the observation region. Beam neutralization at the slits is a possibility but we would suspect that this is small, particularly at this energy, but perhaps should not be discounted.

It is to be noted that the ^1S states are strongly excited. It is too bad that a better comparison with the ^1P states cannot be made. We would strongly suspect that the upper limits shown for the $3\,^1P$ and $4\,^1P$ levels represent figures that are better than an order of magnitude higher than the true cross section because of the imprisonment of resonance radiation.[4] It is particularly bad in our case because of the large (4 in.) diameter of our collision chamber and the fact that the lowest pressure at which we could take data was 2 or 3 μ. The apparent cross sections for the $2\,^1S-n\,^1P$ lines were still dropping rapidly at these pressures.

The measured cross section for the simultaneous ionization and excitation of helium[5] into the $n=4$ state of He$^+$ agrees fairly well with a rough extrapolation of Mapleton's calculations[6,7] and the measured charge-exchange cross section into the $n=4$ state of hydrogen is small but at least the right order of magnitude from what is expected from roughly extrapolating his recent calculations[7] on charge exchange; however, it is difficult to draw conclusions.

ACKNOWLEDGMENTS

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[4] A. V. Phelps, Phys. Rev. **110**, 1362 (1958).

[5] Hydrogen transition probabilities are taken from H. A. Bethe and E. E. Salpeter, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1951), Vol. 35.

[6] R. A. Mapleton, Phys. Rev. **109**, 1166 (1958).

[7] R. A. Mapleton (private communication). Mapleton estimates for the simultaneous ionization and excitation cross section into the $n=4$ state and the charge-exchange cross section into the $n=4$ state to be 3.5×10^{-20} cm^2 and 3×10^{-20} cm^2, respectively.

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Electron Capture from He$(1s^2)$ by Protons

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The two equivalent forms of Born's approximation, prior and post, are used to calculate the electron capture cross section for protons incident on He$(1s^2)$. These cross sections are calculated for capture into eleven different final states in the energy range 12.5 kev to 1 Mev. Although a rather crude wave function, $(Z^3/\pi a_0^3)\exp[-(Z/a_0)(r_1+r_2)](Z=1.6875)$, is used for He, the prior and post total capture cross sections do not differ by more than twenty percent over the energy range investigated. Estimates of the sum of the cross sections for capture into all s states of the hydrogen atom for the two residual ions, He$^+(1s)$ and He$^+(2s)$, are obtained from an adaption of the s-state sum rule as given in the paper of Jackson and Schiff. As in this work of Jackson and Schiff, it is found that the s states provide the major contribution to the total capture cross section. The calculated cross sections agree fairly well with the experimental values. The cross sections for capture into the state He$^+(1s)+$H$(1s)$, is roughly 2.5 times larger than the values obtained by Bransden, Dalgarno, and King.

THE cross section for the following process (A) has been calculated in Born approximation by Bransden, Dalgarno, and King.[1]

$$\mathrm{H}^+ + \mathrm{He}(1s^2) \rightarrow \mathrm{H}(1s) + \mathrm{He}^+(1s). \qquad \text{(A)}$$

In their calculation, the prior interaction was used and the Born matrix element was evaluated approximately. In the present paper, the cross section for reaction (A) is calculated in Born approximation with both forms of the interaction, prior and post. In addition, the prior and post Born cross sections are calculated for capture into ten other final states. A comparison of the results of this paper and those of BDK will be presented later. It is a well-established fact that the prior and post cross sections are equal provided that exact atomic wave functions are used in the Born matrix elements.[2,3] Since only inexact atomic wave functions exist for atoms other than hydrogen, it is not known which of these two cross sections agree more closely with the exact Born cross section. Although the wave function, $(Z^3/\pi a_0^3)\exp[-(Z/a_0)(r_1+r_2)]$ $(Z=1.6875)$, used for He is rather crude, the prior and post total capture cross sections of this paper do not differ by more than twenty percent over the energy range investigated; moreover, they are in fair agreement with the experimental values. The reason for this apparent success

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[1] B. H. Bransden, A. Dalgarno, and N. M. King, Proc. Phys. Soc. (London) **A67**, 1075 (1954). Future references to this paper are denoted by BDK.

[2] J. D. Jackson and H. Schiff, Phys. Rev. **89**, 359 (1953). Future references to this paper are denoted by JS.

[3] E. Gerjuoy, Ann. Phys. **5**, 58 (1958).

appears to emerge from the good representation of the wave function for He over the region of configuration space that provides the major contribution to the cross section for reaction (A), which process provides the major contribution to the total capture cross section. On the basis of this close agreement, it is reasonable to expect that the values of the exact Born cross sections would not differ radically from these approximate values. It is tempting to conjecture that the prior-post numerical discrepancy (hereafter called *i-f-d*) would not be much larger for the analogous capture process from more complicated atoms provided that an equally good wave function is used for the atom in question. One would expect a somewhat larger discrepancy for this latter case since both the atom and its residual ion would be represented by approximate wave functions, whereas the wave functions for He$^+$ is known exactly. This conjecture relevant to the smallness of the *i-f-d* provides a practical bias in favor of the prior cross section since there are fewer electrostatic terms in the prior interaction; wherefore, the evaluation of the prior cross section requires less calculation. The preceding discussion contains the implication that the exact Born cross sections should agree with the measured values. However, current research suggests that the Born approximation for the electron capture problem may fail not only at low energies as is well known, but also at high energies, although the agreement with experiment may be satisfactory over an intermediate energy range.[4,5] The notation is now introduced and the matrix elements are defined.

