Internal Symmetries of Strong Baryon-Meson Interactions*

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A simple model considered previously by Pais in which the Σ and Λ hyperons are regarded as a mass-degenerate supermultiplet in the strong pion interaction is reconsidered. It is shown that recognition of the symmetry exhibited by these hyperons in their pionic coupling leads to certain prescriptions which may be used to break the symmetry via the strong K-meson interactions. The symmetry reduction schemes described make possible the construction of strong baryon-meson interaction Hamiltonian which requires no more than four coupling constants (rather than the customary eight) and which in no way imposes severe restrictions on the strong interactions. Finally, production and scattering amplitudes based on the 4-symmetry are discussed.

I. INTRODUCTION

It has been suggested by Gell-Mann[1] that the strong baryon-pion interactions may possibly possess internal symmetries stronger than those implied by conventional charge independence which are reduced by moderately strong baryon-K meson interactions.[2] The latter interactions are assumed to manifest only isotopic spin invariance. It has also been shown by Pais[3] that the assumption of certain symmetries for the baryon-meson interactions is incompatible with experiment, a conclusion which is not in disagreement with Gell-Mann's original proposal. On the other hand, if there is any validity in the assumption of a symmetry stronger than charge independence for the pion interactions, it would seem that one could reasonably expect to find vestiges of this strong symmetry also in the moderately strong K-meson interactions; otherwise the introduction of additional symmetry is hardly of any value. Stated in another way, if the pion interactions indeed manifest stronger symmetries, one would expect these symmetries to be reduced in a somewhat definite manner, or alternatively, one may regard the K-meson interactions as being derivable from certain symmetry reduction procedures implied by the original symmetries. Transformation groups[4] which are amenable to successive symmetry reductions and which also give rise to the observed baryon mass spectrum in a natural way, however, have not been found thus far to properly describe, e.g., the global symmetry theory of Gell-Mann[1] and Schwinger.[5]

With the intent of gaining some insight into the general question of symmetry reduction, a simple model considered previously by Pais,[3] in which only the Σ and Λ hyperons are regarded as a mass-degenerate multiplet in the strong pion interaction, is reconsidered in this paper. We shall be concerned primarily with the question of reducing the symmetry as well as that of lifting the mass degeneracy of the bare (Σ—Λ) hyperons—which will be referred to collectively as the M baryons. In particular, it will be shown that recognition of the symmetry exhibited by the M baryons in their pionic coupling leads to certain prescriptions which may be used to reduce that symmetry via the K-meson interactions. Since adequate means of handling strong interactions are still wanting, our discussion of necessity will be based purely on symmetry considerations and perturbation theoretic arguments. Furthermore, by virtue of the Pais theorem,[6] we do not expect selection rules stronger than the well-known ones to emerge from the reduced symmetry. However, it will be shown that some arbitrariness exists in the reduction of the symmetry exhibited by the strong baryon-pion interactions. Although one must ultimately resort to phenomenology for the selection of the physically acceptable avenue of reduction, considerable economy in coupling constants may be achieved as a consequence of the assumed symmetry.

In Sec. II the symmetry appropriate to the present model as given, e.g., by Pais[7] is reviewed, and various coupling schemes which may be used to reduce the symmetry exhibited by the baryon-pion interaction are described. In Sec. III we make use of empirical evidence for the selection of currently acceptable baryon-K meson interactions. The mass splitting which results from two simplifying assumptions will also be discussed in the lowest order of perturbation theory. The assumed mass and coupling constant relations, to be sure, conceivably could originate from symmetries higher than those adopted for this paper. However, we take the point of view that if one expects the removal of the baryon mass degeneracy to occur via the strong baryon-meson interactions, any higher symmetry introduced must be broken in a manner analogous to the scheme described in Sec. II.

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[1] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).
[2] We, of course, assume strangeness conservation to hold for all strong baryon-meson interactions. In certain cases one may alternatively say that the symmetry of the stronger interaction is reduced by the weaker interaction. This, as will be shown, is strictly a matter of definition of the initial symmetry used.
[3] A. Pais, Phys. Rev. **110**, 574 (1958).
[4] For symmetry considerations which go beyond four dimensions, see, e.g., J. Tiomno, Nuovo cimento **6**, 69 (1957); R. E. Behrends, Nuovo cimento **11**, 424 (1959); D. C. Peaslee, Phys. Rev. **117**, 873 (1960).
[5] J. Schwinger, Phys. Rev. **104**, 1164 (1957); Ann. Phys. **2**, 407 (1957).
[6] A. Pais, Phys. Rev. **110**, 1480 (1958).
[7] A. Pais, Phys. Rev. **112**, 624 (1958). See also Y. Shimamoto, Phys. Rev. Letters **1**, 463 (1958).

In Sec. IV, consequences of the assumed symmetry in production and scattering processes will be discussed. It will be shown in particular that, if special circumstances prevail, one may obtain expressions relating amplitudes of processes involving Σ^0, Σ^-, and Λ^0 hyperons without resorting to the full 4-symmetry for the baryon-meson interactions.

