Photothermal Effect in Semiconductors*

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When a sample of semiconducting material is illuminated, pairs of excess carriers are generated which diffuse through the material according to the density gradients established. Each pair carries an energy approximately equal to the band gap of the material. This energy is deposited where the excess electron recombines with a hole and causes local heating of the lattice. A temperature distribution will therefore be established in the sample which depends on the characteristics of optical absorption and bulk and surface recombination in and on the sample. This establishment of a temperature distribution in a solid by optically excited diffusing and recombining carriers is called the photothermal effect. The paper gives a formulation of the theory governing the photothermal effect, and the case of small temperature elevations in an infinite slab is worked out in detail.

I. DEFINITION OF THE EFFECT

WHEN a sample of semiconducting material (see Fig. 1) is illuminated with photons of sufficient energy, electron-hole pairs are generated which diffuse through the crystal from the place of their generation to regions of lower excess-pair concentration. Each pair transports an amount of energy approximately equal to the band separation, thus contributing to the thermal conductivity. This energy is deposited where the pair undergoes nonradiative recombination, increasing the local (measurable) temperature of the lattice. A non-uniform temperature distribution is therefore established in the sample which depends on the nature of the incident radiation, on the characteristics of optical absorption, on the bulk and surface recombination mechanism, on the boundary conditions for temperature and energy flux, and on the ordinary thermal conductivity of the material. This establishment of a temperature distribution in a solid by optically excited diffusing and recombining carriers and the attendant modification in the thermal conductivity is called the photothermal (PT) effect. The related effect with applied magnetic field is called the photomagnetothermal (PMT) effect and shall be the subject of a future publication.

In the following, a simple theory of the PT effect is

FIG. 1. Analysis of the PT effect.

developed which appears applicable to germanium and silicon.

II. BASIC FORMULATIONS

We first consider the various components of energy flux inside and outside the sample:

Components of Energy Flux Outside Front Surface

The energy flux f_f incident upon the front surface (energy per unit time and area) is carried by radiation, conduction, and convection, and is conveniently split into seven components:

$$f_f = f_T + f_R + f_{AC} + f_{AL} + f_{EC} + f_{EL} + f_{CC}. \quad \text{(II.1)}$$

Here f_T is the radiated flux transmitted through the sample. f_R is the radiated flux reflected by the sample. f_{AC} is the radiated flux absorbed exclusively under carrier generation in the bulk or on the surface of the sample; it is directly related to the optical carrier-generation rates. f_{AL} is the radiated flux absorbed exclusively by the lattice and by the equilibrium carriers; it consists of absorbed photons of insufficient energy to excite an electron-hole pair and of the excess energy of photons which have excited electron-hole pairs. f_{EC} is the flux emitted by radiative recombination of carriers.[1]
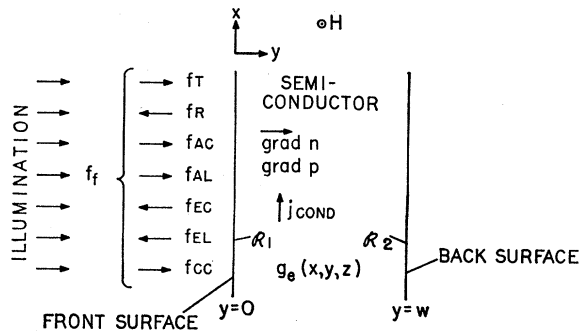
[1] P. Aigrain and C. Benoit à la Guillaume, J. phys. radium 17, 709 (1956); C. Benoit à la Guillaume, J. Phys. Chem. Solids 8, 150 (1959); C. Benoit à la Guillaume and O. Parodi, J. Electronics and Control 6, 356 (1959); M. Bernard and J. Loudette, Compt. rend. 246, 1177 (1958); P. H. Brill and R. F. Schwarz, Phys. Rev. 112, 330 (1958); P. H. Brill and R. F. Schwarz, J. Phys. Chem. Solids 8, 75, 85 (1959); A. G. Chynoweth and K. G. McKay, J. Appl. Phys. 29, 1103 (1958); A. G. Chynoweth and K. G. McKay, J. Appl. Phys. 30, 1811 (1959); A. G. Chynoweth and K. G. McKay, Phys. Rev. 102, 369 (1959); A. G. Chynoweth and K. G. McKay, Phys. Rev. 106, 418 (1957); L. W. Davies, Phys. Rev. Letters 4, 11 (1960); W. D. Dumke, Phys. Rev. 105, 139 (1957); W. D. Dumke, Phys. Rev. 118, 938 (1960); A. Gosnet, O. Parodi, and C. Benoit à la Guillaume, Compt. rend. 248, 1628 (1959); J. R. Haynes, Phys. Rev. 98, 1866 (1955); J. R. Haynes and H. B. Briggs, Phys. Rev. 86, 647 (1952); J. R. Haynes, M. Lax, and W. F. Flood, J. Phys. Chem. Solids 8, 392 (1959); S. H. Koenig and R. D. Brown, III, Phys. Rev. Letters 4, 170 (1960); J. T. Nelson and J. C. Irvin, J. Appl. Phys. 30, 1847 (1959); R. Newman, Phys. Rev. 91, 1313 (1953); R. Newman, Phys. Rev. 100, 700 (1955); R. Newman, W. C. Dash, R. N. Hall, and W. E. Burch, Phys. Rev. 98, 1536(A)

