netism measurements, we point out that the NMR technique may be used for probing the transient, quasi-static magnetic fields in a plasma. The NMR apparatus yields $\int \delta B_z(t)dt$ directly; the integration is done by the spin "memory" of the protons, and not by an external circuit. Of course, δB_z should not contain important Fourier components above ω_{spins}. A considerable advantage of this method resides in the fact that the observations may be postponed until the noisy transient plasma has been deactivated.

ACKNOWLEDGMENTS

The authors wish to thank Professor C. P. Slichter for his suggestion on the nuclear resonance aspect of this experiment and R. A. Kawcyn for his experimental assistance and contribution to the electronic design.

PHYSICAL REVIEW VOLUME 122, NUMBER 2 APRIL 15, 1961

Electrical Breakdown in Hydrogen at Low Pressures*

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(Received December 13, 1960)

Experimental and theoretical determinations of the static voltage-current characteristics, extending from the region of the Townsend (self-maintained) discharge to the normal glow discharge, have been carried out in hydrogen at low pressures, (7–25 mm Hg). The calculations, made on an electronic computer, were based on the distortion of the electric field by space charge, and used the experimentally determined variation of both the primary and secondary Townsend ionization coefficients on the ratio of the field to pressure. Good agreement is obtained between the measured and calculated breakdown and glow voltages, and both the experimental and theoretical curves of the characteristic are of similar shape.

I. INTRODUCTION

The breakdown potential of a gas between parallel plate electrodes was derived by Townsend, who gave a criterion which expressed the condition that internal ionization processes, in the gas and at the cathode, just replace the externally initiated current.[1] However, the actual breakdown transition, i.e., the lowering of the potential with the increase of current, is less well understood.

The voltage-current characteristic of a discharge in this region of transition from breakdown to glow voltage is difficult to obtain experimentally. Mathematical calculations for this region are complex because of the behavior of the space-charge field and its subsequent effect on the discharge parameters. However, with the aid of an electronic computer, Ward[2] has calculated static characteristics which extend well into the glow region at low pressures. The present paper introduces certain improvements into these calculations and compares such theoretical characteristics with those determined experimentally in hydrogen at low pressures. Measurements and calculations extend from breakdown to the normal glow discharge.

A report on a similar comparison for hydrogen at high pressure, but restricted to the breakdown region and below, has been published recently.[3]

II. DESCRIPTION OF APPARATUS

(a) Discharge Tube and Vacuum System

The experiments were carried out in a simple two-electrode parallel plate discharge tube. The electrodes were made of copper in the shape of disks 3 cm in diam, rounded at the edges to prevent local field intensification, mounted parallel and 0.195 cm apart. The electrodes were first cleaned by washing in grease solvents and then carefully polished. The final polishing was carried out with a Selvyt cloth with micro-alumina as the polishing agent. After further washing in distilled water and drying, the electrodes were sealed into a borosilicate glass envelope which was quickly sealed into a vacuum system and pumped down to a pressure of the order of 10^{-6} mm Hg with the aid of an oil diffusion pump and a conventional backing pump. Liquid air traps were always used to prevent back diffusion of oil into the discharge tube.

The gas used in the experiments was hydrogen produced by electrolysis of pure barium hydroxide dissolved in distilled water. The gas was dried by many hours

* Supported in part by the Office of Ordnance Research.
† Now at CERN, European Organization for Nuclear Research, Geneva, Switzerland.
[1] See, for example, F. Llewellyn-Jones, *Ionization and Breakdown in Gases* (Methuen and Company, London, 1957).
[2] A. L. Ward, Phys. Rev. **112**, 1852 (1958).
[3] D. J. De Bitetto, L. H. Fisher, and A. L. Ward, Phys. Rev. **118**, 920 (1960).

contact with phosphorous pentoxide and was then passed through a heated palladium thimble and into the discharge tube via a liquid air trap. Gas pressures were measured with an oil manometer and this too was isolated from the discharge tube by means of a liquid air trap.