II. REDUCTION OF THE INTERNAL SYMMETRY OF THE BARYON-PION INTERACTION

As usual we assume (a) the validity of charge independence in the conventional sense and the strangeness rule, (b) spin $\frac{1}{2}$ for all baryons and spin 0 for all mesons, (c) even relative parity for $(\Sigma—\Lambda)$, and (d) completeness of the known baryon and meson mass spectrum. If one assumes further, (e) the equality of the bare Σ and Λ masses, it has been shown[1,3] that one may write the strong baryon-pion interaction as follows[8]:

$$[\pi]=i\{G_1\bar{N}_1\gamma_5\boldsymbol{\tau}N_1+G[\bar{N}_2\gamma_5\boldsymbol{\tau}N_2+\bar{N}_3\gamma_5\boldsymbol{\tau}N_3]+G_4\bar{N}_4\gamma_5\boldsymbol{\tau}N_4\}\cdot\boldsymbol{\pi}, \quad (1)$$

where

$$N_1=\begin{pmatrix}p\\n\end{pmatrix},\ N_2=-\begin{pmatrix}\Sigma^+\\Y^0\end{pmatrix},\ N_3=\begin{pmatrix}Z^0\\\Sigma^-\end{pmatrix},\ N_4=\begin{pmatrix}\Xi^0\\\Xi^-\end{pmatrix}, \quad (2)$$

$$Y^0=2^{-\frac{1}{2}}(\Lambda^0-\Sigma^0),\quad Z^0=2^{-\frac{1}{2}}(\Lambda^0+\Sigma^0).$$

In Eq. (1), the M-baryon term corresponds to taking the coupling constant relation,

$$G_+:\quad G=G_{\Sigma\Sigma\pi}=G_{\Lambda\Sigma\pi}, \quad (3)$$

which seems to be in agreement with experiment.[9]

The internal symmetry exhibited by the baryons as well as mesons in Eq. (1) may be regarded as direct product representations (i,k) of the four-dimensional real orthogonal group characterized by the quantum numbers i and k of two three-dimensional-like operators $\mathcal{I}^2$ and $\mathcal{K}^2$, respectively.[7] We use the notation i_1 and i_2 for $\mathcal{I}$ spin up and $\mathcal{I}$ spin down, respectively, and similarly k_1, k_2 for $\mathcal{K}$ doublets. Making use of the fact that if the doublet (k_1,k_2) undergoes a unitary unimodular transformation U_k in the $\mathcal{K}$ space, then so does $(-\bar{k}_2,\bar{k}_1)$, where the bar denotes complex conjugates, while $(\bar{k}_1,\bar{k}_2)$ undergoes the complex conjugate transformation $\bar{U}_k$, we make the following correspondence.

$$\begin{array}{llll} & (i,k) & & \\ N_1: & (\frac{1}{2},0), & p\sim i_1, & n\sim i_2; \\ N_4: & (\frac{1}{2},0), & \Xi^0\sim i_1, & \Xi^-\sim i_2; \end{array} \quad (4)$$

$$\begin{array}{lll} N_2, N_3: & (\frac{1}{2},\frac{1}{2}), & \Sigma^+\sim i_1\bar{k}_2\sim -i_1k_1, \\ & & Y^0\sim i_2\bar{k}_2\sim -i_2k_1, \\ & & Z^0\sim i_1\bar{k}_1\sim \ \ i_1k_2, \\ & & \Sigma^-\sim i_2\bar{k}_1\sim \ \ i_2k_2; \end{array} \quad (5)$$

$$\begin{array}{lll} \pi: & (1,0), & \pi^+\sim\sqrt{2}\bar{\imath}_2i_1, \\ & & \pi^0\sim(\bar{\imath}_1i_1-\bar{\imath}_2i_2), \\ & & \pi^-\sim\sqrt{2}(\bar{\imath}_1i_2), \end{array} \quad (6)$$

where $\sim$ denotes "transforms like." We also need

$$K=\begin{pmatrix}K^+\\K^0\end{pmatrix},\quad K^G=\begin{pmatrix}-\bar{K}^0\\\bar{K}^+\end{pmatrix}, \quad (7)$$

with

$$\begin{array}{llll} K: & (\frac{1}{2},0), & K^+\sim k_1, & K^0\sim k_2, \\ K^G: & (\frac{1}{2},0), & -\bar{K}^0\sim k_1, & \bar{K}^+\sim k_2. \end{array} \quad (8)$$

The eigenstates of the rotation operator $\mathbf{T}^2$, where $\mathbf{T}=\mathcal{I}+\mathcal{K}$, in the three-dimensional subspace of the 4-space, are then identified as the conventional isotopic spin states.