f_{EL} is the radiated flux emitted by the lattice and the equilibrium carriers (usual "radiative" surface conditions, Stefan-Boltzmann law). f_{CC} is the flux carried by conduction and convection.

Components of Energy Flux Inside the Sample

Inside the sample we may split the energy flux f_i into eight components:

$$f_i = f_T + f_{AC} + f_{AL} + f_{EC} + f_{EL} + f_C + f_P + f_{PC}, \quad (II.2)$$

where f_T, f_{AC}, f_{AL}, f_{EC}, f_{EL} have been defined previously and f_C is the heat flux carried by ordinary conduction:

$$f_C = -\kappa \operatorname{grad} T. \quad (II.3)$$

κ is the thermal conductivity of the material in the customary sense, T is the absolute local temperature. This flux component contains in particular lattice conduction and conduction by carriers which are in local thermal equilibrium with the lattice, including "ambipolar diffusion" with negligible lifetime.[2]

f_P is the energy flux transported by individual particles which are not in local thermal equilibrium with the lattice and whose motion may to a first order be independent of $\operatorname{grad} T$. This flux component is primarily responsible for the PT effect; an analytical expression for this component can be found when certain assumptions about the carrier transport mechanism in the semiconductor are made. To obtain a definite description of the PT effect which is particularly applicable to germanium and silicon single crystals, we follow the widely used formulations given by van Roosbroeck,[3,4] and the interested reader is referred to these papers for a discussion of the underlying assumptions. The analytic expression for f_P could be based on the "differential transport velocity" mentioned by van Roosbroeck,[3] but a more direct approach is provided by the "current density Δj associated with added carrier concentration Δp" ($\Delta p \equiv p - p_0 = n - n_0 \equiv \Delta n$), also given by van Roosbroeck[4]:

$$\Delta j = (q\mu_n n_0 j_p - q\mu_p p_0 j_n)/(q\mu_n n_0 + q\mu_p p_0). \quad (II.4)$$

q is the electronic charge, μ_n and μ_p are the electron and hole drift mobilities, n_0 and p_0 are the equilibrium electron and hole densities, n and p are the actual electron and hole densities, $\Delta n = \Delta p$ is the density of excess pairs, and j_p and j_n are the hole and electron current densities. Trapping[5] will be neglected here but it will in general considerably modify the following discussions. It may, however, be analyzed along analogous lines. Since every excess pair carries an amount of energy approximately

equal to the band gap E_G, we find an expression for f_P from Eq. (II.4):

$$f_P = (E_G/q)\Delta j = E_G(\mu_n n_0 j_p - \mu_p p_0 j_n)/$$
$$(q\mu_n n_0 + q\mu_p p_0). \quad (II.5)$$

f_{PC} is the energy flux transported by particle complexes not in thermal equilibrium with the lattice.

