Intermediate Vector Boson and Radiative Lepton Decay of the K Meson*

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K meson decay into electron, neutrino, and photon is analyzed in the lowest order perturbation with respect to weak and electromagnetic interactions, but without making any approximation regarding the strong interaction. The weak interaction is assumed to be transmitted by a single charged intermediate vector meson, which interacts with the weak current in the conventional way. It is pointed out here that a certain angular and momentum distribution of decay particles could reveal almost unequivocally whether the intermediate vector meson does exist. It is shown also that other lepton decays of the K meson, which includes μ mesons and π mesons, cannot be used for the same purpose.

I. INTRODUCTION

ONE of the current problems in weak interactions is whether the intermediate vector meson (called B meson hereafter) is really responsible for the weak interaction.[1] We investigate in this paper three-body decays of the K meson into leptons to answer this question. The reasons for this choice are as follows: β decay and pion decay are not suitable, since the energy available in decays is not quite high enough. Two-body lepton decays of the K meson do not reveal anything regarding the B meson. Decays of hyperons and non-leptonic decays involve either unknown details of weak interactions or severe complications with respect to strong interactions. Thus, only three-body lepton decays of the K meson are left as long as we disregard decays which may be discovered in the future. The relevant experiments are not quite feasible at this moment, but will be manageable in the near future when higher intensity K meson beams are attained, for example after the completion of the new Argonne machine.

The decay of the K meson takes place, according to most of the schemes presumed so far, through a group of particles interacting strongly. Therefore, the main difficulty in this problem is how to separate the information on the B meson from the unknowns due to strong interactions. In all the previous works[2,3] on the decays of the K meson (and pion) there are presented only theoretical conjectures and/or assumptions concerning the unknowns in question. In fact, as we shall see later, it is not possible to separate the effect of the B meson from the unknowns due to strong interactions. We found, however, that there is one way which almost unambiguously allows us to extract the information on the speculated vector meson. The purpose of the present paper is to report the details of the proposed analysis (Sec. 3).

The basic assumption which we make is to introduce a single charged vector meson which interacts with the weak current of the usual type, and to assume that this is solely responsible for weak interactions. We then evaluate the decay matrix element in the lowest order perturbation with respect to both weak and electromagnetic interactions, without making any approximation regarding strong interactions (Sec. 2). We explain also why decays other than $K\rightarrow e+\nu+\gamma$ cannot be used for the same purpose (Sec. 4).

2. GENERAL EXPRESSION FOR DECAY MATRIX ELEMENT

We assume the following interaction Lagrangian:

$$L'=\{-ig(j_\mu{}^{(l)}+j_\mu{}^{(s)})\phi_\mu{}^*+\text{H.c.}\}+e(J_\mu{}^{(l)}+J_\mu{}^{(B)}+J_\mu{}^{(s)})A_\mu+\cdots, \quad (1)$$

where only the weak (first term) and the electromagnetic (second term) interactions are given; ϕ_μ represents the charged B meson with mass M, and the j_μ's and J_μ's are the weak and electromagnetic currents, respectively. The superscripts l, s, and B stand for leptons, strongly interacting particles, and B meson, respectively. We further assume that $j_\mu{}^{(l)}$ includes a lepton current $\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu$, where ψ_e and ψ_ν are electron and neutrino fields, respectively. We, however, do not have to assume any form for $j_\mu{}^{(s)}$. All we require is that the interaction Lagrangian (1) be invariant under time inversion.

