Nuclear Interactions of Protons, Neutrons, and Shower Particles of Very High Energy in Nuclear Emulsion*

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Eighty-four interactions of protons and neutrons were located in a 22 liter stack of nuclear emulsion by tracing back showers of minimum-ionizing particles to their origins. The distribution of the number of shower particles, and the number of heavily ionizing prongs are presented for 57 events with dip angles $<17^{\circ}$. The average energy of these events is 3.5×10^{12} ev. The average number of shower particles emitted in nucleon-nucleon collisions at this energy is 15 ± 5 , as estimated from 8 interactions without heavy prongs. The angular distributions of the shower particles are presented for the 57 events. They can be transformed into a system in which the angular distribution is roughly symmetric. This is true even for the collisions with heavy target nuclei $(N_h > 5)$. The degree of anisotropy of the angular distributions is in disagreement with a hydrodynamical model of nucleon-nucleus

primary particles of 20 cm in emulsion was obtained. By scanning the forward cone of the primary interactions, 76 secondary interactions of charged and neutral shower particles were found. The distribution of the prong numbers, of the energy, and the characteristics of their angular distribution are presented. The best estimate of the ratio of secondary collisions produced by neutral particles, and the number produced by charged particles is: $N_n/N_{\rm ch} = 0.40 \pm 0.11$. Adding this result to other published data, it is concluded that $30{\pm}6\%$ of the particles produced in collisions having a primary energy of several Tev are not π mesons. A collision mean free path of 41 ± 8 cm was found for the forwardcone shower particles.

collisions. A lower limit for the collision mean free path of the

1. INTRODUCTION

URING the last decade, many papers have been published about high-energy $(E > 10^{12} \text{ ev})$ meson showers observed in nuclear emulsion. Due to the small flux of particles of high energy in the cosmic radiation, much of our present experimental knowledge is based on observations made on small numbers of events. The purpose of the present investigation is to present data obtained from a large number of interactions found and analyzed in a uniform way.

2. EXPERIMENTAL PROCEDURE

A 22 liter stack of Ilford G 5 emulsion, consisting of 200 pellicles, each 60 cm \times 30 cm \times 600 μ , was exposed over Texas at an altitude of 116 000 ft for 13 hours.

Each plate was scanned for showers of parallel minimum tracks along the lines indicated in Fig. 1. The total magnification used was about 300. All showers having more than 10 parallel tracks in one field of view were traced back to the origin, which can be one of the following possibilities: (1) a nuclear interaction caused by a neutral or singly charged particle, or by a heavy nucleus; (2) a high-energy electron-positron pair; (3) a shower entering the stack from outside.

The scanning criterion puts a lower limit of about 100 Bev for the energy contained in the electron-photon component of the showers. The scanning loss has not

No. of events	L (cm)	$\langle l \rangle$ (cm)
39	>30	17.1
20	>40	22.5

TABLE I. Interaction mean free path $\lambda = \langle l \rangle$ of primary protons.

been evaluated but it is probably quite large for showers having an energy release of the order of 100 Bev.

Altogether, 78 nuclear interactions produced by singly charged particles, and 6 interactions produced by neutral particles were found. Since the exposure was made very near the top of the atmosphere, it can be assumed that the primary particles of these showers were protons and neutrons.

Fifty-seven events with dip angles $<17^{\circ}$ were analyzed. The average primary energy of this sample is about 3.5 Tev (see Sec. 5).

3. INTERACTION MEAN FREE PATH OF PRIMARY PROTONS

In principle, the interaction mean free path of the protons producing the high-energy showers can be



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death.