Heat Flux Outside Back Surface

Heat flux may leave the back surface of the sample by conduction, convection, and radiation.

Temperature Distribution Inside Sample; Continuity Equation for Heat Energy

A volume element of solid is gaining heat energy Q at a rate

$$\partial Q/\partial t = A - \operatorname{div} f_i. \quad (II.6)$$

Q is the heat energy density in w sec m^{-3}; A is the spontaneous heat generation-annihilation rate in w m^{-3} (Joule heating, radioactive decay, chemical reactions, etc.; in general, A may enhance the energy density of the lattice and equilibrium carriers as well as of the excess carriers, but in the interest of a concise treatment, we assume in the following $A = 0$); f is the heat flux inside the sample defined earlier. Q may be split into three parts,

$$Q = Q_1 + Q_2 + Q_3, \quad (II.7)$$

where Q_1 is the heat energy density of the lattice and the equilibrium carriers, and is associated with the measured *temperature* of the sample,

$$\partial Q_1/\partial t = sd(\partial T/\partial t), \quad (II.8)$$

where s is the specific heat and d is the density of the material.

Q_2 is the energy density of the excess carriers in the volume element which under the assumptions made earlier[3,4] is given by

$$Q_2 = E_G \Delta p. \quad (II.9)$$

Q_3 contains all other forms of energy density, such as that of an electromagnetic field. For conciseness of formulation, we assume in the following that $Q_3 = 0$.

To keep the treatment as simple as possible, we assume in addition that

$$f_{AL} = f_{EC} = f_{EL} = f_{PC} = \operatorname{div} f_T = 0. \quad (II.10)$$

One thus obtains from Eq. (II.6)

$$\frac{\partial Q}{\partial t} = \frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t} = -\operatorname{div} f_{AC} - \operatorname{div} f_C - \operatorname{div} f_P. \quad (II.11)$$

The individual terms are derived from Eqs. (II.3), (II.5), (II.8), (II.9) and

$$\operatorname{div} f_{AC} = -E_G g_e, \quad (II.12)$$

(1955); J. I. Pankove, Phys. Rev. Letters **4**, 20 (1960); N. Sclar and E. Burstein, Phys. Rev. **98**, 1757 (1955); A. Whetstone, Rev. Sci. Instr. **30**, 447 (1959).
[2] P. J. Price, Phil. Mag. **46**, 1252 (1955).
[3] W. van Roosbroeck, Bell System Tech. J. **29**, 560 (1950).
[4] W. van Roosbroeck, Phys. Rev. **91**, 282 (1953).
[5] See, e.g., W. van Roosbroeck, Phys. Rev. **119**, 636 (1960), and Bell System Tech. J. **39**, 515 (1960).

where g_e is the external generation rate (by photons) for excess carrier pairs. Equation (II.12) is consistent with the assumptions about the carrier transport mechanism made earlier.

Substituting these various terms, one obtains from Eq. (II.11),

$$sd\frac{\partial T}{\partial t}-\kappa\nabla^2 T=E_G\left(-\frac{\partial\Delta p}{\partial t}-\frac{1}{q}\operatorname{div}j_p+g_e\right). \quad \text{(II.13)}$$

The continuity equation for excess pairs is given by

$$\frac{\partial\Delta p}{\partial t}=\frac{1}{q}\operatorname{div}\Delta j+g_e-r_p+g_{pi}, \quad \text{(II.14)}$$

where r_p is the rate of hole recombination, g_{pi} is the internal (thermal) generation rate for holes. One thus obtains the following basic equation for the photothermal effect:

$$sd(\partial T/\partial t)-\kappa\nabla^2 T=E_G(r_p-g_{pi}). \quad \text{(II.15)}$$