(b) Voltage and Current Measurement

The voltage source, a stabilized 3-kv power supply, the output of which was varied by means of a potential divider, has been described in detail elsewhere.[4] By continuously monitoring the output, a stability of better than 0.01% was obtained. The voltage across the gap was measured with the very high impedance ($\sim 10^{11}$ ohm) electrometer voltmeter shown in Fig. 1. Such a voltmeter had to be used because of the high impedance of the discharge gap. It proved to be more suitable than an electrostatic voltmeter, the relatively large capacitance of the latter tending to produce oscillations in the discharge. The voltmeter could be calibrated directly by simply applying an accurately known voltage across the resistors and adjusting the sensitivity of the electrometer to give a suitable deflection on the galvanometer marked G in Fig. 1. The voltmeter was found to be linear to well within $\pm 1\%$ and the drift inherent in all electrometer devices was reduced to negligible proportions.

Discharge currents were measured with a Pye scalamp galvanometer (marked I in Fig. 1) and care was taken to ensure that leakage currents were eliminated from the measurements.

III. EXPERIMENTAL PROCEDURE AND RESULTS

The voltage-current characteristics were measured at a given pressure by the following procedure.

Ultraviolet light was allowed to fall on the cathode of the discharge gap and the voltage increased until the current in the gap grew to some tenths of a microampere. At this stage the ultraviolet light was switched off so that the production of externally produced electrons ceased. If the discharge current persisted, and did not oscillate, the experiment was continued by continually increasing the voltage of the power supply in small steps ($\sim$0.1 volt). The measurements of the voltage at the corresponding current flowing in the gap were then carried out using the apparatus described in the preceding section.

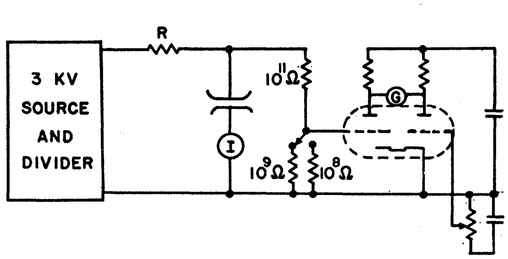

FIG. 1. Block diagram of apparatus used for determining $V-I$ characteristics.

It was usual, in the initial stages of the experiments, to find that the discharge currents oscillated and were not consistently self maintained after the source of initiatory electrons, produced by ultraviolet light, was switched off. This behavior was attributed to an unclean and unstable cathode surface. Thus a long and careful cathode cleaning procedure was evolved which entailed first a baking and heating of the electrodes *in vacuo* and afterwards bombarding the cathode with positive ions by running a glow discharge of 50 ma in hydrogen over a period of several tens of hours. After glow discharging, the discharge tube was evacuated, fresh hydrogen introduced and the voltage-current measurements again attempted. If the discharge was self-sustaining at low current values without oscillations then the voltage-current characteristic was measured. If it was not able to sustain itself, the cathode was again subjected to the heating and glow discharge treatment until the stable self-sustaining conditions were realized. Only under these conditions were the voltage-current characteristics measured.

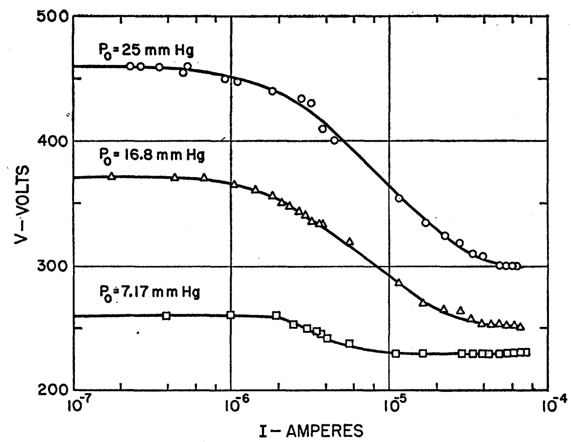

FIG. 2. Experimental voltage-current static characteristics for hydrogen, for a 0.195-cm gap. The pressure corrected to 0°C is shown for each curve.

The results of the experiments are shown in Fig. 2 in which the voltage is plotted versus the current. In these experiments, the cross-sectional areas of the discharges were not measured.