Let us consider a decay $K^+\rightarrow e^++\nu+\gamma$, the four-momenta being q, p, p', and k, respectively, and ϵ being the polarization vector of the photon. The S-matrix

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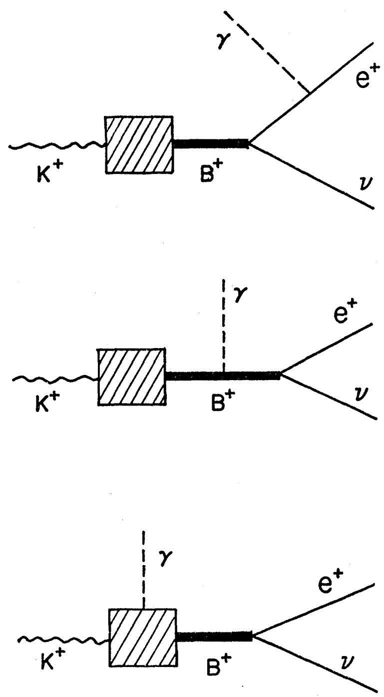

FIG. 1. Three diagrams for $K^+ \to e^+ + \nu + \gamma$ are shown which give rise to three terms of (2), respectively. The unknown constant f is due to the shaded box in the first two diagrams, while the unknown functions h_1 and h_2 come from the shaded area in the last diagram.

element $\langle e^+,\nu,\gamma | K^+\rangle$ is given, to the lowest order with respect to e and g in the frame of reference where the K meson is initially at rest and assuming Coulomb gauge, by

$$\langle e^+,\nu,\gamma | K^+\rangle = i(2\pi)^4\delta(q-p-p'-k)\frac{eg^2m}{2M^2(k_0q_0)^{\frac{1}{2}}} \times \bar{u}(p')(1-\gamma_5)\Bigg\{-(\gamma\cdot\epsilon)f + \frac{(\gamma\cdot\epsilon)(q^2-q\cdot k)}{(q-k)^2+M^2}f - \frac{M^2}{(q-k)^2+M^2}\bigg[(\gamma\cdot\epsilon)h_1 + i(\gamma\cdot\epsilon')\bigg(\frac{q\cdot k}{m^2}\bigg)h_2\bigg]\Bigg\}v(p), \quad (2)$$

where $u(p')$ and $v(p)$ are Dirac spinors of ν and e^+, k_0 and q_0 are the energies of k and q, and ϵ' is another polarization vector parallel to the vector product of k and ϵ. We have put the electron mass equal to zero in (2). The unknowns due to strong interactions are f, a constant, and h_1 and h_2, both of which are functions of k_0 only. These are defined by

$$\langle 0 | j_\mu^{(s)}(0) | K^+\rangle = \frac{m}{(2q_0)^{\frac{1}{2}}} q_\mu f, \quad (3)$$

and

$$i\int dz\, e^{-ikz}\langle 0 | P\{j_\mu^{(s)}(0), \epsilon_\lambda J_\lambda^{(s)}(z)\} | K^+\rangle = \frac{m}{(2q_0)^{\frac{1}{2}}}\bigg[\epsilon_\mu h_1 + i\epsilon_\mu'\bigg(\frac{q\cdot k}{m^2}\bigg)h_2\bigg], \quad (4)$$

where m, the mass of the K meson, is introduced to make f, h_1, and h_2 dimensionless, and all operators and states on the left-hand sides refer to the Heisenberg representation. P in (4) is the chronologically ordered product. The three terms in (2) come from diagrams in which the photon is emitted, respectively, by electron, B meson, and something else, as shown in Fig. 1.

The approach yielding (2), (3), and (4) is virtually the same as in work by previous authors.[2] The essential differences are as follows: First, we have introduced the B meson, while they assumed the direct four-Fermion scheme. Consequently, we have a new term, the second term in (2) or the middle diagram of Fig. 1. This, however, does not add any new unknown in the lowest order of weak and electromagnetic interactions.

Secondly, we have used Low's technique to obtain (2), (3), and (4), while their arguments were based upon the Dyson-Feynman technique. Therefore, our last diagram of Fig. 1 includes both (b) and (c) of Fig. 2 of Treiman and Wyld's paper.[2] The advantage of our procedure is that it leads to expressions like (3) and (4). These expressions can immediately be used to prove the reality of the unknowns when the weak interaction is invariant under time inversion. These could also be used to carry out the dispersion-theoretical estimation of these unknowns.