Under the assumption of constant lifetime (monomolecular recombination), one has

$$-r_p+g_{pi}=-\Delta p/\tau_p, \quad \text{(II.16)}$$

and Eq. (II.15) assumes the form

$$sd(\partial T/\partial t)-\kappa\nabla^2 T=E_G\Delta p/\tau_p. \quad \text{(II.17)}$$

To determine the temperature distribution in the sample, it is thus necessary to calculate the distribution of excess carriers. These carrier distributions are calculated from a set of differential equations which consists of Maxwell's equations, expressions for the conduction-current densities, and continuity equations for the carrier densities.[6]

The expressions for conduction-current densities are:

$$j_{\text{cond}}=j_n+j_p, \quad \text{(II.18)}$$

$$j_n=j_n^*+\tan\theta_n(j_n\times k), \quad \text{(II.19)}$$

$$j_p=j_p^*+\tan\theta_p(j_p\times k), \quad \text{(II.20)}$$

$$j_n^*\equiv q\mu_n n E+qD_n\operatorname{grad}n+\lambda_n\operatorname{grad}T, \quad \text{(II.21)}$$

$$j_p^*\equiv q\mu_p p E-qD_p\operatorname{grad}p+\lambda_p\operatorname{grad}T. \quad \text{(II.22)}$$

The continuity equations for the carrier densities are:

$$\partial n/\partial t=g_n-r_n+(1/q)\operatorname{div}j_n, \quad \text{(II.23)}$$

$$\partial p/\partial t=g_p-r_p-(1/q)\operatorname{div}j_p. \quad \text{(II.24)}$$

The symbols in these equations have the following meanings: j_{cond} is the total conduction current density; θ_n and θ_p are the Hall angles for electrons and holes, respectively; k is the unit vector in the direction of the magnetic field; E is the electric field; D_n and D_p are the

diffusion constants for electrons and holes; λ_n and λ_p characterize the contributions to the electron and hole currents caused by the temperature gradient alone, and present even in the absence of an electric field and any carrier-density gradients; g_n, g_p, r_n, r_p are the generation and recombination rates for electrons and holes, respectively. The generation rates g may be separated into two parts, one, g_i, describing internal thermal generation, the other g_e, describing external generation such as by light:

$$g_n=g_{ni}+g_{ne}, \quad \text{(II.25)}$$

$$g_p=g_{pi}+g_{pe}. \quad \text{(II.26)}$$

One may then define a net internal recombination rate r equal to r_n-g_{ni} for electrons or r_p-g_{pi} for holes. To obtain the temperature distribution in the crystal, one must in general solve the simultaneous system consisting of Eqs. (II.15), (II.18) through (II.24), and Maxwell's equations. To render this problem tractable, it is obviously necessary to make simplifying assumptions. Among several possibilities we select the following special case for further analytical treatment.

III. SPECIAL CASE: THE PT EFFECT IN THE INFINITE SLAB

We assume (1) local charge neutrality, $\Delta n=\Delta p$, $\operatorname{grad}n=\operatorname{grad}p$; (2) a plane parallel homogeneous slab of semiconductor infinite in the x and z directions (see Fig. 1); (3) steady state; (4) open-circuit conditions; (5) small Hall angles; (6) small injection levels; (7) constant lifetime; (8) external generation rate g_e only a function of y. The minority-carrier density is then described[6] by a simple diffusion equation which in the case of N-type material reads,

$$d(Dd\Delta p/dy)/dy-\Delta p/\tau=-g_{pe}, \quad \text{(III.1)}$$

where D is the ambipolar diffusion constant given by $D\equiv(n+p)/(n/D_p+p/D_n)$ and frequently well approximated by $D_p=$ const. To obtain the distribution of excess pairs, one must solve Eq. (III.1) for a given temperature distribution, a given external generation function g_{pe}, and under the prevailing boundary and auxiliary conditions.