Let the masses be defined as follows: M=proton; M_n=helium nucleus; m=electron; $M_1=M+m$; $M_{n1}=M_n+m$; $M_{n2}=M_n+2m$; $\mu_1=MM_{n2}/M_T$; $M_T=M+M_{n2}$; $\mu_2=M_1M_{n1}/M_T$. The laboratory particle coordinates are $\mathbf{r}_p$=proton; $\mathbf{r}_n$=helium nucleus; $\mathbf{r}_{1,2}$=electrons; the prior (primed) and post (unprimed) relative coordinates are

$$\mathbf{x}_1'=\mathbf{r}_1-\mathbf{r}_n;\quad \mathbf{x}_2'=\mathbf{r}_2-\mathbf{r}_n=\mathbf{x}_3;$$

$$\mathbf{R}'=\mathbf{r}_p-\frac{M_n\mathbf{r}_n+m(\mathbf{r}_1+\mathbf{r}_2)}{M_{n2}};$$

$$\mathbf{x}_1=\mathbf{r}_1-\mathbf{r}_n;\quad \mathbf{x}_2=\mathbf{r}_2-\mathbf{r}_p;$$

$$R=\frac{M\mathbf{r}_p+m\mathbf{r}_2}{M_1}-\frac{M_n\mathbf{r}_n+m\mathbf{r}_1}{M_{n1}}.$$

Other relevant quantities are defined as follows: $\hbar$= Planck's constant divided by 2π; e= electron charge; $a_0=me^2/\hbar^2$= first Bohr radius; K and K_{mn} are the magnitudes of the initial and final wave vectors of relative motion, respectively; $v=\hbar K/\mu_1$; $v_{mn}=\hbar K_{mn}/\mu_2$; $\mathbf{A}_1=(m/M_{n1})\mathbf{A}_3$; $\mathbf{A}_2=(M/M_1)\mathbf{K}_{mn}-\mathbf{K}$; $\mathbf{A}_3=\mathbf{K}_{mn}-(M_{n1}/M_{n2})\mathbf{K}$. The binding energy of He($1s^2$), He$^+{}_m$ in the state labeled by the principal quantum number m, and H in the state labeled by the principal quantum number n, are, respectively, ϵ', ϵ_m', and ϵ_n. Conservation of energy is given by $\frac{1}{2}\mu_1v^2-\epsilon'=\frac{1}{2}\mu_2v_{mn}^2-\epsilon_m'-\epsilon_n$. The normalized wave functions that represent He$^+{}_m$, H$_n$, and He($1s^2$) are denoted, respectively, by ϕ_m, ϕ_n, and $(Z^3/\pi a_0^3)\exp[-(Z/a_0)(x_1+x_3)]$. The integration is simplified by the use of the set $(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ as independent variables.[2] The prior and post matrix elements for the reaction, $\mathrm{H}^++\mathrm{He}(1s^2)\rightarrow\mathrm{H}_n+\mathrm{He}^+{}_m$, are labeled, respectively, by $g_{mn}{}^i$ and $g_{mn}{}^f$. These quantities are defined in (1a).[1]

$$g_{mn}{}^{i,f}=\frac{\mu_2e^2Z^3}{2\pi^2\hbar^2a_0{}^3}\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\,\phi_m{}^*(\mathbf{x}_1)\phi_n{}^*(\mathbf{x}_2)V_{i,f}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\exp[-(Z/a_0)(x_1+x_3)]$$
$$\times\exp\{i[\mathbf{A}_1\cdot\mathbf{x}_1+\mathbf{A}_2\cdot\mathbf{x}_2-\mathbf{A}_3\cdot\mathbf{x}_3]\},\tag{1a}$$
$$V_i=2|\mathbf{x}_3-\mathbf{x}_2|^{-1}-|\mathbf{x}_3-\mathbf{x}_1-\mathbf{x}_2|^{-1}-|\mathbf{x}_2|^{-1}=V_{pn}+V_{p1}+V_{p2},$$
$$V_f=2|\mathbf{x}_3-\mathbf{x}_2|^{-1}+|\mathbf{x}_3-\mathbf{x}_1|^{-1}-2|\mathbf{x}_3|^{-1}-|\mathbf{x}_3-\mathbf{x}_1-\mathbf{x}_2|^{-1}=V_{pn}+V_{21}+V_{2n}+V_{p1}.$$