Finally, it is the pion decay that was investigated in the previous works.[2] However, the same approach can be used in case of K meson decay, as long as we assume the weak interaction scheme (1).

It is important to notice that all these unknowns do not depend upon the B meson. They could, however, depend upon whether the decaying particle is a pion or a K meson. Previous authors[2] have argued in the pion case that h_2 and h_1', defined by

$$f + h_1 = -m^{-2}(q\cdot k)h_1', \quad (5)$$

could very well be constant with respect to the photon energy k_0. We presuppose a similar behavior also in the case of the K meson, and seek a method to check this conjecture empirically without elaborating further theoretically on this matter.

3. PROPOSED EXPERIMENTAL ANALYSIS

The differential decay probability $W(k,\theta)$ as a function of photon energy k (denoted as k_0 previously) and the angle θ between photon and electron is given by

$$W(k,\theta)=\frac{e^2}{4\pi^3}\frac{g^4}{M^4}\frac{B(k)p^2k^3}{m[m-k(1-\cos\theta)]}$$
$$\times\{(h^2+h_2^2)[m-k(1-\cos\theta)-p\sin^2\theta]$$
$$-2hh_2[m\cos\theta+k(1-\cos\theta)]\}, \quad (6)$$

where

$$B(k)=M^4/(M^2-m^2+2mk)^2,$$
$$h=h_1'+(m/M)^2f, \quad (7)$$

and

$$p=\frac{m(m-2k)}{2[m-k(1-\cos\theta)]}$$

is the electron energy when the electron mass is ignored. We have taken all the unknowns to be real in (6) since we can prove that the relative phases of f, h_1, and h_2 are zero if (1) is invariant under time inversion.

It is clear from (6) and (7) that the information on B meson is included only in $B(k)$, and, consequently, it is impossible to see the effect of $B(k)$ only, irrespective of the unknown behaviors of h and h_2. It is, however, possible to eliminate $B(k)$. For example, consider the photon energy spectrum $W(k)$ [the angular integral of $W(k,\theta)$] and $W(k,\theta=\pi)$ [the photon energy spectrum when the electron is emitted backwards with respect to the photon]. These are, from (6),

$$W(k)=\frac{e^2}{12\pi^3}\frac{g^4}{M^4}B(k)k^3(m-2k)(h^2+h_2^2), \quad (8)$$

$$W(k,\theta=\pi)=\frac{e^2}{16\pi^3}\frac{g^4}{M^4}B(k)k^3m(h+h_2)^2. \quad (9)$$

Therefore, the ratio becomes

$$\frac{W(k)}{W(k,\theta=\pi)}=\frac{4}{3}\frac{(h^2+h_2^2)}{(h+h_2)^2}\left(1-\frac{2k}{m}\right), \quad (10)$$

which does not include $B(k)$.

The expression (10) suggests the following empirical check on the possible constant behavior of h and h_2 as functions of k (see the discussions at the end of Sec. 2): Plot $W(k)/W(k,\theta=\pi)$ against k and see if the data fall on a straight line. Suppose they do over a certain energy range. This implies that the ratio of h to h_2 is constant in the same region, unless either h or h_2 vanishes. It is then likely that h and h_2 are constant individually because they are originally due, respectively, to the axial vector and vector parts of the weak current (or vice versa depending upon the K meson parity) and, consequently, would not be simply related. The only conceivable reason for the possible constancy is the extreme high energy involved in the last shaded box of Fig. 1. If so, h and h_2 should be independently constant.

If, therefore, the data concerning the ratio (10) actually fall on a straight line at least over a certain energy range, we then plot, for example, $W(k)/k^3(m-2k)$ against k. Any deviation from the horizontal behavior in the same energy region is now attributable to $B(k)$. In Fig. 2 are shown $B(k)$ as functions of k for some values of M. This analysis could check the intermediate vector meson hypothesis and even determine the mass of the B meson.

