Potentials in a Conductor of Varying Cross Section

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(Received November 21, 1960)

A Bernoulli voltage, V_B, proportional to the square of the current I, may be expected in a conductor of varying cross section. Previous experiments to detect V_B are discussed. It is pointed out that in such experiments a Hall voltage, $V_H \propto I^2$, due to the magnetic field of the current I ("eigen-Hall effect" EHE), is superposed on V_B. The ratio V_H/V_B is calculated. Experiments are performed on a bismuth sample of varying cross section. The dependence of the measured voltages upon temperature and cross-sectional area shows that the EHE is dominant over the Bernoulli effect.

THE current carriers in a conductor may sometimes be considered as a fluid. For example, in the case of superconductors Bopp[1] has discussed the equation of motion of a nonviscous electronic fluid and has derived in this way London's equations with an additional Bernoulli field,

$$\mathbf{E}_B = (m/2e)\,\mathrm{grad}(\mathbf{v}^2) = (m/2e)\,\mathrm{grad}(\mathbf{J}^2/n^2e^2), \quad (1)$$

[m = electron mass, e = charge (<0), n = concentration, $\mathbf{v}$ = drift velocity, $\mathbf{J}=ne\mathbf{v}$ = current density]. This field is, however, not observable on the outside of the superconductor.[1,2]

Several authors have reported experiments on normal conductors to detect the Bernoulli votage,

$$V_B = V_1 - V_2 = -(m/2n^2e^3)(J_1^2 - J_2^2), \quad (2)$$

between two points where the current densities have the values J_1 and $J_2 \ll J_1$. The samples had, in all cases, a constriction in the cross section, their shape recalling the hydrodynamic Venturi tube. On passage of an alternating current, a dc voltage $\propto J^2$ was looked for. Ivashchenko[3] observed in iron, lead, and mercury voltages of about 10^{-6} volt (roughly proportional to the resistivity ρ) at $J \leq 2.5\times10^6$ amp/m². But, according to Eq. (2), we would expect $\leq 10^{-19}$ volt by assuming $m=m_0$ = free electron mass and $1/ne = 10^{-10}$ m³/amp sec. Dorfman and Kagan[4] interpreted the voltages observed by Ivashchenko as thermoelectric forces; after some precautions to avoid these had been taken, the repetition of the experiment showed no voltages above 10^{-6} volt in tin foils at $J \leq 6\times10^7$ amp/m². Recently, Chester[5] carried out experiments on thin film samples of bismuth with $J \leq 10^{10}$ amp/m². To get rid of the discrepancy between his experimental data and Eq. (2), Chester proposed that the right side of Eq. (2) be divided by an effective mass of the order $m^*/m = 10^{-4}$. This procedure requires closer consideration. Further, the value of m^*/m for bismuth is somewhat low in comparison with the known absolute values of the elements of the effective mass tensor, which vary between 2.4×10^{-3} and 2.5.[6]

In connection with these problems, we wish to point out the importance of the Hall field $\mathbf{E}_H = R\mathbf{B}\times\mathbf{J}$ (R = Hall coefficient, $\mathbf{B}$ = magnetic field). Even if no external magnetic field is present, a current flowing through a conductor gives rise to a Hall effect due to its own magnetic field.[7] This effect was named "eigen-Hall effect" (EHE)[8] and used for quantitative measurements in samples of various configurations.[8,2] The EHE generates, in a thin plate, a voltage [see ref. 2, Eq. (3.35)]

$$V_H = -(\ln 4)(\mu_0/4\pi)RFJ^2, \quad (3)$$

($\mu_0/4\pi = 10^{-7}$ volt sec/amp m). This Hall voltage is proportional to J^2 as is the Bernoulli voltage, V_B. Both voltages are superposed and their ratio (with $R \approx 1/ne$) is

$$V_H/V_B = (2\ln 4)(\mu_0/4\pi)(e/m)(F/R). \quad (4)$$

A rough estimate for bismuth films with $F=10^{-9}$ m², $R=+5\times10^{-8}$ m³/amp sec at room temperature[9] and $m=m_0$ yields $V_H/V_B=10^3$. In this case the EHE is dominant, even if we replace m by an effective mass $m^* < m_0$.