The second form of $V_{i,f}$ associates the particles with the interactions. Since helium is initially in a singlet state, and spin-dependent forces are neglected, the final state must also be a singlet in the electron coordinates. A permutation of the laboratory electron coordinates leads to a new set of post relative coordinates which are given by

$$\mathbf{x}_{12}=\mathbf{r}_2-\mathbf{r}_n,\quad \mathbf{x}_{22}=\mathbf{r}_1-\mathbf{r}_p,$$

$$\mathbf{R}_2=\frac{M\mathbf{r}_p+m\mathbf{r}_1}{M_1}-\frac{M_n\mathbf{r}_n+m\mathbf{r}_2}{M_{n1}}.$$

A repetition of the previous calculation leads to the identical expression for the capture of electron 1 as was obtained for the capture of electron 2. The two amplitudes are added and multiplied with the singlet normalization factor, $2^{-\frac{1}{2}}$, to obtain $\sqrt{2}g_{mn}{}^{i,f}$ for the singlet scattering amplitude.[1] The cross section for capture is given by[6]

$$Q_{mn}{}^{i,f}=\frac{4\pi v_{mn}}{v}\int_0^\pi|g_{mn}{}^{i,f}|^2\sin\theta d\theta.$$

The expressions for these various amplitudes are

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[4] R. H. Bassel and E. Gerjuoy, Phys. Rev. **117**, 749 (1960). Future references to this paper are denoted by BG.

[5] D. R. Bates, Proc. Roy. Soc. (London) **A247**, 294 (1958).

[6] W. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1949), 2nd ed.

long and only the ground-state case is worked out in detail. The amplitudes for capture into excited states are all derivable from the ground-state case by means of parametric differentiation.[7] The spectroscopic notation will be used to label the amplitudes and the cross sections. Dimensionless units are introduced with a_0, the unit of length, and $\epsilon_1 = me^4/2\hbar^2$, the unit of energy. The wave functions $\phi_{1s}(\mathbf{x}_1)$ and $\phi_{1s}(\mathbf{x}_2)$ are replaced, respectively, by $(8/\pi)^{\frac{1}{2}}e^{-\alpha x_1}$ and $e^{-\beta x_2}/\sqrt{\pi}$. It is seen that the functional forms of all excited states are derivable from $\exp[-\alpha x_1 + i\mathbf{A}_1\cdot\mathbf{x}_1]$ and $\exp[-\beta x_2 + i\mathbf{A}_2\cdot\mathbf{x}_2]$ by means of parametric differentiation with respect to α, β, and the rectangular components of $\mathbf{A}_{1,2}$, after which operation the appropriate values of (α,β) are inserted and the factors $\exp[i\mathbf{A}_j\cdot\mathbf{x}_j]$ are removed. The notation employed for the electrostatic interaction terms is used to label the various components of the matrix elements.

$$\alpha=2,\quad \beta=1,\quad m=n=1.$$

$$g_{1s;1s}{}^{i}=ZC[2\beta V_{pn}-V_{p1}-\beta V_{p2}],\quad g_{1s;1s}{}^{f}=\beta C[2ZV_{pn}+ZV_{21}-2V_{2n}-ZV_{p1}],$$

$$C=\frac{2^{17/2}Z^3(\alpha+Z)\mu_2a_0}{m},\quad Z=1.6875,\quad V_{pn}=[(\alpha+Z)^2+A_1{}^2]^{-2}\int_0^1 dx\,x(1-x)\left(\frac{\partial}{\partial U_1}+\frac{\partial}{\partial V_1}\right)^2 U_1{}^{-1}V_1{}^{-\frac{1}{2}},$$

$$V_{p1}=\int_0^1 dx\,x(1-x)\int_0^1 dy\,y(1-y)^3\left(\frac{\partial}{\partial U_3}+\frac{\partial}{\partial V_3}\right)^4 U_3{}^{-1}V_3{}^{-\frac{1}{2}},$$

$$V_{p2}=[(\alpha+Z)^2+A_1{}^2]^{-2}[\beta^2+A_2{}^2]^{-1}[Z^2+A_3{}^2]^{-2},$$

$$V_{21}=[\beta^2+A_2{}^2]^{-2}\int_0^1 dx\,x(1-x)\left(\frac{\partial}{\partial U_2}+\frac{\partial}{\partial V_2}\right)^2 U_2{}^{-1}V_2{}^{-\frac{1}{2}},$$

$$V_{2n}=[(\alpha+Z)^2+A_1{}^2]^{-2}[\beta^2+A_2{}^2]^{-2}[Z^2+A_3{}^2]^{-1},\quad U_1=(\beta^2+A_2{}^2)(1-x)+x(Z^2+A_3{}^2),$$

$$V_1=\beta^2(1-x)+Z^2x+x(1-x)|\mathbf{A}_3-\mathbf{A}_2|^2,\quad U_2=(Z^2+A_3{}^2)(1-x)+x[(\alpha+Z)^2+A_1{}^2],$$