IV. CALCULATIONS

(a) Formulation

The general method of calculation is given in reference 2. The two differential equations for the electron current density, J_-, and the electric field, E, which have been solved with the aid of the computer are

$$dJ_-/dx=\alpha(x)J_-(x), \tag{1}$$

[4] C. G. Morgan, Phys. Rev. **104**, 566, (1956).

and

$$\frac{dE}{dx}=\frac{1}{\epsilon_0}\left[\frac{J-J_-(x)}{W_+(x)}-\frac{J_-(x)}{W_-(x)}\right], \tag{2}$$

where x is the distance measured from the cathode, J is the total current density, ϵ_0 is the permittivity of free space, and W_+ and W_- are the drift velocities of the positive ions and electrons, respectively. The dependence of Townsend's first ionization coefficient, α/p_0, on E/p_0 is given by

$$\alpha/p_0=A_1\exp(-B_1p_0/E),\quad E/p_0\leq K_1, \tag{3a}$$

$$\alpha/p_0=A_2\exp(-B_2p_0/E),\quad E/p_0>K_1, \tag{3b}$$

where A_1, A_2, B_1, B_2, and K_1 are constants and p_0 is the gas pressure corrected to 0°C. The boundary conditions are $J_-(d)=J$ (d is the anode distance), and

$$J_-(0)=\frac{J_0+(\omega/\alpha)J}{1+(\omega/\alpha)}, \tag{4}$$

where ω/α is the general secondary ionization coefficient and J_0 is the externally initiated current density.

An important improvement has been made over previously reported calculations. The dependence of the positive ion mobility on E/p_0 has been expressed as a function of the form

$$W_+=C_1p_0^{-1}E(1-D_1p_0^{-1}E),\quad E/p_0\leq K_2, \tag{5a}$$

$$W_+=C_2p_0^{-\frac{1}{2}}E^{\frac{1}{2}}(1-D_2p_0^{\frac{3}{2}}E^{-\frac{3}{2}}),\quad E/p_0>K_2, \tag{5b}$$

where C_1, C_2, D_1, D_2, and K_2 are constants. The electron mobility, the variation of which plays a comparatively minor role in the calculations, has been assumed constant.

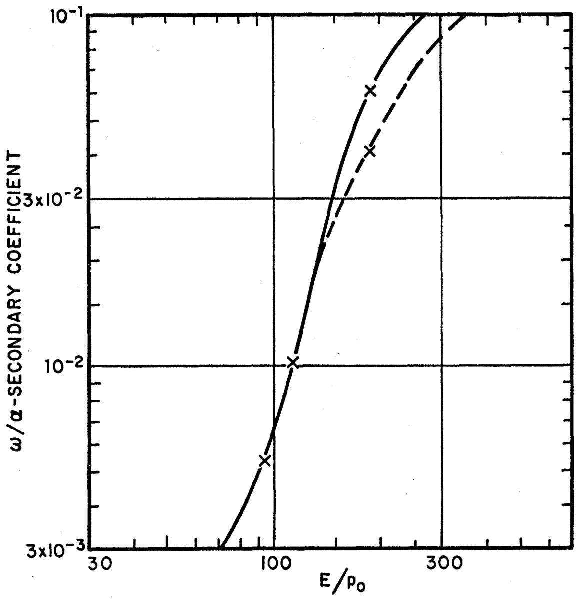

FIG. 4. Secondary coefficients, ω/α, calculated from breakdown fields are shown as points. Solid and dashed lines show the two assumed variations which were used in calculations.

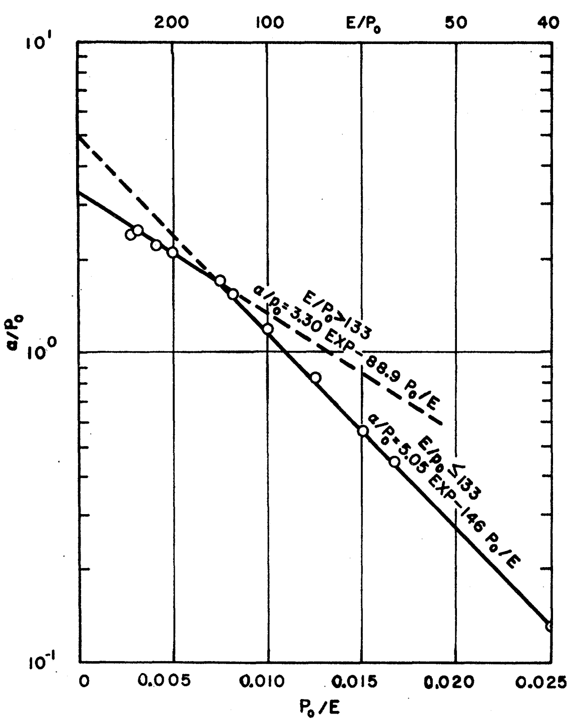

FIG. 3. Variation of Townsend's first coefficient, α, as a function of the field, E. Points shown are experimental data; straight lines indicate calculation parameters.