Note now that according to (5) one may introduce, instead of the doublet N_2 and N_3, two other doublets, viz.,

$$M_1=\begin{pmatrix}-\Sigma^+\\Z^0\end{pmatrix}\sim k \text{ doublet},\quad i_1;$$
$$M_2=\begin{pmatrix}-Y^0\\\Sigma^-\end{pmatrix}\sim k \text{ doublet},\quad i_2. \quad (9)$$

With the use of these doublets, one may write the M-baryon terms of Eq. (1) also as

$$[\pi]_M=iG\{\sqrt{2}\bar{M}_1\gamma_5M_2\pi^++\sqrt{2}\bar{M}_2\gamma_5M_1\pi^-+(\bar{M}_1\gamma_5M_1-\bar{M}_2\gamma_5M_2)\pi^0\}, \quad (10)$$

or more concisely as

$$[\pi]_M=iG\bar{M}\gamma_5\boldsymbol{\tau}M\cdot\boldsymbol{\pi}, \quad (11)$$

where the $\mathcal{I}$ spin operators $\boldsymbol{\tau}$ act on the "$\mathcal{I}$ doublet"

$$M=\begin{pmatrix}M_1\\M_2\end{pmatrix}. \quad (12)$$

By virtue of the separate conservation of $\mathcal{I}^2$, $\mathcal{I}_3$, $\mathcal{K}^2$, and $\mathcal{K}_3$, the M baryons remain degenerate in the strong pion interaction.

On the other hand, the M-baryon-pion interaction may be made to exhibit only $\mathbf{T}^2$, T_3 invariance by replacing M_1 and M_2 in Eq. (10) by N_2, N_3, respectively, i.e.,

$$[\pi]_{M'}=iG'\{\sqrt{2}\bar{N}_2\gamma_5N_3\pi^++\sqrt{2}\bar{N}_3\gamma_5N_2\pi^-+(\bar{N}_2N_2-\bar{N}_3N_3)\pi^0\}=iG'\bar{M}'\gamma_5\boldsymbol{\kappa}M'\cdot\boldsymbol{\pi}, \quad (13)$$

[8] Particle symbols will be used to denote the corresponding annihilation operators. Note also that N_2 in this paper is taken to be $-N_2$ of Pais, reference 3.

[9] See, e.g., R. H. Dalitz, in *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

where the $\mathcal{K}$ spin operator acts on the "$\mathcal{K}$ doublet"

$$M'=\begin{pmatrix} N_2 \\ N_3 \end{pmatrix}. \tag{14}$$

One also has

$$[\pi]_{M'}=iG'[\bar{M}_1\gamma_5\boldsymbol{\kappa}M_1+\bar{M}_2\gamma_5\boldsymbol{\kappa}M_2]\cdot\boldsymbol{\pi}. \tag{15}$$

It is evident that Eq. (13) corresponds to a scalar coupling of a vector **I** and a vector **K**. It may be verified by expanding either Eq. (13) or (15) that [see Eq. (3)]

$$G_-:\quad G'=G_{\Sigma\Sigma\pi}=-G_{\Lambda\Sigma\pi}. \tag{16}$$

It is to be noted that 4-space invariance may be restored in Eqs. (13) and (15) if one interchanges the role of i and k in (5) and (9). The $\mathcal{I}$ and $\mathcal{K}$ spin assignment of the other particles, of course, remains unchanged. Thus if one adopts the correspondence

$$\begin{aligned} \Sigma^+ &\sim k_1\bar{i}_2\sim -k_1i_1,\\ Y^0 &\sim k_2\bar{i}_2\sim -k_2i_1,\\ Z^0 &\sim k_1\bar{i}_1\sim \quad k_1i_2,\\ \Sigma^- &\sim k_2\bar{i}_1\sim \quad k_2i_2, \end{aligned} \tag{17}$$

instead of the one given by (5), and interchanges the spin operators $\boldsymbol{\tau}\leftrightarrow\boldsymbol{\kappa}$ in (11) and (13), one may say that the coupling corresponding to G_+ transforms like $\mathbf{I}\cdot\mathbf{K}$, while the coupling corresponding to G_- behaves like a scalar in both the $\mathcal{I}$ and $\mathcal{K}$ spaces.

Hereafter we shall speak of the "first representation" when referring to the correspondence given by (5) and of the "second representation" when making reference to the correspondence given by (17).

In the remaining portion of this section, the validity of the coupling constant relation G_+ will be assumed for the sake of definiteness and the M baryons will be treated in the first representation.

Turning now to the moderately strong K-meson interaction, we note that either N_1 or N_4 may be coupled to the M baryons and K mesons via one of four coupling schemes.