To obtain the temperature distribution, including the consequences to the PT effect, one must in general solve Eqs. (II.17) and (III.1) simultaneously. Under the assumption, however, that the temperature differences caused by the PT effect are small enough to leave D, p_0, and τ in Eq. (III.1) essentially constant throughout the sample, one may first solve Eq. (III.1) for Δp (with $T=$const) and then obtain the temperature gradient due to the PT effect by solving Eq. (II.17) for T.

A particularly simple case prevails if the external bulk generation rate g_{pe} is zero (surface generation only) and carrier and temperature gradients lie in the y direction only; then

$$T(x,y,z)=-(E_G D_p/\kappa)\Delta p(y)+c_1 x+c_2 y+c_3 z+c_4, \quad \text{(III.2)}$$

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[6] This formulation is patterned after that for the photomagneto-electric effect given by W. van Roosbroeck, Phys. Rev. 101, 1713 (1956), and a generalization given by W. W. Gärtner, Phys. Rev. 105, 823 (1957).

where c_1 through c_4 are constants of integration to be determined by the boundary conditions for temperature and energy flux.

Under the assumption that D_p, p_0, and τ_p are constant throughout the sample, one obtains the following expression[7] for Δp:

$$\Delta p = \alpha e^{y/L} + \beta e^{-y/L} - (L/D) \int_0^y g_{pe}(\eta)\, \sinh[(y-\eta)/L]d\eta, \tag{III.3}$$

where

$$\alpha = \frac{(\mathfrak{D}+s_1)\mathfrak{R}_2 + (\mathfrak{D}-s_2)\mathfrak{R}_1 \exp(-w/L) + (\mathfrak{D}+s_1)\Gamma}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)\exp(w/L) - (\mathfrak{D}-s_1)(\mathfrak{D}-s_2)\exp(-w/L)}, \tag{III.4a}$$

$$\beta = \frac{(\mathfrak{D}-s_1)\mathfrak{R}_2 + (\mathfrak{D}+s_2)\mathfrak{R}_1 \exp(w/L) + (\mathfrak{D}+s_1)\Gamma}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)\exp(w/L) - (\mathfrak{D}-s_1)(\mathfrak{D}-s_2)\exp(-w/L)}, \tag{III.4b}$$

$$\Gamma = \frac{1}{D}\int_0^w g_{pe}(\eta)\left[D\cosh\left(\frac{w-\eta}{L}\right) + s_2 L \sinh\left(\frac{w-\eta}{L}\right)\right]d\eta, \tag{III.4c}$$

and $\mathfrak{D}=D/L$; $L=(D\tau_p)^{\frac{1}{2}}$ is the diffusion length for excess pairs; $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are the external (optical) pair generation rates on the front and back surface, respectively; s_1 and s_2 are the corresponding surface recombination velocities; w is the thickness of the slab. This solution is valid even when g_{pe} is an arbitrary function of y, i.e., when the incident light contains components of different absorption coefficients, and when internal reflections occur. Many special cases are discussed by Gärtner.[7]

Substituting these solutions for Δp into Eq. (II.17) and solving for $T(y)$, one obtains the temperature distribution in the sample which contains the contribution from the PT effect. For the case where $\mathfrak{R}_1=\mathfrak{R}_2=0$ and monochromatic radiation,

$$g_{pe}=Ik\,\exp(-ky), \tag{III.5}$$

where I is the photon flux density and k is the absorption coefficient of the incident monochromatic radiation, one finds the following expression for $T(x,y,z)$:

$$T(x,y,z) = -\frac{E_G L^2 Iw}{\kappa\tau D}\frac{K}{W^2-K^2}\left[\frac{(K-S_2)(W-S_1)e^{-K}-(K+S_1)(W+S_2)e^W}{(W+S_1)(W+S_2)e^W-(W-S_1)(W-S_2)e^{-W}}e^{-Y}\right.$$
$$\left. +\frac{(K-S_2)(W+S_1)e^{-K}-(K+S_1)(W-S_2)e^{-W}}{(W+S_1)(W+S_2)e^W-(W-S_1)(W-S_2)e^{-W}}e^Y+e^{-(K/W)Y}\right]+c_1x+c_2y+c_3z+c_4, \tag{III.6}$$