					250 Bev events (reference 1)			
	$N_h=0$	$N_h \leq 5$	$N_h > 5$	All events	$N_h \leq 5$	$N_h > 5$	All events	
No. of events $\langle N_h \rangle$ $\langle n_s \rangle$	$\overset{8}{\overset{0}{_{15\pm5}}}$	38 2.2 \pm 0.4 20 \pm 3.5	$19 \\ 15 \pm 3.5 \\ 28 \pm 6.5$	$57 \\ 6.5 \pm 0.9 \\ 22.5 \pm 3$	28 10.9±1.9	24 16.5 \pm 3.4	$52 \\ 8.5 \pm 1.2 \\ 12.9 \pm 1.8$	

TABLE II. Average multiplicities of the primary proton and neutron interactions.



FIG. 2. Distribution of N_h and n_s of the 57 primary interactions.

obtained from the distribution of the shower origins inside the stack. In order to get a good result, however, the stack must be large compared to the interaction mean free path. This condition is not quite fulfilled in our case. The integral distribution of the length l the primary particle traveled in the stack before interacting has been plotted for two values of the potential length L of the whole event in the stack, L>30 cm and L>40cm. L is the total length of the primary track plus the length of the shower in the stack. The value of the mean free path λ obtained for the two different choices of Lis listed in Table I. λ is largely determined by the geometry of the stack, and by L, since λ increases with L. Our statistics allow us to get only a lower limit for the mean free path. We get

$\lambda > 20$ cm.

This should be compared with the geometrical mean free path in emulsion of 28 cm.

4. GENERAL CHARACTERISTICS OF THE INTERACTIONS

All the information regarding the distribution of the number of thin tracks n_s , and the number of grey and black tracks N_h , is contained in Fig. 2, for the 57 showers having dip angles <17°. Each point represents one event. The average number of shower particles increases with the average number of heavy prongs. Since interactions with $N_h > 8$ must have occurred on heavy target nuclei (Ag or Br), this means that the multiplicity n_s rises with increasing mass of the target nucleus. The number of thin tracks n_s in an individual shower shows very large fluctuations. Average multiplicities are given in Table II. Again, the influence of the mass of the target nucleus on the mean multiplicity can be seen. The values quoted for the 8 events with $N_h=0$ is our best estimate for the multiplicity expected for nucleonnucleon collisions for energies of several Tev. Since the criterium $N_h=0$ is not sufficient for isolating nucleonnucleon collisions, the value given is expected to overestimate the true multiplicity for nucleon-nucleon collisions slightly. A discussion, and a comparison of multiplicities obtained at different energies have been given elsewhere.¹

5. ANGULAR DISTRIBUTION OF THE SHOWER PARTICLES

The angle θ_L of all shower particles with respect to the line of flight of the primary particle was measured



FIG. 3. Angular distribution of the shower particles from the primary interactions plotted over $\log_{10} \tan \theta_L$. Events with $N_h = 0$.

¹ E. Lohrmann, M. W. Teucher, and M. Schein, this issue [Phys. Rev. 122, 672 (1961)].

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Fig. 4. Angular distribution of the shower particles from the primary interactions plotted over $\log_{10} \tan \theta_L$: (a) events with $N_h \leq 5$, $n_s > 5$; (b) events with $N_h > 5$, $n_s > 5$; (c) events with $n_s \leq 5$.

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FIG. 5. Combined angular distribution of shower particles, normalized to the same primary energy according to Eqs. (2) and (3). $N_h=0$.

for the 57 events. In the following, the angular distribution of the shower particles will be discussed in terms of the variable

$$x = \log_{10} \tan \theta_L.$$
 (1)

Figures 3, 4(a), 4(b), and 4(c), show the detailed angular distributions of all our events plotted in the x coordinate.