(1) Scalar coupling of an $\mathcal{I}$ doublet I (N_1 or N_4) with the i components of the M baryons, and of a $\mathcal{K}$ doublet K (K or K^G) with the k components of the M baryons. Introducing a 2×2 matrix

$$\mathfrak{M}=\begin{pmatrix} \mathfrak{M}_1{}^1 & \mathfrak{M}_1{}^2 \\ \mathfrak{M}_2{}^1 & \mathfrak{M}_2{}^2 \end{pmatrix}=\begin{pmatrix} Z^0 & \Sigma^+ \\ \Sigma^- & Y^0 \end{pmatrix}, \tag{18}$$

with $\mathfrak{M}_\alpha{}^\beta\sim i_\alpha\bar{k}_\beta$, where the superscript denotes complex conjugate doublet components, one has[10]

$$\bar{I}\mathfrak{M}K=\bar{I}N_3k_1-\bar{I}N_2k_2. \tag{19}$$

Equation (19) gives rise to the coupling constant relation,

$$F_+(I):\quad F_{I\Sigma K}=F_{I\Lambda K}, \tag{20}$$

[10] Space-time factor, 1 or $i\gamma_5$, will hereafter be suppressed since they are not relevant to our discussion.

where the I refers to either N_1 or N_4. If this coupling scheme is used for both the nucleons and the cascade particles, then $\mathcal{I}^2$, $\mathcal{K}^2$, $\mathcal{I}_3$, and $\mathcal{K}_3$ are separately conserved also in the K-meson interaction and the reduction of the M baryons into the observed isotopic multiplets becomes strictly forbidden.[11] One then obtains selection rules stronger than those implied by conventional charge independence and strangeness conservation.

If the remaining three coupling schemes are used, separate conservation of $\mathcal{I}^2$, $\mathcal{K}^2$, $\mathcal{I}_3$, and $\mathcal{K}_3$ is destroyed and only $\mathbf{T}^2$ and T_3 become the conserved quantities.

(2) Scalar coupling of the $\mathcal{I}$ doublet with the k components of $\mathfrak{M}$, and of the $\mathcal{K}$ doublet with the i components of $\mathfrak{M}$, i.e.,

$$\bar{I}\omega\mathfrak{M}^T\omega K=\bar{I}M_2k_1-\bar{I}M_1k_2, \tag{21}$$

where the superscript T denotes the transpose and

$$\omega=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{22}$$

The use of this coupling scheme for both N_1 and N_4 corresponds to taking the coupling constant relation

$$F_-(I):\quad F_{I\Lambda K}=-F_{I\Sigma K}. \tag{23}$$

It is to be noted that if one adopts the coupling scheme G_- for the strong M-baryon–pion interaction together with the coupling scheme given here for the K-meson interaction, four-dimensional symmetry may be restored in the baryon-meson interactions by the use of the second representation. Then separate conservation of $\mathcal{I}^2$, $\mathcal{K}^2$, $\mathcal{I}_3$, and $\mathcal{K}_3$ again forbids the reduction of the M baryons into their isotopic components and stronger selection rules than charge independence come into play.

(3) A vector coupling of the $\mathcal{I}$ doublet with the i components of $\mathfrak{M}$ combined in a charge-conserving manner with a vector coupling of the $\mathcal{K}$ doublet with the k components of $\mathfrak{M}$, i.e., taking a scalar product of a vector **I** with a vector **K**. Explicitly one has

$$\bar{I}\tau_+\mathfrak{M}\kappa_-K+\bar{I}\tau_-\mathfrak{M}\kappa_+K+\bar{I}\tau_3\mathfrak{M}\kappa_3K, \tag{24}$$

where $\boldsymbol{\tau}$, and $\boldsymbol{\kappa}$ are i and k spin operators, respectively. In this case we have the coupling constant relation,

$$F_-{}^3(I):\quad F_{I\Lambda K}=-3F_{I\Sigma K}. \tag{25}$$

(4) A scalar coupling of two vectors as in scheme (3), but now we couple the $\mathcal{I}$ doublet with the k components of $\mathfrak{M}$ and the $\mathcal{K}$ doublet with the i components of $\mathfrak{M}$, i.e.,

$$\bar{I}\rho_+\omega\mathfrak{M}^T\omega\rho_-K+\bar{I}\rho_-\omega\mathfrak{M}^T\omega\rho_+K+\bar{I}\rho_3\omega\mathfrak{M}^T\omega\rho_3K, \tag{26}$$

where $\boldsymbol{\rho}$ are spin operators. This coupling scheme gives rise to the coupling constant relation,

$$F_+{}^3(I):\quad F_{N\Lambda K}=3F_{N\Sigma K}. \tag{27}$$

[11] Clearly 4-space invariance also implies 3-space ($\mathbf{T}^2$) invariance. It is, however, the former which dominates over the latter invariance.

TABLE I. Summary of transformation properties of the coupling schemes.