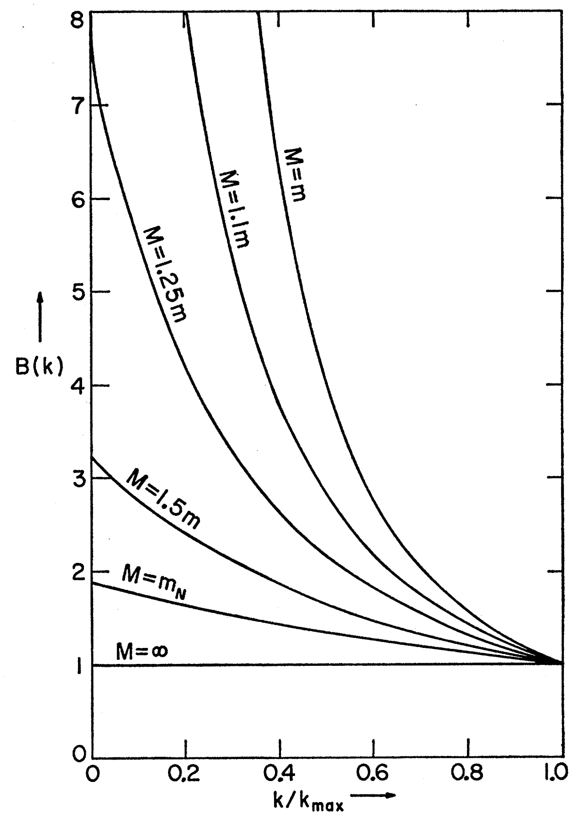

FIG. 2. The function $B(k)$ in (7) is plotted against k, the photon energy, in units of the maximum photon energy $(m/2)$, for some values of M, the mass of the B meson. m is the K meson mass.

If the ratio (10) happens to exhibit linear behavior over the entire range of k and consequently h and h_2 may be regarded as constant for all k, then the peak of $W(k)$ will occur at $k=(3/4)k_{\max}$ ($k_{\max}=m/2$) if $M=\infty$ and will be shifted towards the lower energy side as M approaches m, the likely minimum limit of M. Some forms of $W(k)$ are shown in Fig. 3 for some values of M. If the over-all shape of $W(k)$ is consistent with (8), M may be determined most accurately by observing the ratio R defined by

$$R=\int_0^{k_{\max}/2}W(k)dk\bigg/\int_{k_{\max}/2}^{k_{\max}}W(k)dk, \quad (11)$$

since R is very sensitive to M as is shown in Fig. 4, as long as M is not too large.

If the data indicate too small $W(k,\theta=\pi)$, which

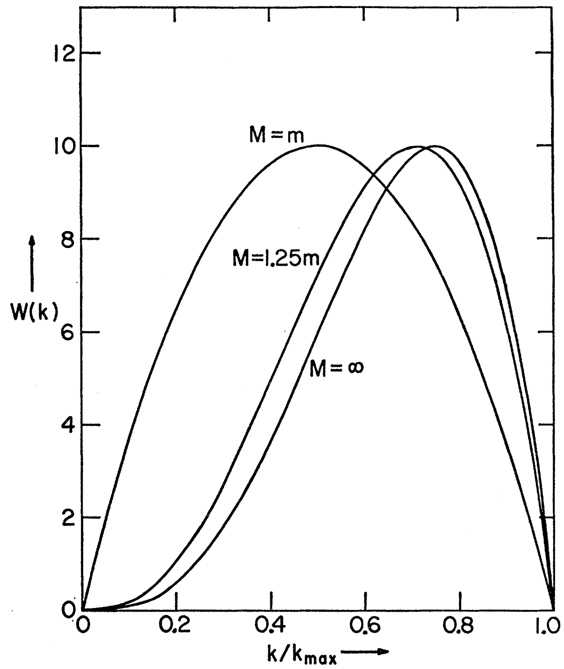

FIG. 3. The energy spectrum $W(k)$, Eq. (8), for a photon with energy k, is plotted against k in units of the maximum photon energy $(m/2)$ for three values of M, the mass of the B meson, assuming that all unknowns (h and h_2) are constant. The ordinate is in arbitrary units. m is the K meson mass.