An estimate of the primary energy E_0 can be obtained from the angular distribution by the relations²

$$\log_{10}\gamma_c = -\langle x \rangle, \tag{2}$$

$$E_0 = (2\gamma_c^2 - 1)Mc^2. \tag{3}$$

The value E_0 obtained in this way will for an individual case give only a very rough estimate of the true primary energy.¹ E_0 is included in Figs. 3, 4(a), and 4(b) for the 54 events with $n_s > 5$. In order to compare the various angular distributions, they have been normalized to the same primary energy. The only estimate available for the primary energy is that given by Eqs. (2) and (3). We have, therefore, combined the angular distribution of the 54 events with $n_s > 5$ by plotting them over the coordinate

$$y = x - \langle x \rangle = \log_{10}(\gamma_c \tan \theta_L) = \log_{10} (\tan \theta_L / \tan \theta_c). \quad (4)$$

In order to investigate the dependence of the angular distribution on the primary energy and the target nucleus, we have divided our events into five groups



FIG. 6. Combined angular distribution of shower particles, normalized to the same primary energy according to Eqs. (2) and (3). $N_h \leq 5$, high-energy group.

according to N_h and according to the primary energy as estimated from Eqs. (2) and (3). The combined angular distributions for the five groups of events are shown in Figs. 5, 6, 7, 8, and 9.

The shape of the five distributions is roughly symmetric. This indicates that there is a system of reference in which the angular distribution is symmetric. This system will be called the symmetry system (s system). The Lorentz factor γ_s of this system is given in a good approximation¹ by

$$\gamma_s = 1.3 \gamma_c. \tag{5}$$

In order to transform the angular distributions into this system, the momentum of the shower particles must also be known. This was estimated from the known distribution of the transverse momentum of shower particles. (See for example Nishimura,³ and Milekhin and Rosental.⁴ A list of some of the experimental results is given by Schein *et al.*⁵) A more detailed description of the procedure has been given elsewhere.¹ One obtains



FIG. 7. Combined angular distribution of shower particles, normalized to the same primary energy according to Eqs. (2) and (3). $N_h \leq 5$, low-energy group.

in this way a correction for the well-known formula

$$\tan(\theta_s/2) = \gamma_s \tan\theta_L,\tag{6}$$

which is only valid if the velocity of the *s* system is equal to the velocity of the particles emitted in this system. Figures 10 and 11 give the angular distribution in the c.m. system for events with $N_h > 5$ and $N_h \le 5$ separately. The two energy groups from Figs. 5–9 have been combined for this purpose. The symmetry in the *s* system can be tested by a χ^2 test as applied to a comparison between two experimental histograms.³ The distribution for events with $N_h \le 5$ is quite compatible with the assumption of symmetry. For showers with $N_h > 5$, the χ^2 test indicates a significant deviation from

³ J. Nishimura, quoted by Z. Koba, Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics (Interscience Publishers, Inc., New York, 1956), Vol. IV, p. 46. ⁴ G. A. Milekhin and I. L. Rosental, Soviet Phys.—JETP 33(6),

² C. Castagnoli, G. Cortini, D. Moreno, C. Frazinetti, and A. Manfredini, Nuovo cimento 10, 1539 (1953).

^{154 (1957).}

⁶ M. Schein, D. M. Haskin, E. Lohrmann, and M. W. Teucher, Phys. Rev. **116**, 1238 (1959).

symmetry of about 3 standard deviations. Looking at Fig. 11, it can be seen, however, that the actual deviation from symmetry is quite small. The reason for the result of the χ^2 test is probably the large number of tracks involved, which allows one to detect a minor deviation from symmetry. This near-symmetry in collisions with $N_h > 5$ is quite remarkable, since the majority of these collisions are produced on heavy target nuclei. The same symmetry was found in other independent observations at similar energies⁶ and at about 250 Bev.1

The angular distribution in the *s* system is roughly of the form

$$N(\theta_s)d\Omega \approx \sin^{-1}\theta_s d\Omega. \tag{7}$$

The near symmetry for the collisions with heavy nuclei would find an explanation in terms of a hydrodynamical model proposed by Belenki and Landau,7 and Milekhin.8 They assume that the incident nucleon interacts with a roughly cylindrical part of the target nucleus as a whole, and that the angular distribution of the mesons



FIG. 8. Combined angular distribution of shower particles, normalized to the same primary energy according to Eqs. (2) and (3). $N_h > 5$, high-energy group.

produced is symmetric in the center-of-momentum system. We shall now give a more detailed comparison of our data with this model.