with $W=w/L$, $Y=y/L$, $K=kw$, $S_1=s_1w/D$, and $S_2=s_2w/D$. For the case where $(1/L)=k$, the solution for T, Eq. (III.6), assumes the indeterminate form $0/0$. Solving Eq. (III.1) for $1/L=k\,(W=K)$ then yields

$$T(x,y,z) = -\frac{E_G L^2 Iw}{2\kappa\tau D}\left\{\left[\frac{K+S_2+(K-1-S_2)(K-S_1)}{(K+S_1)(K+S_2)e^K-(K-S_1)(K-S_2)e^{-K}}e^{-K}+\frac{Y}{K}\right]e^{-Y}\right.$$
$$\left.+\frac{K-S_2+(K-1-S_2)(K+S_1)}{(K+S_1)(K+S_2)e^K-(K-S_1)(K-S_2)e^{-K}}e^{Y-K}\right\}+c_1x+c_2y+c_3z+c_4. \tag{III.7}$$

For the case of front-surface generation only,

$$\mathfrak{R}_1\neq0, \quad \mathfrak{R}_2=0, \quad g_{pe}=0, \tag{III.8}$$

one finds

$$T(x,y,z) = -\frac{E_G L^2 \mathfrak{R}_1}{\kappa\tau}\frac{(\mathfrak{D}-s_2)e^{Y-W}+(\mathfrak{D}+s_2)e^{-(Y-W)}}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^W-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}+c_1x+c_2y+c_3z+c_4. \tag{III.9}$$

The constants of integration, c_1 through c_4, must be determined from the boundary conditions for temperature and/or heat flux. There are obviously many different possibilities for the type of boundary conditions one might employ, depending on the assumptions about the various energy-flux components (see Fig. 1) and the temperature at front and back surfaces. We shall discuss here only a few special cases.

In the following discussions we assume that

$$f_T=f_R=f_{AL}=f_{EC}=f_{EL}=0. \tag{III.10}$$

[7] W. W. Gärtner, Phys. Rev. 105, 823 (1957).

The simplest boundary conditions are then given by

$$T(0)=T_1=\text{const} \quad \text{at} \quad y=0,$$
$$T(w)=T_2=\text{const} \quad \text{at} \quad y=w. \tag{III.11}$$

This condition may, for example, be realized by bringing the front surface of the sample in contact with a stirred transparent liquid (to permit simultaneous illumination).

It is now of interest to determine the heat flux though the sample for this temperature difference which is equivalent to determining the "apparent thermal conductivity" of the crystal under the influence of the photothermal effect. As an illustration, we shall do this for the case characterized by Eq. (III.8). The constants in Eq. (III.9) are then given by

$$c_1=c_3=0, \tag{III.12a}$$

$$c_2=\frac{T_2-T_1}{w}+\frac{E_GL^2\mathfrak{R}_1}{\kappa\tau}\frac{1}{w}\frac{2\mathfrak{D}-(\mathfrak{D}-s_2)e^{-W}-(\mathfrak{D}+s_2)e^{W}}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^{W}-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}, \tag{III.12b}$$

$$c_4=T_1+\frac{E_GL^2\mathfrak{R}_1}{\kappa\tau}\frac{(\mathfrak{D}-s_2)e^{-W}+(\mathfrak{D}+s_2)e^{W}}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^{W}-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}. \tag{III.12c}$$

The heat flux carried through the slab by the lattice and electrons and holes is in the steady state equal to the heat flux f_B leaving the back surface. It consists of two contributions:

$$f_B=f_C(w)+f_P(w), \tag{III.13}$$

where from Eq. (II.3)

$$f_C(w)=-\kappa\frac{\partial T}{\partial y}\bigg|_{y=w}, \tag{III.14}$$

and

$$f_P(w)=\frac{E_G}{q}j_p(w)=-E_GD\frac{\partial p}{\partial y}\bigg|_{y=w}. \tag{III.15}$$