would be the case if $h+h_2\approx 0$, we suggest using $W(k, \pi>\theta>\pi/2)$, the photon energy spectrum when the electron is emitted into the backward hemisphere, which is given by

$$W(k,\pi>\theta>\pi/2)=\frac{e^2}{24\pi^3}\frac{g^4}{M^4}B(k)mk^3\left(\frac{m-2k}{m-k}\right)\times\left[\left(1-\frac{k(m-2k)}{4(m-k)^2}\right)(h^2+h_2^2)+\frac{3}{2}\left(\frac{m-2k}{m-k}\right)hh_2\right]. \quad (12)$$

If we plot $[(m-k)W(k,\pi>\theta>\pi/2)/W(k)]$ against $(m-2k)/(m-k)$, then we expect, if h and h_2 are really constant, nearly linear behavior [note that the coefficient of $h^2+h_2^2$ in (12) is nearly unity within a few percent error].

It is added that $W(k, \theta=0)$ cannot be used, even though it assumes a very simple form. The reason is that those terms, which are proportional to electron mass in $W(k,\theta)$ and therefore have been dropped in (6), may become non-negligible only near the forward direction.

As is clear by now, the entire analysis collapses if the data for the ratio (10) fail to exhibit linear behavior over an appreciable energy range. Even if the data for the ratio (10) fall on a straight line, the previous analysis breaks down if the data on $W(k,\theta)$ given by (6) indicate that either h or h_2 is negligibly small compared with the other.

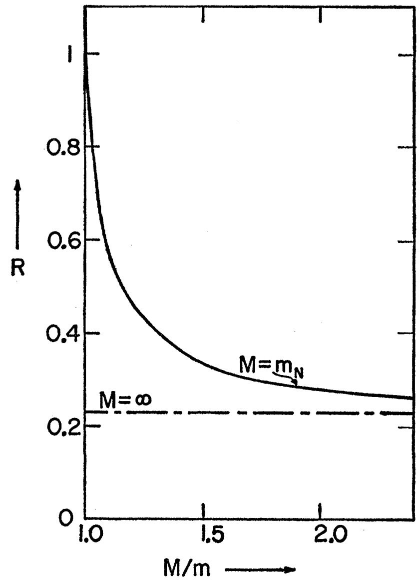

FIG. 4. The ratio R defined by (11) is plotted against M, the mass of the B meson, in units of the K-meson mass, m. The arrow corresponds to the nucleon mass, m_N.

4. OTHER THREE-BODY LEPTON DECAYS OF THE K MESON

So far we have discussed only $K^+\to e^++\nu+\gamma$. We have investigated all the three-body lepton decays of the K meson and found that $K^+\to e^++\nu+\gamma$ is the only one which allows an analysis of the type explained in Sec. 3.

When the μ meson is involved ($K^+\to\mu^++\nu+\gamma$ and $K^+\to\mu^++\nu+\pi^0$), the decay probability becomes too complicated to allow any practicable analysis, simply because the terms proportional to the charged lepton mass are no longer negligibly small. On the other hand, the decay probability for $K^+\to e^++\nu+\pi^0$ becomes too simple to allow any analysis. To illustrate this situation, suppose that the term with hh_2 in $W(k,\theta)$ given by (6) is missing. Then two unknowns $B(k)$ and $(h^2+h_2^2)$ are always combined into a single factor, and we can never see the behaviors of these unknowns separately. A calculation similar to those outlined in Sec. 2 leads to such an expression in the case of $K^+\to\pi^0+e^++\nu$. In the paper by Oneda and Pati,[3] the authors simply assume that the unknowns involved are constant to make definite predictions on the B meson. We, however, don't see much validity in their assertion, since there is no empirical way of checking this conjecture.

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