The shape of the angular distribution can be described by the dispersion of the distribution in the *x* coordinate:

$$\sigma = \langle (x - \langle x \rangle)^2 \rangle^{\frac{1}{2}} = \langle y^2 \rangle^{\frac{1}{2}}.$$
 (8)

An isotropic angular distribution would give $\sigma = 0.39$. σ increases with increasing collimation of the shower in the direction of the shower axis. Values of σ for the various groups of showers are given in Table III. Included for comparison are σ values for showers of about 250 Bev. These values are smaller than the ones given in reference 1, since the angular distributions were normalized to the same true primary energy in reference



FIG. 9. Combined angular distribution of shower particles, normalized to the same primary energy according to Eqs. (2) and (3). $N_h > 5$, low-energy group.

(1), whereas here they have been normalized to the same energy as found from Eqs. (2) and (3).

Table III shows that σ increases slowly with energy. This means that with increasing primary energy the angular distribution of the showers gets more peaked in the forward and backward directions in the *s* system. The value of σ obtained by us is in good agreement with the result of the Polish group⁶ at about the same energy.

Our data can be compared with the calculations of Milekhin.⁸ The value expected for σ from the hydrodynamical model is

$$(2.3\sigma)^2 = 0.56 \log_{10}(E_0/M) + 1.6 \log_{10} \lceil 2/(n+1) \rceil + 1.6, \quad (9)$$

where n is the number of nucleons contained in the column in the target nucleus, E_0 the primary energy, and *M* the nucleon mass. We have used n = 3.4 for the events with $N_h > 5$ and n = 2 for the events with $N_h \le 5$. The theoretical values following from Eq. (9) are also given in Table III. Our data are in disagreement with the result of the hydrodynamical theory given by Eq. (9). The same disagreement exists for the result of the Polish group. An analysis of our angular distributions along different lines shows⁹ that also the detailed shape of the distribution is not in agreement with the hydrodynamical theory, which predicts a normal distribution over the x coordinate.

We conclude that the present version of the hydro-



⁹ J. Gierula, D. M. Haskin, and E. Lohrmann, following paper [Phys. Rev. 122, 626 (1961)]

⁶ J. Bartke, P. Ciok, J. Gierula R. Hołynski, M. Miesowicz, and T. Saniewska, Nuovo cimento 15, 18 (1960). ⁷ S. S. Belenki and L. D. Landau, Suppl. Nuovo cimento 3,

 ¹⁵ (1956).
 ⁸ G. A. Milekhin, Zhur. Eksp. i Theoret. Fiz. 35, 1185 (1958) [translation: Soviet Phys.—JETP 35(8), 829 (1959)].

<u>.</u>	$N_h=0$	$N_h \leq 5$ $\gamma_c > 55 \gamma_c < 55$	$N_{h} > 5$ $\gamma_{o} > 40 \gamma_{o} < 40$	All energies $N_h \leq 5 N_h > 5 N_h \geq 0$	250-Bev events (reference 1) $(n_s > 5)$ $N_h \le 5$ $N_h > 5$
No. of events $\langle \sigma \rangle$ σ expected from Eq. (9)	8 0.79 1.08	$\begin{array}{ccc} 17 & 18 \\ 0.81 & 0.76 \end{array}$	10 9 0.78 0.73	35 19 54 0.78 0.76 0.78 1.02 0.97	$\begin{array}{cccc} 23 & 23 \\ 0.53 & 0.54 \\ 0.88 & 0.81 \end{array}$

TABLE III. Average σ values.

dynamical model cannot explain all the experimental data about angular distributions.