The last equation is obtained by substituting the expression for the Dember field [see, e.g., reference 7, Eq. (18)] into Eqs. (II.20) and (II.22). From Eqs. (III.9) and (III.12) one finds

$$\frac{\partial T}{\partial y}\bigg|_{y=w}=\frac{T_2-T_1}{w}+\frac{E_GL\mathfrak{R}_1}{\kappa\tau}\frac{2s_2+[2\mathfrak{D}-(\mathfrak{D}-s_2)e^{-W}-(\mathfrak{D}+s_2)e^{W}]w^{-1}}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^{W}-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}. \tag{III.16}$$

The second term on the right-hand side of Eq. (III.16) describes the modification in the temperature distribution due to the PT effect, and it vanishes for $\mathfrak{R}_1=0$, or $\tau\to 0$, or $s_1\to\infty$, or any combination of these conditions.

$(\partial p/\partial y)$ may, for example, be calculated from Eqs. (10) and (11) in reference 7, and one obtains

$$\frac{\partial p}{\partial y}\bigg|_{y=w}=-\frac{\mathfrak{R}_1}{L}\frac{2s_2}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^{W}-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}. \tag{III.17}$$

Substitution of Eqs. (III.14) through (III.17) into (III.13) yields the following expression for the heat flux f_B through the slab:

$$f_B=\kappa\frac{T_1-T_2}{w}-\frac{E_GL\mathfrak{R}_1}{\tau W}\frac{2\mathfrak{D}-(\mathfrak{D}-s_2)e^{-W}-(\mathfrak{D}+s_2)e^{W}}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^{W}-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}. \tag{III.18}$$

The second term on the right-hand side is due to the PT effect.

To calculate some numerical examples, we simplify this expression by assuming very long bulk lifetime, i.e.,

$$W\ll 1 \to e^{\pm W}\approx 1\pm W, \tag{III.19}$$

and one finds

$$f_B=\kappa\frac{T_1-T_2}{w}+\frac{E_GL\mathfrak{R}_1}{\tau}$$
$$\times\frac{s_2}{\mathfrak{D}(s_1+s_2)+\mathfrak{D}^2W+s_1s_2W}. \tag{III.20}$$

For high front-surface recombination, $s_2/s_1\to 0$, the second term on the right-hand side and thus any evidence of the PT effect vanishes. For extremely long lifetime, $W\to 0$, f_B assumes the form

$$f_B(W\to 0)=\kappa\frac{T_1-T_2}{w}+\frac{E_G\mathfrak{R}_1s_2}{s_1+s_2}, \tag{III.21}$$

and for low front-surface recombination and high back-surface recombination, $s_1/s_2\to 0$, there results

$$f_B(W\to 0, s_1/s_2\to 0)=\kappa\frac{T_1-T_2}{w}+E_G\mathfrak{R}_1. \tag{III.22}$$

TABLE I. Relative apparent thermal conductivity κ_a/κ.

$(T_1-T_2)/w$ in w cm⁻²	$E_G\mathfrak{R}_1$ in w cm⁻²	s_1 in cm sec⁻¹	s_2 in cm sec⁻¹	κ_a/κ
1	0.01	100	10⁴	1.00047
0.01	0.01	100	10⁴	1.047
0.0001	0.01	100	10⁴	5.7
0.0001	0.1	100	10⁴	4.8
0.0001	0.1	10⁴	100	1.047
0.0001	0.01	100	100	4.7
0	0.01	100	10⁴	∞
−0.001	0.01	100	10⁴	−3.7

The last equation indicates that under the conditions mentioned before, the entire incident radiated flux, $f_{AC}=E_G\mathfrak{R}_1$, is transported through the sample by the photothermal effect, although the crystal is completely opaque to the incident light.