Coupling scheme	Transformation property: First representation	Second representation	Doublets used in coupling
G_+: $G_{\Sigma\Sigma\pi}=G_{\Lambda\Sigma\pi}$	I_0K_0	$\mathbf{I}\cdot\mathbf{K}$	N_2, N_3 or M_1, M_2
G_-: $G_{\Sigma\Sigma\pi}=-G_{\Lambda\Sigma\pi}$	$\mathbf{I}\cdot\mathbf{K}$	I_0K_0	N_2, N_3 or M_1, M_2
$F_+(I)$: $F_{I\Lambda K}=F_{I\Sigma K}$	I_0K_0	$\frac{1}{2}[I_0K_0+\mathbf{I}\cdot\mathbf{K}]$	N_2, N_3
$F_-(I)$: $F_{\Sigma\Lambda K}=-F_{I\Sigma K}$	$\frac{1}{2}[I_0I_0+\mathbf{I}\cdot\mathbf{K}]$	I_0K_0	M_1, M_2
$F_+{}^3(I)$: $F_{I\Lambda K}=3F_{I\Sigma K}$	$\frac{1}{2}[3I_0K_0-\mathbf{I}\cdot\mathbf{K}]$	$\mathbf{I}\cdot\mathbf{K}$	M_1, M_2
$F_-{}^3(I)$: $F_{I\Lambda K}=-3F_{I\Sigma K}$	$\mathbf{I}\cdot\mathbf{K}$	$\frac{1}{2}[3I_0K_0-\mathbf{I}\cdot\mathbf{K}]$	N_2, N_3

We also note that in the second representation, this coupling scheme transforms like $\mathbf{I}\cdot\mathbf{K}$.

The symmetry of the M baryon-meson couplings in the 4-space may, therefore, be summarized as follows (see Table I):

(i) The structure of the coupling schemes characterized by G_+ and G_- is simple, or irreducible, in the sense that the coupling involved is of the form ($\mathcal{G}$ scalar) ($\mathcal{K}$ scalar) or $\mathbf{I}\cdot\mathbf{K}$ in the two representations. The two doublet sets (N_2,N_3) and (M_1,M_2) may be used to describe the pionic coupling.

(ii) The coupling schemes F_+ and $F_-{}^3$ are irreducible (reducible) in the first (second) representation. These couplings may be expressed only via the use of the doublet set (N_2,N_3).

(iii) The coupling schemes F_- and $F_+{}^3$ are irreducible (reducible) in the second (first) representation. These couplings may be expressed only via the use of the doublet set (M_1,M_2).

III. BARYON-K MESON INTERACTIONS

In the previous section it was shown that the symmetry exhibited by the M baryons in their pionic coupling may be reduced in a definite manner if one assumes any one of three coupling schemes based on the original 4-symmetry to hold true for the moderately strong K-meson interactions. The fact that the assumed symmetry provides a rather flexibile means of obtaining strong baryon-meson interactions requiring no more than the five coupling constants (instead of the customary eight) is not entirely unwelcome in view of the present state of our knowledge of the strongly interacting particles. In particular, if one assumes the strong baryon-meson couplings to be responsible not only for all strangeness conserving processes, but also for the gross structure of the observed baryon mass spectrum, the absence of compelling reasons which dictate the use of any one of the reduction schemes clearly can be used to advantage. It is now of interest to see whether or not any of the K-meson coupling schemes described above are favored by experiment, assuming as in Sec. II the validity of G_+.

The available experimental information on the photoproduction from protons of K^+ mesons provides a suitable basis for the selections of the nucleon–K-meson interaction. Here the reaction is not as complicated as the other known processes involving K mesons, and one may rely on perturbation calculations for low energies.

It is reported by the Cornell group[12] that the cross sections for the reactions $\gamma+p\rightarrow\Lambda^0+K^+$ and $\gamma+p\rightarrow\Sigma^0+K^+$ at comparable energies above the corresponding thresholds are of comparable magnitudes. It is also reported that[13] in the analyses of the reaction $\gamma+p\rightarrow\Lambda^0+K^+$ based on the perturbation calculation of Kawaguchi and Moravcsik[14] with the inclusion of the transition moment $\langle\Lambda^0|\mu|\Sigma^0\rangle$ introduced by Capps,[15] only the case with the assumption $F_{N\Sigma K}/F_{N\Lambda K}=-1$ may be considered to fit the data at all. It seems therefore that we are tentatively justified in assuming the validity of coupling scheme (2) for the nucleon–K-meson interaction and obtain

$$[NK]=i\sqrt{2}f\{\bar{N}_1\gamma_5M_2K^+-\bar{N}_1\gamma_5M_1K^0\}+\text{Herm. conj.},\quad(28)$$

where $f=F_{N\Sigma K}=-F_{N\Lambda K}$.

As has already been mentioned, if the second representation is adopted for the M baryon, 4-space symmetry is restored in (28) and the pions become the agent which destroy this symmetry via Eq. (10). Then, while the resulting selection rules $\Delta\mathcal{G}=\Delta\mathcal{K}=\Delta\mathcal{G}_3=\Delta\mathcal{K}_3=0$ for processes not involving external pions suggest, e.g., that the charge exchange scattering $K^0+p\rightarrow K^++n$ is forbidden, and this indeed is the case in lowest order as may be easily verified, the process becomes allowed with the direct intervention of the pions in intermediate states. Furthermore, reactions such as $\pi^++p\rightarrow\Sigma^++K^+$ and $K^-+p\rightarrow\Sigma^++\pi^-$, which would otherwise be forbidden if the full 4-space dominate both π and K couplings, are now allowed.