The dispersion σ appears to depend little on the mass of the target nucleus, as can be seen from the comparison of the groups of events with $N_h \leq 5$, and $N_h > 5$. This is again in agreement with observations made at 250 Bev¹ and by the Polish group.⁶

Several authors¹⁰⁻¹² have pointed out that certain features of the angular distribution of jets suggest a model, in which the mesons are emitted isotropically from two centers ("fireballs"), which move with opposite velocities in the center-of-mass system. A detailed discussion of our data with respect to this model will be given elsewhere.9

6. AVERAGE PRIMARY ENERGY

Figure 12 shows the integral energy spectrum of events with $N_h \leq 5$. The energies were obtained from Eqs. (2) and (3). At high energies, the distribution is of the expected form $E^{-1.7}$. At lower energies, the distribution deviates from the power law. This must be attributed to the fluctuations of the energy estimate from Eqs. (2) and (3), and to scanning loss. In order to obtain the average energy of our showers, several corrections must be applied. The assumptions under which Eqs. (2) and (3) are derived, are too simplified. A better estimate of the primary energy is obtained if one uses instead of Eq. (3)

$$E = n(2\gamma_c^2 - 1)Mc^2.$$
 (10)

At primary energies around 250 Bev, n has been deter-



¹⁰ P. Ciok, T. Coghen, J. Gierula, R. Holynski, A. Jurak, M. Miesowicz, T. Saniewska, O. Stanisz, and J. Pernegr, Nuovo cimento 8, 166 (1958). ¹¹ G. Cocconi, Phys. Rev. 111, 1699 (1958). ¹² K. Wie Niewscimente 10, 004 (1959).

¹² K. Niu, Nuovo cimento 10, 994 (1958).

mined experimentally¹:

n=1.8 for events with $N_h > 5$,

n=0.75 for events with $N_h \leq 5$.

The same values of n were used in this work. The average energies of the showers with $N_h \leq 5$ and $N_h > 5$ are the same. A second correction must be made, since the large fluctuation of the angular distribution method together with the shape of the primary energy spectrum produces a systematic increase of the mean energy as estimated from Eqs. (2) and (10). This last correction depends also on the scanning bias and cannot be accurately made.

The average energy of our showers is, therefore, somewhat uncertain. The best estimate is 3.5 Tev. This value is derived from the geometrical mean energy. The flux of cosmic rays, as calculated from our data at energies >6 Tev where scanning loss is unimportant, agrees within a factor of 1.5 with a spectrum of the form

$$N(>E) = 2.5 \times 10^{-5} (E/10^{12} \text{ ev})^{-1.7} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$$
. (11)

Equation (11) was obtained by extrapolation between the measurements of the proton flux at the equator, and the data of the extensive air showers.

7. INELASTICITY

The inelasticity of a collision is in the laboratory system given by

n=

$$=\sum E_m/E_0,\qquad(12)$$



 TABLE IV. Secondary interactions of particles in the forward cone of the primary events.

N_h	n _s	E ₀	N _h	ns	E_{0}	1	Vл (b)	ns Neut	E ₀ ral pri-
	(a)	Charged	primary pa	article	s		ma	ary pa	rticles
0	8	2250	9	8	82		0	15	330
0	15	1800	9	11	180		1	5	3500
1	11	1440	9	11	60		1	15	4700
1	13	2650	9	11	16		1	18	940
2	5	50	9	26	21		1	27	250
2	14	246	10	14	70		2	13	65
3	7	29	10	18	110		2	13	12
3	14	200	11	7	60		2	16	52
3	15	350	11	9	60		4	5	500
4	11	925	12	26	58		4	5	5
4	11	390	13	15	30		4	7	680
4	13	108	16	6	5		5	10	2
5	7	2400	16	7	7		5	14	1030
5	7	8	16	12	16		6	5	25
5	12	11	16	18	60		7	6	1600
5	13	220	17	8	12		7	9	22
5	19	100	17	10	4		8	15	435
6	5	93	19	9	230		9	11	310
6	6	13	19	15	16	1	.2	17	24
6	10	45	23	31	57	1	3	8	[°] 6
6	17	70	33	12	6	1	4	15	4
7	8	72				1	.6	16	5
7	10	350				1	.6	20	100
7	18	44				2	:0	5	9
7	35	180				2	:0	10	9
8	9	34							
8	10	255							
8	10	60							
8	30	140							
9	7	220							