$$V_2=Z^2(1-x)+(\alpha+Z)^2x+x(1-x)|\mathbf{A}_1-\mathbf{A}_3|^2,$$

$$U_3=[1-y][(1-x)(\beta^2+A_2{}^2)+x(Z^2+A_3{}^2)]+y[(\alpha+Z)^2+A_1{}^2],$$

$$\begin{aligned}V_3=[1-y][(1-x)\beta^2+xZ^2]+y(\alpha+Z)^2+\{1-y\}\{yA_1{}^2+[1-x][1-(1-x)(1-y)]A_2{}^2\\ +x[1-x(1-y)]A_3{}^2-2y(1-x)\mathbf{A}_1\cdot\mathbf{A}_2-2xy\mathbf{A}_1\cdot\mathbf{A}_3-2x(1-x)(1-y)\mathbf{A}_2\cdot\mathbf{A}_3\}.\end{aligned}\tag{1b}$$

$$\alpha=2,\quad \beta=\tfrac{1}{2},\quad m=1,\quad n=2.$$

$$g_{1s;2s}=\frac{1}{4\sqrt{2}}\left(2+\frac{\partial}{\partial\beta}\right)g_{1s;1s}.\tag{2a}$$

In this relation, and all subsequent relations, it is understood that the same superscript, i or f, should appear on both sides of the equation.

$$g_{1s;2p}{}^{0;\pm1}=\frac{-i}{4\sqrt{2}}\left(\frac{\partial}{\partial A_{2z}};\frac{1}{\sqrt{2}}\frac{\partial}{\partial A_{2\pm}}\right)g_{1s;1s},$$
$$\frac{\partial}{\partial A_{2\pm}}=\left(\frac{\partial}{\partial A_{2x}}\pm i\frac{\partial}{\partial A_{2y}}\right).\tag{2b}$$

The differential cross section for this p state is obtained from the following expression:

$$\sum_{m=-1}^{1}|g_{1s;2p}{}^{m}|^2=|C_2\mathbf{A}_2-C_3\mathbf{A}_3|^2.$$

In the derivation of the preceding formula, all terms that have A_1, or a power of A_1, as a factor are dropped. This is a very good approximation since $A_1\approx10^{-4}A_3$. (A_1 is retained in the quantities that comprise $g_{1s;1s}$.) C_2 and C_3 are linear combinations of quantities derived from the terms in $g_{1s;1s}$. These remarks apply to all p and d states.

$$\alpha=2,\quad \beta=\tfrac{1}{3},\quad m=1,\quad n=3.$$

$$g_{1s;3s}=\frac{1}{81\sqrt{3}}\left(27+18\frac{\partial}{\partial\beta}+2\frac{\partial^2}{\partial\beta^2}\right)g_{1s;1s}.\tag{2c}$$

$$g_{1s;3p}{}^{0;\pm1}=\frac{-i\sqrt{2}}{81}\left(6+\frac{\partial}{\partial\beta}\right)\left(\frac{\partial}{\partial A_{2z}};\frac{1}{\sqrt{2}}\frac{\partial}{\partial A_{2\pm}}\right)g_{1s;1s}.\tag{2d}$$

$$g_{1s;3d}{}^{0}=\frac{1}{81\sqrt{6}}\left(-2\frac{\partial^2}{\partial A_{2z}{}^2}+\frac{\partial^2}{\partial A_{2+}\partial A_{2-}}\right)g_{1s;1s},$$
$$g_{1s;3d}{}^{\pm1}=-\frac{1}{81}\frac{\partial}{\partial A_{2z}}\frac{\partial}{\partial A_{2\pm}}g_{1s;1s},\tag{2e}$$
$$g_{1s;3d}{}^{\pm2}=\frac{-1}{162}\left(\frac{\partial}{\partial A_{2\pm}}\right)^2 g_{1s;1s}.$$

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[7] E. Corinaldesi and L. Trainor, Nuovo cimento **9**, 940 (1952).

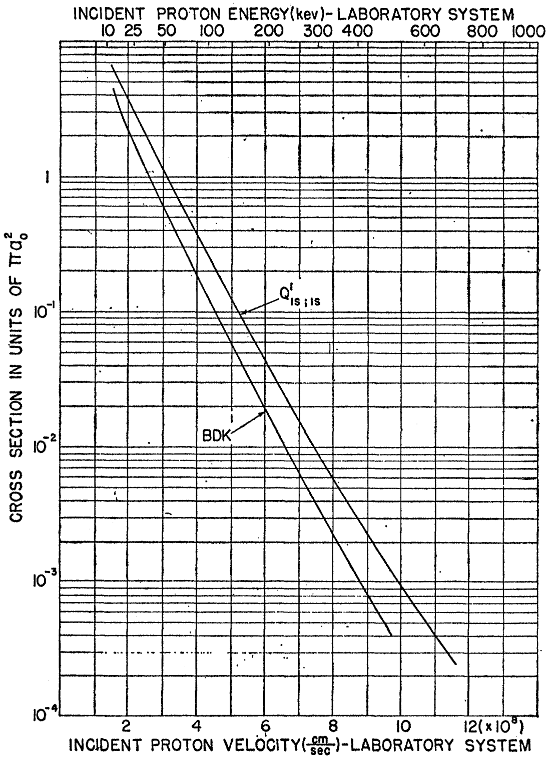

FIG. 1. Capture cross sections into the state $He^+(1s)+H(1s)$ calculated with the prior interaction. BDK refers to the calculations of Bransden *et al.*