(b) Parameters for Calculations

The coefficient α was measured in the same gas system as that in which the static characteristics were measured.[5] The results are shown in Fig. 3, where the slopes and intercepts yield the values of the constants: $A_1=5.05$ cm^{-1} (mm Hg)$^{-1}$, $B_1=146$ volt cm^{-1} (mm Hg)$^{-1}$, $A_2=3.30$ cm^{-1} (mm Hg)$^{-1}$, B_2 was calculated internally by the computer to satisfy the continuity of α, and $K_1=133$ volt cm^{-1} (mm Hg)$^{-1}$. Calculations were also made for $K_1=10^5$ volt cm^{-1} (mm Hg)$^{-1}$, i.e., only the constants A_1 and B_1 were used in this case.

The secondary coefficient, ω/α, at breakdown was determined from these constants determining α/p_0, and the breakdown potential, V_s, at each pressure, assuming no space charge distortion. The Townsend breakdown condition is then

$$1+(\omega/\alpha)^{-1}=\exp[Ap_0d\exp(-Bp_0d/V_s)], \tag{6}$$

where the values used for A and B depend on whether V_s/p_0d is greater or less than K_1. A plot of ω/α vs E/p_0 determined in this manner is shown as a solid line in

[5] E. Jones and F. Llewellyn-Jones, Proc. Phys. Soc. (London) **72**, 363 (1958).

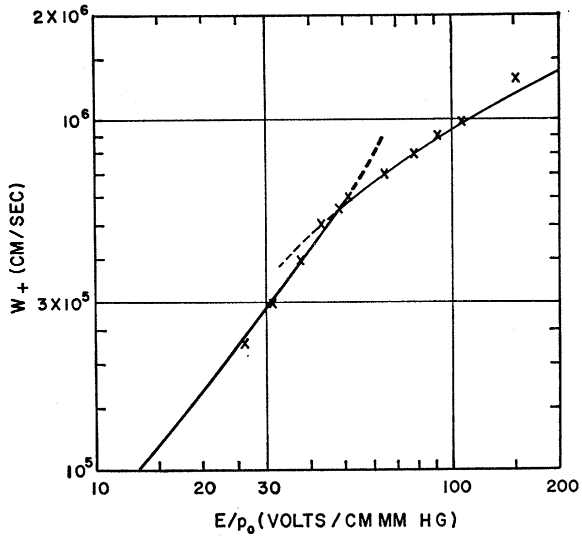

FIG. 5. Variation of positive-ion velocity, W_+, with field E. Points are from reference 6; solid lines show variation used in calculations.

Fig. 4. The curve using entirely the constants A_1 and B_1, even for $V_s/p_0d > 133$ volt cm^{-1} (mm Hg)$^{-1}$, is shown as a dashed line in Fig. 4. The extrapolation to higher values of E/p_0 is arbitrary, and will receive later comment.

The externally initiated current density, J_0, was assumed to be 10^{-12} amp cm^{-2}. A change of two orders of magnitude in either direction for J_0 does not noticeably (less than 0.1 volt at most) change the calculated voltage in the region of the experimental measurements.

The measurements of Rose[6] were used to determine the constants of Eq. (5) for the variation of the positive ion drift velocity with E/p_0. The constants chosen were $C_1 = 6\times10^3$ cm^2 (mm Hg) volt^{-1} sec^{-1}, $D_1 = -1.97\times10^{-2}$ volt^{-1} cm (mm Hg), $C_2 = 1.0\times10^5$ cm$^{\frac{3}{2}}$ (mm Hg)$^{-\frac{1}{2}}$ volt^{-1} sec^{-1}, and $K_2 = 48$ volt cm^{-1} (mm Hg)$^{-1}$. D_2 was calculated internally by the computer. The fit to Rose's data is shown in Fig. 5. The electron mobility was chosen as 3.5×10^5 cm^2 volt^{-1} sec^{-1} at 1 mm Hg.