where E_0 is the primary energy and $\sum E_m$ can be obtained from the angular distribution of the shower particles, since their transverse momentum is quite independent of the angle of emission.³⁻⁵ One gets in a good approximation

$$\sum E_m = 1.65 \langle p_t \rangle \sum \sin^{-1} \theta_i. \tag{13}$$

The factor 1.65 takes into account the production of neutral particles. In calculating this factor, we have assumed that 30% of the particles produced are not π mesons (see Sec. 9) and that one-half of them are neutral. The value of the average transverse momentum $\langle p_t \rangle = 0.32$ Bev/c was taken from several published measurements.⁵

The primary energy E_0 can in our case only be estimated from the angular distribution Eqs. (2) and (10). Applied to an individual case this method is not very reliable. Therefore, only the inelasticity averaged over all the events gives a meaningful result. Even so, the uncertainties of the corrections discussed in Sec. 6 introduce rather large limits of error. The limits of error were obtained by making extreme assumptions regarding the scanning loss, and by flux arguments. We obtain for the average inelasticity

 $\langle \eta \rangle \approx 0.50$,

with a lower limit of 0.25.

8. INTERACTIONS OF THE SHOWER PARTICLES

A search for secondary interactions of the shower particles was carried out by scanning the forward cone of events with a dip angle $<17^{\circ}$, and 5 events of very high energy, which had a dip angle $>17^{\circ}$. The scan was conducted in a cone of half-opening angle of 1.0×10^{-2} rad around the shower axis for 5 cm. All interactions were recorded, which had 5 or more shower particles, collimated in the forward direction of the original interaction. The characteristics of the events found in this way are given in Table IV. Table IV contains also the primary energy of the interactions as estimated from the angular distribution of the shower particles by means of Eqs. (2) and (3). Fifty-one interactions produced by charged particles, and 25 interactions produced by neutral particles were found. Since the neutral interactions cannot be produced by π^0 mesons, they must be originated by neutrons, antineutrons, or neutral strange particles. No difference between these interactions, and those produced by charged particles, mostly π mesons, can be seen. Mean values of the prong number are given in Table V.

The geometrical mean energy of the shower particles in the forward cone, as estimated from the transverse momentum, and also from the angular distribution of the interactions is about 140 Bev.

Also included in Table V is the value of σ for the angular distribution of the secondary interactions. Comparison with Table III shows that the value of σ is about the same as for proton and neutron collisions of the same average energy. The average multiplicity $\langle n_s \rangle$ seems to be somewhat larger than the corresponding value for proton or neutron interactions of the same energy. However, the values of $\langle n_s \rangle$ and of $\langle N_h \rangle$ listed in Table V will overestimate the true multiplicity due to the restriction $n_s \geq 5$ used in the scanning, and due to scanning loss of events with $N_h=0$. A more careful scan without this restriction was made (see Sec. 10). The mean value of n_s resulting from this scan should be free from this systematic effect. It is also listed in Table V.

The energy distribution of the secondary interactions, as found from Eqs. (2) and (3), is shown in Fig. 13.



FIG. 13. Energy distribution of secondary interactions.