It can be shown that the following relation is valid.

$$\sum_{m=-2}^{2} |g_{1s;3d}{}^m|^2 \propto C_1^2A_2^4+C_2^2\left[\frac{3A_2^2A_3^2+(\mathbf{A}_2\cdot\mathbf{A}_3)^2}{4}\right]$$
$$+C_3^2A_3^4-2C_1C_2A_2^2(\mathbf{A}_2\cdot\mathbf{A}_3)+C_1C_3[3(\mathbf{A}_2\cdot\mathbf{A}_3)^2$$
$$-A_2^2A_3^2]-2C_2C_3A_3^2(\mathbf{A}_2\cdot\mathbf{A}_3).$$

This expression has the familiar form of a linear combination of scalar products of spherical tensors of rank two, and this form is consistent with the properties of the spherical harmonics that are implicit in the d-state wave function.[8]

$$\alpha=1,\quad \beta=1,\quad m=2,\quad n=1.$$

$$g_{2s;1s}=\frac{1}{4\sqrt{2}}\left(2+2\frac{\partial}{\partial\alpha}\right)g_{1s;1s}. \tag{3a}$$

$$g_{2p;1s}{}^{0;\pm1}=\frac{-i}{2\sqrt{2}}\left(\frac{\partial}{\partial A_{1z}};\frac{1}{\sqrt{2}}\frac{\partial}{\partial A_{1\pm}}\right)g_{1s;1s}. \tag{3b}$$

$$\alpha=1,\quad \beta=\tfrac{1}{2},\quad m=2,\quad n=2.$$

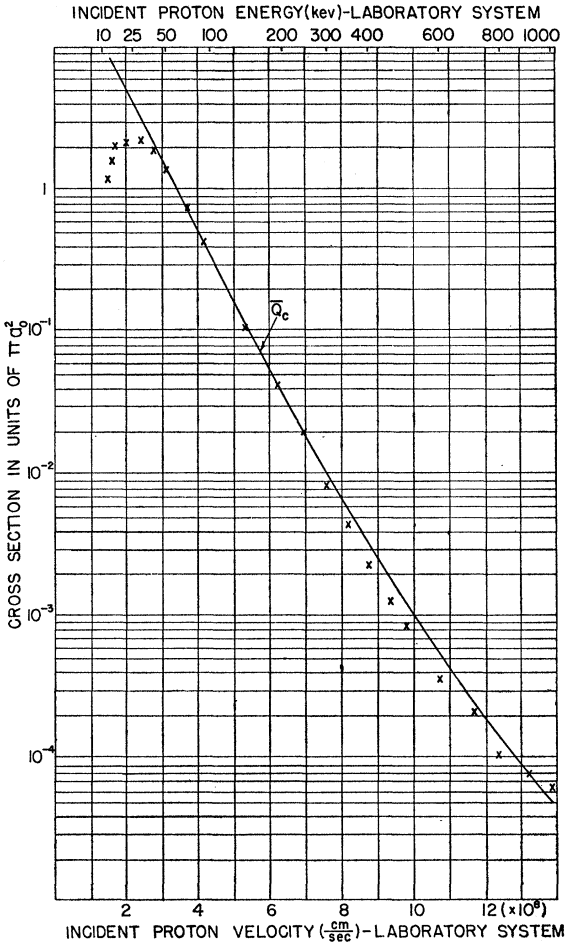

FIG. 2. $\bar{Q}_c$=arithmetic average of prior and post total capture cross sections. (See text for discussion of $\bar{Q}_c$.) × =experimental values.

$$g_{2s;2s}=\frac{1}{32}\left(2+2\frac{\partial}{\partial\alpha}\right)\left(2+\frac{\partial}{\partial\beta}\right)g_{1s;1s}. \tag{3c}$$

$$g_{2s;2p}{}^{0;\pm1}=\frac{-i}{32}\left(2+2\frac{\partial}{\partial\alpha}\right)\left(\frac{\partial}{\partial A_{2z}};\frac{1}{\sqrt{2}}\frac{\partial}{\partial A_{2\pm}}\right)g_{1s;1s}. \tag{3d}$$

$$g_{2p;2s}{}^{0;\pm1}=\frac{-i}{16}\left(\frac{\partial}{\partial A_{1z}};\frac{1}{\sqrt{2}}\frac{\partial}{\partial A_{1\pm}}\right)\left(2+\frac{\partial}{\partial\beta}\right)g_{1s;1s}. \tag{3e}$$

In Fig. 1, the cross sections, $Q_{1s;1s}{}^i$, are compared with the results of BDK. The energy dependence is nearly the same, but the cross sections of this paper are roughly 2.5 times larger than those of BDK. In Fig. 2, the quantity, $\bar{Q}_c=\frac{1}{2}(Q_c{}^i+Q_c{}^f)$, is compared with the measured values of the cross sections.[9] $Q_c{}^{i,f}$ is the sum of the calculated cross sections, and the arithmetic average is plotted since it is not known which cross

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[8] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), 1st ed., Chap. 5.