With the choice of (28) for the nucleon-K meson interaction, the M baryons are reduced into their isotopic components, although the mass degeneracy remains in the lowest order approximation in perturbation theory. To make possible the removal of this mass degeneracy in lowest order, we may, e.g., couple the cascade particles to the K mesons via either coupling

[12] B. D. McDaniel, A. Silverman, R. R. Wilson, and G. Cortellessa, Phys. Rev. Letters **1**, 109 (1958).
[13] B. D. McDaniel, A. Silverman, R. R. Wilson, and G. Cortellessa, Phys. Rev. **115**, 1039 (1959).
[14] M. Kawaguchi and M. Moravcsik, Phys. Rev. **107**, 563 (1957).
[15] R. Capps, Phys. Rev. **114**, 920 (1959).

scheme (3) or (4). Coupling scheme (3) gives

$$[\Xi K]=-\sqrt{2}g\{\sqrt{2}\bar{N}_4\gamma_5\rho_+N_2\bar{K}^0+\sqrt{2}\bar{N}_4\gamma_5\rho_-N_3\bar{K}^+ +\bar{N}_4\gamma_5\rho_3N_2K^+-\bar{N}_4\gamma_5\rho_3N_3\bar{K}^0\} +\text{Herm. conj.}, \quad (29)$$

with $3g=3F_{\Xi\Sigma K}=-F_{\Xi\Lambda K}$, while coupling scheme (4) gives

$$[\Xi K]=-\sqrt{2}g\{\sqrt{2}\bar{N}_4\gamma_5\rho_+M_1\bar{K}^0+\sqrt{2}\bar{N}_4\gamma_5\rho_-M_2\bar{K}^+ +\bar{N}_4\gamma_5\rho_3M_1\bar{K}^+-\bar{N}_4\gamma_5\rho_3M_2\bar{K}^0\} +\text{Herm. conj.}, \quad (30)$$

with $3g=3F_{\Xi\Sigma K}=F_{\Xi\Lambda K}$.

For an empirical determination of the $\Xi-K$ coupling, the production rates of cascade particles in hydrogen bubble chamber by K^- mesons, $K^-+p\rightarrow\Xi^0+K^0$ and $K^-+p\rightarrow\Xi^-+K^+$, seem to be relevant. Here the particles in both the initial and final states may be characterized by $\mathcal{I}^2$, $\mathcal{K}^2$, $\mathcal{I}_3$, and $\mathcal{K}_3$, and if one neglects pionic contributions from G_+ and treats the $\Xi-K$ meson interaction in perturbation theory, one obtains[16]

(a) $F_-(\Xi)$: $A_{\Xi^-K^+}=0$,

(b) $F_+{}^3(\Xi)$: $A_{\Xi^-K^+}\approx-2A_{\Xi^0K^0}$.

One also infers from the lowest order Feynman diagrams that

(c) $F_+(\Xi)$: $A_{\Xi^0K^0}=0$,

(d) $F_-{}^3(\Xi)$: $A_{\Xi^0K^0}\approx-2A_{\Xi^-K^+}$.

It is reported by the Berkeley hydrogen bubble chamber group[17] that the cross sections for the processes in question for 1.15-Bev/c K^- mesons are $\sigma_{\Xi^0K^0}\approx 50\ \mu$b and $\sigma_{\Xi^-K^+}\leq 17\ \mu$b. On the basis of our crude estimate and available information, we are therefore tempted to favor (29) for the $\Xi-K$ interaction at this time.

To see whether or not the qualitative features of the observed baryon mass spectrum can be accounted for by the interactions given by (1), (28), and (29) or (30), we now consider the lowest order contributions to the baryon self energies due to these interactions. For this purpose we make the following simplifying assumptions:

(i) $m_0(N_1)=m_0(N_4)$, $G_1=G_4$;

and

(ii) the relative parity of the nucleons and cascade particles is even.

One then obtains[18]

$$\begin{aligned}\Delta m(\Xi)&=-\{3G_1^2F(II\pi)+12g^2F(IMK)\},\\ \Delta m(\Sigma)&=-\{3G^2F(MM\pi)+2(f^2+g^2)F(IMK)\},\\ \Delta m(\Lambda)&=-\{3G^2F(MM\pi)+2(f^2+9g^2)F(IMK)\},\\ \Delta m(N)&=-\{3G_1^2F(II\pi)+4f^2F(IMK)\},\end{aligned} \quad (31)$$

where

$$F(II\pi)=F(\Xi\Xi\pi)=F(NN\pi),$$
$$F(MM\pi)=F(\Sigma\Sigma\pi)=F(\Sigma\Lambda\pi)',$$
$$F(IMK)=F(N\Sigma K)=F(\Xi\Sigma K),\text{ etc.}$$

From Eqs. (31) it is seen that the gross structure of the observed baryon mass spectrum already manifests itself in the lowest order approximation in perturbation theory provided that $g^2<f^2$. Furthermore to the extent that higher order terms may be neglected, an estimate of the ratio $g^2/f^2\sim 1/23$ may be obtained from the interval ratio

$$m(\Sigma)-m(\Lambda)/m(\Xi)-m(N)\sim\tfrac{1}{5}.$$

The situation clearly becomes more complicated if assumptions (i) and (ii) do not hold.