To prove that the voltage measured is in fact due to Hall effect, it can be shown that the EHE and the Hall effect generated in an external magnetic field, correspond quantitatively.[2] Additionally, the dependence of the measured voltage upon the temperature, T, and upon the geometrical dimensions of the sample can be investigated: In solid cylinders $V_B \propto R(T)^2I^2/F^2$, whereas $V_H \propto R(T)I^2/F$ (I = current intensity, F = cross section). Disturbing thermoelectric effects due to Joule heating which are proportional to $\rho(T)$ and furthermore to I^2, must be carefully avoided. In our experiments we use temperature baths and a low power input. The alternating potential of frequency 2ω is measured

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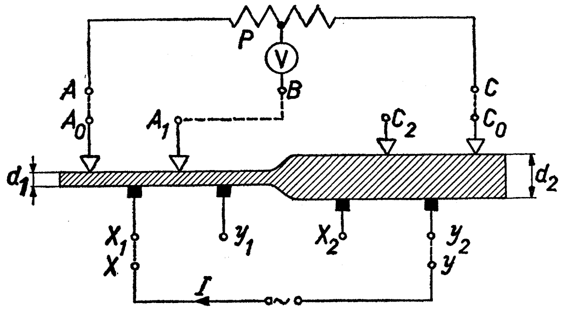

FIG. 1. Sample and circuit diagram. (Connections for measurement No. 1 are dashed.)

when an alternating current of frequency $\omega = 2\pi \times 30$ sec^{-1} passes through the sample. The measurement of the simultaneous dc voltage is less accurate. (For details see reference 2.)

We performed experiments on a sample of polycrystalline bismuth having the shape illustrated in Fig. 1 (diameter of the cylindrical portions $d_1 = 2.3 \times 10^{-3}$ m and $d_2 = 3.8 \times 10^{-3}$ m, total length 0.1 m). The measured voltages were proportional to I^2. We obtained the values in Table I for $I_{\text{rms}} = 1.5$ amp, the sample being in liquid air. The voltages at 80°K exceed those at room temperature by a factor of about six.

The Ohmic potential drop (of frequency ω) is compensated by means of the potentiometer P in Fig. 1. The probes outside the current terminals are at the potential $V_B = V_H = 0$, the probes A_1 and C_2 are at potentials V_1 and V_2, respectively. Measurements No. 1 and 3 yield the same value of V_1, within the limits of error, No. 2 and 4 likewise for V_2. The mean ratio of voltages $V_1/V_2 = 2.59$ is near the ratio of cross sections $F_2/F_1 = 2.73$ [within the experimental error, whereas $(F_2/F_1)^2 = 7.44$]. The Hall coefficient calculated according to

$$R = -2^{\frac{1}{2}}(4\pi/\mu_0)FV_{\text{rms}}/I^2_{\text{rms}} \tag{5}$$

has a mean value $R = -5.4 \times 10^{-6}$ $\text{m}^3/\text{amp sec}$ and a temperature dependence in agreement with published data.[10] Hence in our bulk bismuth sample of the configuration shown by Fig. 1, we measure the EHE, whereas the Bernoulli voltage is not observable (approximately $V_B = 10^{-11}$ volt for $I = 1.5$ amp and $m = m_0$ according to Eq. (2); extreme values of the effective mass are excluded). Further studies are necessary to prove the existence of the Bernoulli effect in a conductor.

TABLE I. Measurements on a bismuth sample of the configuration shown by Fig. 1 ($I_{\text{rms}} = 1.5$ amp), $T = 80$°K.

Measurement No.	Current terminals X Y	Potential probes A B C	Measured rms voltages V_1 (in 10^{-8} volt)	V_2
1	X_1 Y_2	A_0 A_1 C_0	19.8	
2	X_1 Y_2	A_0 C_2 C_0		7.5
3	X_1 Y_1	A_0 A_1 C_2	20.5	
4	X_2 Y_2	A_1 C_2 C_0		8.1

ACKNOWLEDGMENTS

It is a pleasure to thank Professor G. A. Busch, Head of the Laboratory of Solid State Physics of the Swiss Federal Institute of Technology, and Consultant to the IBM Research Laboratory Zurich, for valuable discussions.

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