	Charged primary $(n_s \ge 5)$			Neutral primary $(n_s \geq 5)$			Charged primary $(n_s \ge 0)$		
	$N_h \leq 5$	$N_h > 5$	All events	$N_h \leq 5$	$N_h > 5$	All events	$N_h \leq 5$	$N_h > 5$	All events
$egin{aligned} & \mathrm{No. \ of \ events} \ \langle N_h angle \ \langle n_s angle \ \sigma \ (\mathrm{charged+neutral}) \end{aligned}$	$\begin{array}{r} 17 \\ 3.1 \pm 0.7 \\ 11.5 \pm 2.8 \\ 0.58 \end{array}$	$\begin{array}{r} 34\\ 11.7{\pm}2.0\\ 13.5{\pm}2.3\\ 0.54\end{array}$	$51 \\ 8.8 \pm 1.2 \\ 12.8 \pm 1.8 \\ 0.55$	13 2.5 \pm 0.7 12.6 \pm 3.4	$12 \\ 12.3 \pm 3.5 \\ 11.4 \pm 3.3$	$25 \\ 7.2 \pm 1.4 \\ 12.0 \pm 2.4$	13 1.9 \pm 0.5 9.0 \pm 2.5	25 11.4 \pm 2.3 10.5 \pm 2.1	$38 \\ 8.4 \pm 1.4 \\ 10.0 \pm 1.6$

TABLE V. Characteristics of secondary interactions.

There is a tail in the energy distribution extending to very high energies. Nine secondary interactions have energies >1000 Bev, four of which have a neutral, and five of which have a charged primary. This indicates that the particles carrying away the highest energies are not π mesons, but most probably nucleons.

The transverse momentum p_t of the shower particles produced in meson-nucleus collisions was measured in 8 cases with favorable geometry. No restriction with respect to N_h was applied. Only events with a charged primary, and not too high energy, E < 500 Bev, were selected for measurement. Therefore, the number of nucleons among the particles producing the showers selected in this way should be quite small. The distribution of p_t is shown in Fig. 14. The mean value is

$$\langle p_t \rangle = 0.27 \pm 0.05 \text{ Bev}/c.$$

Both the distribution and the mean value of p_t agree with measurements of events initiated by nucleons.⁵

From the known value of the transverse momentum in meson-induced interactions one can calculate the inelasticity η in the same way as in Sec. 7. However, the mean primary energy of the interactions can again only be determined from the angular distribution of the shower particles, and is, therefore, uncertain. Our best estimate for the inelasticity is

$\eta \gtrsim 0.60.$

The energy of secondary interactions can in general only be estimated from the angular distribution of the



FIG. 14. Distribution of the transverse momentum of shower particles emitted from secondary interactions.

shower particles emitted in these collisions. An independent check of this method would be very desirable. Table VI shows the comparison of values for the primary energy obtained from the angular distribution and Eqs. (2) and (3) with scattering measurements of the track of the primary particles. The angular distribution can overestimate as well as underestimate the true primary energy.

The estimate from the angular distribution will in an individual case not be very accurate. For more quantitative results bigger statistics are needed.

9. NATURE OF THE SHOWER PARTICLES

Direct mass measurements of the shower particles emitted from the high-energy primary interactions of protons and neutrons are in most of the cases not possible. One has to rely, therefore, on indirect evidence about the nature of the shower particles. One of the methods consists in comparing the number of secondary interactions produced by charged secondary particles $N_{\rm ch}$ with the number produced by neutral particles N_n . One makes the following two assumptions:

(i) 50% of the particles which are not π mesons are charged, the rest are neutral. It is reasonable to assume that these particles are nucleons, antinucleons, and strange particles. In this case, condition (i) should be valid in a good approximation.

(ii) The interaction mean free path of neutral and charged secondary particles, and the characteristics of the secondary collisions are equal (compare also Table V).

The fraction of charged particles among the shower particles which are not π mesons is then

$$N_x/N_s = N_n/N_{\rm eh}$$

In order to reduce the low-energy background, and the influence of tertiary interactions and knock-on

TABLE VI. Primary energies of secondary collisions obtained from the angular distribution by using Eqs. (2) and (3): E_0 , and from scattering measurements: E_{so} .