To get a quantitative idea of the magnitude of the PT effect as compared to ordinary thermal conductivity, we define an "apparent thermal conductivity," κ_a, as the ratio of the actual heat flux f_B divided by the temperature gradient $(T_1-T_2)/w$ which would prevail in the absence of the PT effect:

$$\kappa_a = f_B w/(T_1-T_2). \quad (III.23)$$

Table I shows the "relative apparent thermal conductivity" κ_a/κ,

$$\frac{\kappa_a}{\kappa} = 1 + \frac{w}{\kappa(T_1-T_2)}E_G\mathfrak{R}_1\frac{L}{\tau}$$
$$\times\frac{s_2}{\mathfrak{D}(s_1+s_2)+\mathfrak{D}^2W+s_1s_2W}, \quad (III.24)$$

for various values of temperature difference between front and back surface, incident light intensity, and front and back surface recombination velocities. The fixed parameters are (for germanium) $\kappa=0.64$ w deg⁻¹ cm⁻¹, $D=50$ cm² v⁻¹ sec⁻¹, $W=10(\gg1)$, $L=1$ cm, $\tau=20$ msec. The table indicates that strong radiation and small temperature differences result in a high value of κ_a which is also increased by small values of s_1 and high values of s_2. One observes that in extreme cases the heat transported by the photothermal effect may be several times that carried by ordinary conduction. Furthermore, κ_a may become infinite which means that heat is transported through the slab although no temperature difference exists between opposite faces of the slab. Finally, the apparent thermal conductivity may even become negative, indicating that heat flows against the temperature gradient. Obviously, the apparent thermal conductivity is strongly anisotropic and shows the strongest deviation from the ordinary thermal conductivity in the direction perpendicular to the faces of the slab.

The boundary conditions for the PT effect may, of course, be more complicated than the ones given by Eq. (III.11). One may, for example, still have

$$T(w) = T_2 = \text{const} \quad \text{at} \quad y=w, \quad (III.25)$$

whereas the front surface may emit a heat flux f_{EL} which is a function of its temperature T_1. In the simplest case one may take

$$f_{EL} = -\alpha(T_1-T_0), \quad (III.26)$$

where α is the "exterior thermal conductivity" and T_0 is an external reference temperature. Equation (III.26) may, for example, be the linear approximation to the Stefan-Boltzmann radiation law. In the steady state, the incoming heat flux f_{AC} must then equal the heat flux leaving the sample (f_{EL},f_B), i.e.,

$$f_{AC}+f_{EL}+f_B=0. \quad (III.27)$$

Substituting f_{EL} from Eq. (III.26) and f_B from Eq. (III.18) [under assumption (III.8) about carrier absorption where $f_{AC}=E_G\mathfrak{R}_1$], one finds the following expression for the front-surface temperature T_1

$$T_1 = \left\{\alpha T_0 - \frac{\kappa}{w}T_2 + f_{AC}\left[1 - \frac{L}{\tau W}\right.\right.$$
$$\left.\left.\times\frac{2\mathfrak{D}-(\mathfrak{D}-s_2)e^{-W}-(\mathfrak{D}+s_2)e^W}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2)e^W-(\mathfrak{D}-s_1)(\mathfrak{D}-s_2)e^{-W}}\right]\right\}$$
$$\times\left(\frac{\kappa}{w}-\alpha\right)^{-1}. \quad (III.28)$$

The second term in the square bracket describes the modification of the front-surface temperature due to the PT effect as compared to the case where the entire incident radiation is absorbed on the surface by the lattice and the equilibrium carriers only.

Many other special cases of the PT effect may be calculated in analogy to the treatment of the Dember effect in references 6 and 7.

IV. CONCLUSIONS

The photothermal effect is readily analyzed for the case of small injection and monomolecular recombination. For small temperature gradients it may lead to large deviations of the apparent thermal conductivity from the ordinary thermal conductivity of the bulk material. Its measurement, however, seems to be delicate. It appears easier to observe the related photomagnetothermal effect, just as it is simpler to measure the photomagnetoelectric effect than the Dember effect. The photomagnetothermal effect, whose theory may be worked out from the basic equations given in this paper, will be the subject of a future publication.