[9] S. K. Allison, Revs. Modern Phys. **30**, 1137 (1958); P. M. Stier and C. F. Barnett, Phys. Rev. **103**, 896 (1956); C. F. Barnett and H. K. Reynolds, Phys. Rev. **109**, 355 (1958).

TABLE I. Cross sections, $Q_{m;n}$, in units of $\pi a_0^2=8.79\times10^{-17}$ cm² for capture into the state $\mathrm{He}^+{}_m+\mathrm{H}_n$; i=prior, f=post. Q_c=sum of calculated cross sections. $Q_s=\sum_n (Q_{1s;ns}+Q_{2s;ns})$. $Q_e=Q_s$+other calculated values. E=incident energy of the proton in kev—laboratory system.

$m;n$ \ $E=$		7.03	12.5	22.2	39.5	70.3	125	222	395	703	1000
1s;1s	i	12.5	6.77	3.47	1.53	5.44×10^{-1}	1.41×10^{-1}	2.51×10^{-2}	2.94×10^{-3}	2.32×10^{-4}	4.16×10^{-5}
	f	13.2	7.55	4.02	1.75	5.77×10^{-1}	1.34×10^{-1}	2.15×10^{-2}	2.41×10^{-3}	1.99×10^{-4}	3.86×10^{-5}
1s;2s	i	5.50×10^{-1}	3.30×10^{-1}	2.20×10^{-1}	1.31×10^{-1}	5.84×10^{-2}	1.76×10^{-2}	3.37×10^{-3}	4.03×10^{-4}	3.13×10^{-5}	5.52×10^{-6}
	f	5.73×10^{-1}	3.45×10^{-1}	2.45×10^{-1}	1.54×10^{-1}	6.56×10^{-2}	1.74×10^{-2}	2.93×10^{-3}	3.29×10^{-4}	2.66×10^{-5}	5.05×10^{-6}
1s;2p	i	8.05×10^{-1}	5.55×10^{-1}	2.97×10^{-1}	1.28×10^{-1}	4.16×10^{-2}	8.85×10^{-3}	1.12×10^{-3}	8.21×10^{-5}	3.75×10^{-6}	4.71×10^{-7}
	f	8.30×10^{-1}	5.70×10^{-1}	3.03×10^{-1}	1.37×10^{-1}	4.54×10^{-2}	9.01×10^{-3}	1.02×10^{-3}	7.03×10^{-5}	3.29×10^{-6}	4.36×10^{-7}
1s;3s	i	1.4×10^{-1}	8.20×10^{-2}	5.67×10^{-2}	3.6×10^{-2}	1.67×10^{-2}	5.19×10^{-3}	1.01×10^{-3}	1.21×10^{-4}	9.38×10^{-6}	1.65×10^{-6}[a]
	f	1.4×10^{-1}	8.54×10^{-2}	6.21×10^{-2}	4.22×10^{-2}	1.90×10^{-2}	5.20×10^{-3}	8.81×10^{-4}	9.88×10^{-5}	7.97×10^{-6}	1.51×10^{-6}[a]
1s;3p	i	2.1×10^{-1}	1.57×10^{-1}	8.87×10^{-2}	3.99×10^{-2}	1.36×10^{-2}	3.01×10^{-3}	3.89×10^{-4}	2.89×10^{-5}	1.3×10^{-6}	
	f	2.2×10^{-1}	1.62×10^{-1}	9.04×10^{-2}	4.24×10^{-2}	1.49×10^{-2}	3.08×10^{-3}	3.57×10^{-4}	2.48×10^{-5}	1.1×10^{-6}	
1s;3d	i	9.1×10^{-3}	9.37×10^{-3}	6.19×10^{-3}	2.79×10^{-3}	8.76×10^{-4}	1.64×10^{-4}	1.59×10^{-5}	7.95×10^{-7}	2.2×10^{-8}	
	f	9.4×10^{-3}	9.62×10^{-3}	6.14×10^{-3}	2.86×10^{-3}	9.51×10^{-4}	1.68×10^{-4}	1.45×10^{-5}	6.78×10^{-7}	2.0×10^{-8}	
2s;1s	i	1.03×10^{-1}	1.08×10^{-1}	8.61×10^{-2}	4.40×10^{-2}	1.37×10^{-2}	2.91×10^{-3}	4.46×10^{-4}	3.88×10^{-5}	2.09×10^{-6}	3.0×10^{-7}
	f	1.00×10^{-1}	1.03×10^{-1}	8.09×10^{-2}	4.23×10^{-2}	1.36×10^{-2}	2.48×10^{-3}	3.17×10^{-4}	3.45×10^{-5}	3.43×10^{-6}	7.1×10^{-7}
2p;1s	i	3.84×10^{-3}	1.37×10^{-2}	2.50×10^{-2}	2.34×10^{-2}	1.12×10^{-2}	2.77×10^{-3}	3.52×10^{-4}	2.36×10^{-5}	8.59×10^{-7}	8.6×10^{-8}
	f	3.45×10^{-3}	1.15×10^{-2}	1.93×10^{-2}	1.66×10^{-2}	7.77×10^{-3}	2.11×10^{-3}	3.58×10^{-4}	3.85×10^{-5}	2.36×10^{-6}	3.1×10^{-7}
2s;2s	i		5.2×10^{-3}	5.6×10^{-3}	4.0×10^{-3}	1.6×10^{-3}	4.0×10^{-4}	6.4×10^{-5}	5.6×10^{-6}	2.9×10^{-7}	4.0×10^{-8}
	f		4.8×10^{-3}	5.2×10^{-3}	3.76×10^{-3}	1.6×10^{-3}	3.6×10^{-4}	4.4×10^{-5}	4.4×10^{-6}	4.4×10^{-7}	9.2×10^{-8}
2s;2p	i		4.8×10^{-3}	6.0×10^{-3}	4.4×10^{-3}	1.7×10^{-3}	3.1×10^{-4}	3.1×10^{-5}	1.7×10^{-6}	5.6×10^{-8}	
	f		4.8×10^{-3}	6.0×10^{-3}	4.4×10^{-3}	1.8×10^{-3}	3.2×10^{-4}	2.9×10^{-5}	1.4×10^{-6}	5.2×10^{-8}	
2p;2s	i		4.8×10^{-3}	1.4×10^{-3}	1.9×10^{-3}	1.3×10^{-3}	3.9×10^{-4}	5.6×10^{-5}	3.7×10^{-6}	1.3×10^{-7}	
	f		4.0×10^{-4}	1.1×10^{-3}	1.4×10^{-3}	8.8×10^{-4}	2.8×10^{-4}	5.2×10^{-5}	5.6×10^{-6}	3.4×10^{-7}	
Q_c	i		8.05	4.26	1.95	7.05×10^{-1}	1.83×10^{-1}	3.20×10^{-2}	3.65×10^{-3}	2.81×10^{-4}	4.97×10^{-5}
	f		8.84	4.83	2.20	7.50×10^{-1}	1.74×10^{-1}	2.75×10^{-2}	3.02×10^{-3}	2.45×10^{-4}	4.67×10^{-5}
Q_s	i					6.54×10^{-1}	1.73×10^{-1}	3.12×10^{-2}	3.64×10^{-3}	2.86×10^{-4}	5.09×10^{-5}
	f					7.00×10^{-1}	1.65×10^{-1}	2.66×10^{-2}	2.98×10^{-3}	2.64×10^{-4}	4.77×10^{-5}
Q_e	i					7.23×10^{-1}	1.88×10^{-1}	3.31×10^{-2}	3.78×10^{-3}	2.92×10^{-4}	5.15×10^{-5}
	f					7.70×10^{-1}	1.80×10^{-1}	2.84×10^{-2}	3.12×10^{-3}	2.53×10^{-4}	4.85×10^{-5}

