± 1.0	19.9 ± 1.4	18.8 ± 2.0	20.8 ± 1.7
$\bar{n}_s$	12.9	34.0	22.0	55.5	60.0	51.7
Deviation from the normal distribution	>3 st. dev.	~1.5 st. dev.	2 st. dev.	>3 st. dev.	<1 st. dev.	>3 st. dev.
$\bar{\sigma}$	0.85	0.79	0.82		0.76	1.01
Fig. 12	(a)	(b)	(c)	(d)		

[14] J. Bartke, P. Ciok, J. Gierula, R. Hołyński, M. Mięsowicz, and T. Saniewska, Nuovo cimento **15**, 18 (1960).

5. CONCLUSIONS

The compilation of the results reached in this investigation with results presented by GMZ contains 128 collisions of singly charged or neutral particles of primary energy in the region from 1 to about 100 Tev. The angular distributions of these events show in the average a significant deviation from the shape of a normal distribution predicted by hydrodynamical theory.[7] The two-center model of multiple meson production seems to be a better working hypothesis for explaining the general shape of the angular distribution. In addition, the following observations are in agreement with the predictions of this model.

1. The relation between the shape of the angular distribution and its dispersion σ for collisions characterized by low evaporation ($N_h \leqq 5$) and not a very high number of shower particles ($n_s \leqq 20$).
2. The relation between the shape of the angular distribution and the number of shower particles for the same type of collisions as in 1, if the inelasticity is assumed not to depend strongly on n_s (see reference 12).
3. The relation between the inelasticity and the angular distribution for collisions in the 300-Bev energy region characterized by a small number of shower particles ($n_s \leqq 5$).
4. The angular distribution of shower particles in the rest system of individual centers ("fireballs") for events characterized by $N_h \leqq 5$ and high enough dispersion ($\sigma > 0.9$) is roughly isotropic.

However, one observation is in disagreement with the two-center model for nucleon-nucleon collisions. The highly anisotropic angular distributions for events having low evaporation ($N_h \leqq 5$) and high multiplicity ($n_s > 20$) do not show the shape of angular distribution predicted by the two-center model for this degree of anisotropy.

The general shape of the angular distributions for collisions with heavy nuclei of the photographic emulsion is very similar to the shape observed in collisions which are probably nucleon-nucleon collisions. However, in collisions producing a very large number of secondaries (≈ 60) large differences in the individual shapes are observed. The two-center model in its present form is not applicable to this type of collision.

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