From the foregoing discussion, we therefore conclude that the possibility of obtaining strong baryon-meson interactions requiring at most four coupling constants G, G_1, f, and g, and only two bare baryon masses $m_0(M)$ and $m_0(N)=m_0(\Xi)$ exists in the simple symmetry model being considered here. The inclusion of the latter mass relations needs no further justification if one postulates that all fermions with identical transformation properties in the 4-space possess the same bare mass. If one further postulates that these fermions are coupled to the pions with the same coupling constant, we may also take $G_1=G_4$. On the other hand, symmetries higher than 4-space symmetry must be introduced if the relation $G_1=G_4=G$ holds.

It is also seen that the quantity of interest is the ratio g^2/f^2 which is of the order of 1/23. One would expect comparison of K-meson-nucleon scattering and cascade particle productions at comparable energies to provide as crude an estimate as the one given above of the ratio in question.

IV. PRODUCTION AND SCATTERING AMPLITUDES

If coupling schemes other than G_+ and F_+, or G_- and F_- hold within the framework of the 4-space symmetry, it is well known that selection rules stronger than those already implied by charge independence and the strangeness rule do not come into play. It may, e.g., be verified that the baryon-meson couplings discussed in the previous sections are invariant under the following transformation:

$$n\leftrightarrow p,\quad \pi^+\leftrightarrow\pi^-,\quad -\pi^0\leftrightarrow\pi^0,\quad \Xi^0\leftrightarrow\Xi^-,\quad -\Sigma^+\leftrightarrow\Sigma^-,$$
$$Z\leftrightarrow -Y,\quad K^+\leftrightarrow -K^0,\quad \bar{K}^0\leftrightarrow\bar{K}^+. \quad (32)$$

However, the transformation defined by (32) is nothing more than charge symmetry which is already implied by

[16] The explicit forms of the amplitudes will be given in Sec. IV.

[17] L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 2, 215 (1959).

[18] For similar consideration based on somewhat different assumptions, see, e.g., G. Takeda, Progr. Theoret. Phys. (Kyoto) 19, 631 (1958); H. Katsumori, Progr. Theoret. Phys. (Kyoto) 19, 342 (1958); and R. P. Feynman (unpublished, 1958). For mass splitting within an isotopic multiplet, see, e.g., B. H. Bransden and R. G. Moorhouse, Phys. Rev. Letters 2, 431 (1959); R. E. Behrends and L. Landovitz, Phys. Rev. 117, 589 (1960).

charge independence. (Note that according to our definition of charge symmetry, one has $\Sigma^0 \rightarrow \Sigma^0$ and $\Lambda^0 \rightarrow -\Lambda^0$.) Clearly the absence of stronger selection rules implies that one must resort to more detailed considerations if one wants to discover verifiable consequences of the 4-symmetry.

In this section consequences of the 4-symmetry as exhibited in production scattering amplitudes will be discussed. The expression we obtain, in general, will not be directly verifiable. However, it will be shown that if certain conditions hold, one may obtain amplitude relations of the form $\alpha A_{\Sigma^-}+\beta A_{\Sigma^0}+\gamma A_{\Lambda^0}=0$ without invoking the full 4-symmetry. The origin of the conditions, however, will depend on a more detailed treatment than on one used here and will not be considered.

We observe now that if the M-baryon–pion interaction indeed manifests 4-symmetry, one may express the amplitudes related to reactions of practical interest in terms of matrix elements of an expression of the form

$$S=\sum I_0K_0+\sum \mathbf{I}\cdot\mathbf{K}, \tag{33}$$

where $\sum I_0K_0$ denotes all contributions which are a scalar in both the $\mathcal{I}$ and $\mathcal{K}$ spaces, while $\sum \mathbf{I}\cdot\mathbf{K}$ denotes all contributions which transform like a scalar product of an $\mathcal{I}$ vector and a $\mathcal{K}$ vector.[19]

For the purpose of justifying the above statement we assume first that the structure of all the fundamental interactions is irreducible, i.e., they are all of the form ($\mathcal{I}$ scalar)($\mathcal{K}$ scalar) or $\mathbf{I}\cdot\mathbf{K}$ in one of the two representations of Sec. II. Then the validity of the above statement is evident for the lowest order processes. More generally, when one considers contributions due to higher order processes, one will have expressions of the form $[(\mathcal{I}\text{ scalar})(\mathcal{K}\text{ scalar})]^n(\mathbf{I}\cdot\mathbf{K})^m$, which, however, may be reduced to scalar products of irreducible tensors of rank L, $\mathbf{I}_L$, and $\mathbf{K}_L$, via the relations

$$(\mathbf{I}_1'\cdot\mathbf{K}_1')(\mathbf{I}_1\cdot\mathbf{K}_1)=\sum_{L=0}^{2}(-)^L\mathbf{I}_L\cdot\mathbf{K}_L, \tag{34}$$

$$(\mathbf{I}_l\cdot\mathbf{K}_l)(\mathbf{I}_{l'}\cdot\mathbf{K}_{l'})=(-)^{l+l'}\sum_{L=|l-l'|}^{l+l'}(-)^L\mathbf{I}_L\cdot\mathbf{K}_L. \tag{35}$$