[a] These two cross sections were estimated from the approximate relation $Q_{1s;3s}/Q_{1s;2s}(1000)\approx Q_{1s;3s}/Q_{1s;2s}(703)$.

section is the better representation of the exact Born cross section. The author estimates that the inclusion of the capture cross sections for the omitted final states would not increase Q_c by more than 5%, and this increase would not alter the picture that is presented in Fig. 2. Above 40 kev the agreement of the calculated cross sections with the measured values is fairly good. It is not known why the experimental value at 1 Mev exceeds the calculated value; perhaps, as is discussed in BG, this is evidence of a failure of the Born approximation at high energies, or it may be the result of an experimental error, or least likely, for reasons that are given later, it may be the result of the approximate wave function. The most striking feature of Table I is the small *i-f-d* for the cross sections $Q_{1s;n}{}^{i,f}$ and $Q_c{}^{i,f}$. The *i-f-d* is larger for simultaneous charge transfer and excitation, and this is expected since the effective overlap of $\mathrm{He}^+{}_{2s,2p}$ and He cover a region of $\mathbf{x}_1$ configuration space where the wave function for He is less accurate than in the corresponding overlap region of $\mathrm{He}^+{}_{1s}$ and He. Since the configuration spaces, $\mathbf{x}_2$ of H and $\mathbf{x}_3$ of He, are different, the preceding analysis does not apply; however, the numerical results show that the excited states of H are much less effective in producing the *i-f-d* than are the excited states of He⁺. There is a remarkable similarity of the ratios $Q_{1s;2s}/Q_{1s;1s}$, $Q_{1s;2p}/Q_{1s;2s}$, and $Q_{1s;3s}/Q_{1s;2s}$ (the superscripts i or f are understood) to the corresponding ratios (with the first subscript deleted) for the electron capture process in atomic hydrogen as given in JS.[10] For an incident proton energy above 70 kev, there are the following approximate equalities among the ratios:

$$\frac{Q_{1s;2s}}{Q_{1s;1s}}\approx\frac{Q_{2s;2s}}{Q_{2s;1s}};\quad\frac{Q_{1s;3s}}{Q_{1s;2s}}\approx\frac{8}{27}.$$