E ₀ (Bev)	$E_{ m sc}$ (Bev)	Туре
16 97 81 340 45	$\sim 40 \\ > 350 \\ \ge 130 \\ 65 \pm 25 \\ 95_{-40}^{+55}$	5+11p 22+52p 13+18p 0+5p 4+8p

nucleons, we have applied an energy cutoff to the listed in Table IV, namely:

 $E_0 > 5$ Bev, and $E_0 > 10$ Bev if $n_s \leq 10$.

We get then from Table IV

$$N_n/N_{\rm ch} = 19/48 = 0.40 \pm 0.11.$$

Table VII shows a compilation of some other measurements with good statistics.^{13,14} The average value of all measurements is

$$N_n/N_{\rm ch} = N_x/N_s = 0.27 \pm 0.05.$$

This number includes neutrons which were not created in the interaction but which are probably the primary particles continuing after the collision. It is reasonable to expect a 50% probability for a charge exchange of the primary nucleon. Our data are consistent with this view, as has been discussed in Sec. 8. Assuming again the same interaction mean free path, we can obtain a better estimate for the fraction of the strange particles and nucleons-antinucleons produced, if one corrects for the presence of the nucleons coming from the incident particle. One obtains

 $N_x/N_s = 0.24 \pm 0.05.$

From this we conclude that $(30\pm 6)\%$ of all particles (neutral, and charged) produced in showers having an average energy of several Tev are not π mesons.

Taking the combined material of this work, and of reference 14, we have looked for a dependence of $N_n/N_{\rm ch}$ on the characteristics of the interaction. No dependence

TABLE VII. Ratio N_n/N_{ch} of the number of secondary interactions produced by neutral and by charged particles.

	N_n	$N_{ m ch}$	$N_n/N_{ m ch}$
This work	19	48	0.40
Reference 13	12	48	0.25
Reference 14	8	51	0.16
Average (total)	147	39	$0.27 {\pm} 0.05$
Average of this work and reference 14 $n_s > 25$ $n_s < 25$ $N_h > 5$ $N_h \le 5$ E > 4.5 Tev E < 4.5 Tev			0.21 ± 0.06 0.41 ± 0.13 0.17 ± 0.08 0.32 ± 0.07 0.21 ± 0.06 0.36 ± 0.09

¹³ B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. 3, 327 (1958).
 ¹⁴ E. Lohrmann and M. W. Teucher, Phys. Rev. 112, 587 (1958).

on N_s , N_h , or the angle of emission θ_i could be found within the available statistics. The detailed numerical data are given in Table VII.

10. INTERACTION MEAN FREE PATH OF SHOWER PARTICLES

For about $\frac{1}{3}$ of the material discussed in Sec. 8 a very careful scan was conducted for secondary interactions produced by charged particles. The scanning cone was the same, namely a half-angle of 1.0×10^{-2} rad for 5 cm. All interactions having a primary particle coincident with the direction of the beam were accepted. No restriction on n_s was applied. The average energy of the shower particles in this cone was about 100 Bev. Thirty-eight events were found. The following corrections were applied:

Scanning loss, as determined from a double	
scan:	+0.05
Overlooking of interactions with $N_h = 0$:	+0.11
Rejection of events within 30μ from air or	
glass surface:	+0.10
Uncertainty of the location of the shower	
axis:	+0.01
Contribution from tertiary interactions:	-0.19
Correction for electrons among the shower	
particles:	+0.03
Total correction:	+0.11

The statistical error of our result is 16%, the estimated uncertainty of the corrections 12%, producing a total error of 20% for the result. From the total path length of shower particles contained in the inspected cone, and the number of interactions, one gets the following value for the collision mean free path, after applying the corrections:

$$\lambda = 41 \pm 8$$
 cm.

This is in quite good agreement with the mean free path of protons and neutrons¹ measured at about 250 Bev. It indicates that the mean free path does not depend strongly on energy, and that it is roughly the same for mesons and for nucleons.

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