(c) Method of Calculations

The formulation as programmed for computation permits calculations only for a constant ω/α. Figure 4 shows that the experimentally measured ω/α is a strongly-varying function of E/p_0. Consequently, families of static characteristics were calculated at each pressure with ω/α as a parameter. Such a family for $p_0 = 25$ mm Hg is shown in Fig. 6. It was then assumed that the effective ω/α was determined by the field existing at the cathode. This means, in effect, that a positive-ion secondary coefficient was used.

The method of determining the value of J at which each ω/α static curve is valid is somewhat indirect. An example, using $\omega/\alpha = 1.5\times10^{-2}$, will be used to illustrate the procedure. Figure 4 shows that $\omega/\alpha = 1.5\times10^{-2}$ for an E/p_0 value of 125 volts cm^{-1} (mm Hg)$^{-1}$. Hence the $\omega/\alpha = 1.5\times10^{-2}$ static characteristic is valid only for the current density which corresponds to that E/p_0 value at the cathode, i.e., $E_k/p_0 = 125$ volts cm^{-1} (mm Hg)$^{-1}$ in our example. The family of E_k/p_0 vs J curves, with ω/α as parameter, is also plotted in Fig. 6. In our example, the $\omega/\alpha = 1.5\times10^{-2}$ curve has $E_k/p_0 = 125$ for a current density of 1.15×10^{-3} amp cm^{-2}. The current density values thus determined are marked by crosses in Fig. 6. Finally from the static characteristic for $\omega/\alpha = 1.5\times10^{-2}$ one finds that $V = 407$ volts for $J = 1.15\times10^{-3}$ amp cm^{-2}. Other points on the static characteristic for a varying ω/α are calculated in this manner and indicated by open points in Fig. 6. The heavy solid line is drawn through these points. The calculated static characteristics for the three pressures are shown in Fig. 7. The dashed line was calculated using $K_1 = 133$, while the solid line was calculated for $K_1 = 10^5$ volt cm^{-1} (mm Hg)$^{-1}$.

The extrapolation of ω/α in Fig. 4 to higher E/p_0 values than obtained experimentally at breakdown deserves some comment. The extrapolation used was chosen with the guide of published data and specifically to give approximate agreement between the calculated voltage minimum and the experimentally measured normal glow voltage at the highest pressure. If a curve through $\omega/\alpha = 10^{-1}$ at $E/p = 500$ volts cm^{-1} (mm Hg)$^{-1}$ is used, a minimum voltage of 315 volts at $\approx 4\times10^{-2}$ amp cm^2 is calculated for that pressure. Calculations to higher current densities than the highest values plotted are beyond the capability of the computer as the problem is presently programmed.

V. DISCUSSION AND CONCLUSIONS

The above calculations necessarily yield voltage vs *current density* static characteristics, while one must

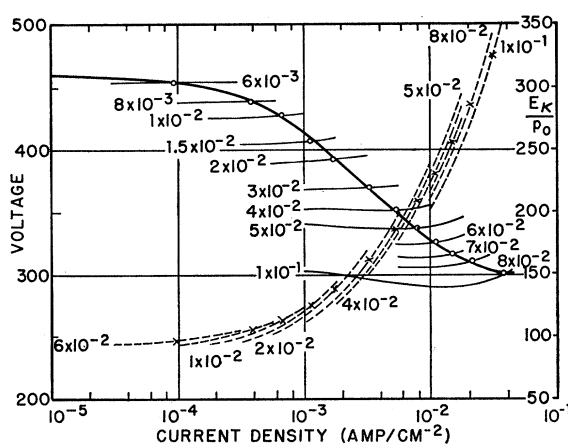

FIG. 6. Calculated voltage vs current density, J, characteristics are shown in light solid lines for the constant ω/α values shown on each curve. The dashed lines show calculated cathode fields, E_k, (right scale) as a function of J. The derived static characteristic for a varying ω/α is shown in the heavy solid line.

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[6] D. J. Rose, J. Appl. Phys. 31, 643 (1960).