By reason of the first approximate equality, it will be assumed that $Q_{2s;3s}/Q_{2s;2s}\approx8/27$ also holds. This 8/27 relation is the first term of the n^{-3} law $[Q_n=(8/n^3)Q_2,\ n\geq3]$ which is discussed in JS. For the purpose of obtaining an estimate of the total contribution from the s-state cross sections of the two ions, $\mathrm{He}^+(2s)$ and $\mathrm{He}^+(1s)$, the n^{-3} law will be assumed. Equation (19) of JS is modified to give the following relations:

$$\sum_{n=1}^{\infty}Q_{1s;ns}=Q_{1s;1s}\left[1+1.616\frac{Q_{1s;2s}}{Q_{1s;1s}}\right];$$

$$\sum_{n=1}^{\infty}Q_{2s;ns}=Q_{2s;1s}\left[1+1.616\frac{Q_{2s;2s}}{Q_{2s;1s}}\right].$$

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[10] The author has calculated the capture cross sections into the states 1s, 2s, 2p, 3s, 3p, 4s, and 5s for protons in atomic hydrogen

These quantities are given in the table along with the cross section, Q_e, which is defined as, $Q_e=\sum_n [Q_{1s;ns}+Q_{2s;ns}]$+other calculated cross sections. Q_e exceeds Q_c by only a few percent. It is unlikely that the inclusion of all other omitted capture cross sections would increase Q_e by more than two percent. A comparison of Q_e with $\sum_n [Q_{2s;ns}+Q_{1s;ns}]$ shows that the s states provide the significant contribution to the total capture cross section, and this fact is in precise accord with the results of JS.

It would be interesting to learn whether a better wave function for He would give cross sections that are in better agreement with the experimental values. It is planned to calculate the cross sections, $Q_{1s;1s}^{i,f}$, with the six-parameter wave function of Hylleras. Since this improved wave function provides a larger binding energy for He, and since He is the target atom, it is believed that the capture cross sections will be reduced, and it is hoped that the *i-f-d* will be smaller. There is some additional evidence to support the remark that the cross sections will be smaller. An improved wave function for H^- in the reaction, $\mathrm{H}+\mathrm{H}\rightarrow\mathrm{H}^-+\mathrm{H}^+$, led to a larger cross section, and this is consistent with the previous conjecture since H^- is formed by the capture process.[11]

The author believes that a calculation of the magnitude that is described in this paper deserves the dignity of a few remarks relevant to the calculational details. A Gaussian quadrature method was used to evaluate the integrals. These integrals were calculated with an accuracy of five (or four) significant figures to obtain cross sections with an accuracy of three (or two) places. Simpson's rule was used to integrate the differential cross sections, the starting angular increment, $\Delta\theta$, being 5×10^{-5} radian. As the incident proton energy decreased, the total angular width of the angular integration had to be increased; for example, the angular range required to obtain $Q_{1s;1s}^{i,f}$ correctly to three figures was 9° at 22.2 kev and 1.5° at 703 kev. The behavior of the electrostatic matrix elements with respect to the scattering amplitude follows the same pattern as in the corresponding charge transfer processes of atomic hydrogen in atomic hydrogen and protons in atomic hydrogen.[10,11] This pattern is illustrated for $g_{1s;1s}^{i}=g$. [See Eq. (1b).] The term $2V_{pn}$ is dominated by $V_{p1}+V_{p2}$ for a small angular range starting at $\theta=0$ so that $g<0$. g goes through a zero and the subsequent decrease of $V_{p1}+V_{p2}$ with increasing angle is much greater than the corresponding decrease of $2V_{pn}$. This large angular tail, as it is called in BG, provides a significant contribution to the cross section, the contribution of this tail increasing as the energy decreases. A final remark is made with reference to the evaluation of the Feynman integrals for the excited states. Some of these integrals required 63 subdivisions for each variable of integration in the interval (0,1) in order to obtain the desired accuracy. The requirement of this small subdivision is caused by the rapid change of the integrand near the end points of the integral. For the case of protons in atomic hydrogen, the situation is worse, and this fact illustrates that these integrals are rather sensitive functions of the parameters of the integrands.

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in order to verify numerically the n^{-3} law for s states as given in JS. (Unpublished work.)

[11] R. A. Mapleton, Phys. Rev. **117**, 479 (1960).