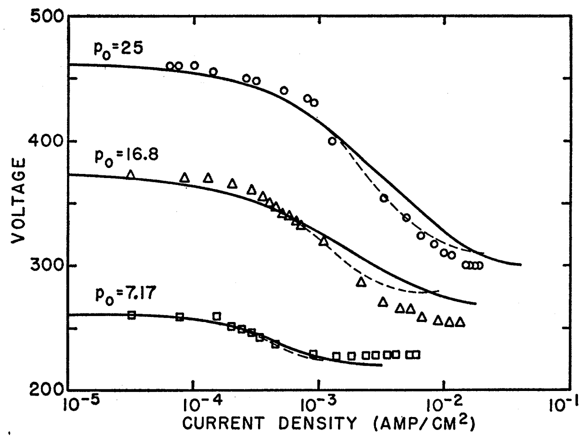

FIG. 7. Solid and dashed lines show calculated voltage-current density static characteristics (see text). The points show corresponding experimental data for comparison, under the assumption of a constant discharge area for each pressure.

experimentally measure voltage vs *current* static characteristics. As mentioned in part III, no attempt to measure discharge areas was made; indeed, such measurements present a number of serious problems.

Completely apart from the question of the discharge area, one may compare the experimental and calculated normal glow voltages. The agreement is quite good, the errors being less than 15 volts for $p_0=16.8$ mm Hg and 10 volts for $p_0=7.17$ mm Hg. Also, the general shape and symmetry of the calculated curves are similar to the experimental ones, though, of course, these are not independent of the discharge area.

In order to display the experimental and calculated curves on the same graph, we have chosen to use the curves for the lowest pressure, since the area changes less with current than at higher pressures. Arbitrarily choosing $V=250$ volts for comparison, one obtains an experimental current from Fig. 2 of 3×10^{-6} amp, and a calculated current density of 2.5×10^{-4} amp cm^{-2} from Fig. 7. One then requires an area of 1.2×10^{-2} cm^2 for agreement. Using this area, the experimental data for the lowest pressure of Fig. 2 is converted to current density values and plotted also in Fig. 7. Since diffusion is inversely proportional to pressure, one may expect the discharge area to be similarly related. Consequently, areas of $(7.17/25)\times1.2\times10^{-2}$ cm^2 and $(7.17/16.8)\times1.2\times10^{-2}$ cm^2 were chosen for use in plotting the experimental data for pressures of 25 and 16.8 mm Hg, respectively, in Fig. 7. One may note that the change of area expected from diffusion gives satisfactory agreement between the calculated and experimental characteristics. Moreover, one may note a steeper slope of experimental data than for the calculated data at the higher pressures, indicating a decrease in discharge area with increasing current, as one observes experimentally.

An effective discharge diameter of a little more than one millimeter at the lowest pressure is indicated by the above comparisons. Recollection of the visible discharges in this pressure range indicate a diameter perhaps five times larger than this. It seems reasonable that visible discharge area might be considerably larger than the region of sensibly constant current density. However, no truly satisfactory explanation seems possible at present. A three-dimensional calculation including diffusion does not seem feasible even with a computer. The only parameter upon which the calculated area depends strongly is the positive-ion mobility, μ_+. No substantial error in μ_+ is thought probable.

An equally serious question is that of the effective secondary coefficient. The calculations assumed that the field at the cathode completely determined ω/α. This is true only for a positive-ion secondary process. Morgan[4] and Jones and Llewellyn-Jones[7] have shown that for $E/p_0\sim100$, approximately 50% of the total secondary coefficient is due to the action of positive ions at the cathode. This result is obtained from a comparison between experimental and calculated temporal growth of discharges. It is difficult to estimate what effect a photoelectric contribution to the secondary coefficient would have upon calculations, when the field where the photons are created varies across the gap. In the light of the present agreement between experimental and calculated characteristics, the assumption of a total ion secondary process seems adequate.

There does not seem to be sufficient difference between results of calculations using $K_1=133$ and $K_1=10^5$ to enable one to make a definite choice for α/p_0 values at high E/p_0.

Experimental measurements of the distribution of current density over the cathode surface would enable a more critical test to be made of the calculations.

ACKNOWLEDGMENTS

The present formulation was encoded for the IBM 704 by Mr. J. P. Menard of the National Bureau of Standards Computation Laboratory.

We wish to thank Professor F. Llewellyn-Jones, in whose Laboratory the experimental work was carried out by one of us (E. J.) in a research programme on electrical breakdown supported by the Electricity Supply Research Council.

[7] E. Jones and F. Llewellyn-Jones, Proc. Phys. Soc. (London) **75**, 762 (1960).