The selection rule[20] associated with expressions of the form Eqs. (34) and (35) are satisfied in reactions of practical interest only by the $L=0$ and 1 terms of (34) by virtue of the strangeness rule, and one therefore obtains (33). On the other hand, if the selection rule associated with the $L=2$ and higher terms can be satisfied, one must of course include these terms also.

We now note that since the assignment of $\mathcal{I}$ and $\mathcal{K}$ spins is strictly a matter of definition, the result one obtains must be independent of the representation used. Furthermore, the fundamental interactions which are irreducible in a given representation may be expressed in terms of sums of the irreducible interactions of the other representation, as is shown in Table I. Thus, if the fundamental interactions which are not irreducible in a representation are expressed in terms of the irreducible interaction of the representation and the general reduction schemes given by Eqs. (34) and (35) are used, one again obtains (33) which of course, should be equivalent to the expression obtained from consideration in the representation in which the interactions are irreducible.

Finally we note that the above procedure is not restricted to strong interactions which possess an irreducible structure in one of the two representations, but applies equally to all charge-independent and strangeness-conserving M baryon–K meson interactions of the form

$$\begin{aligned}(\Sigma)+n(\Lambda)&\equiv\bar{N}_1\boldsymbol{\tau}K\cdot\boldsymbol{\Sigma}+n\bar{N}_1K\Lambda_0+\text{H.c.},\\ &\text{or } \bar{N}_4\boldsymbol{\tau}K^G\cdot\boldsymbol{\Sigma}+n\bar{N}_4K^G\Lambda_0+\text{H.c.},\end{aligned} \tag{36}$$

with an arbitrary n. This follows from the fact that expressions of the form given by Eq. (36) may be reduced into sums of the irreducible interactions in the two representations as follows:

(I) First representation,

$$\begin{aligned}(\Sigma)+n(\Lambda)&=\alpha[(\Sigma)+(\Lambda)]+\beta[(\Sigma)-3(\Lambda)]\\ &\sim\alpha(I_0K_0)+\beta(\mathbf{I}\cdot\mathbf{K}),\\ \alpha&=(3+n)/4,\quad \beta=(1-n)/4.\end{aligned} \tag{37}$$

(II) Second representation,

$$\begin{aligned}(\Sigma)+n(\Lambda)&=\alpha[(\Sigma)-(\Lambda)]+\beta[(\Sigma)+3(\Lambda)]\\ &\sim\alpha(I_0K_0)+\beta(\mathbf{I}\cdot\mathbf{K}),\\ \alpha&=(3-n)/4,\quad \beta=(1+n)/4.\end{aligned} \tag{38}$$

It is evident then that, to the extent that one may neglect the $\Sigma-\Lambda$ mass difference,[21] reaction and scattering amplitudes obtained from (33) are valid to all orders of perturbation theory and reflect the 4-dimensional origin of the strong interactions. On the other hand, the introduction of the irreducible interactions which serve as convenient bases for expressing charge-independent interactions of the form given by Eq. (36) is meaningful if and only if the 4-space provides the underlying symmetry in the strong interactions. Thus, although one must resort to more dynamical means for the determination of the actual coupling scheme in operation, amplitude relations based on Eq. (33) may be used to test the assumption of 4-symmetry in the strong interactions. By virtue of the selection rule $\Delta i(k)=0,\ \pm1;\ 0\leftrightarrow 0$, associated with the $\mathbf{I}\cdot\mathbf{K}$ term, only the I_0K_0 term of (33) contributes, e.g., to pion-nucleon scattering. What we call $\mathcal{I}$ spin, therefore, corresponds to the classical isotopic spin.

[19] Similar considerations, of course, apply if the M baryon–K meson interactions manifest 4-space symmetry and the M baryon–pion interaction is invariant only in the 3-space as has been suggested by J. J. Sakurai, Phys. Rev. **113**, 1769 (1959).

[20] The selection rule used is that the initial $i(k)$, the final $i'(k')$ and the rank of the irreducible tensor L must satisfy a triangular relation.

[21] A. Pais, reference 7.

We now consider a few examples and, in the spirit of the present paper, base our discussion on the assumption of coupling schemes in which the structure of both $M-\pi$ and nucleon-K coupling is irreducible either in the first or second representation.

(a) K^+-Nucleon Scattering

To demonstrate the importance of the $M-\pi$ interaction and the utility of the 4-symmetry, we consider first scattering amplitudes of K^+-nucleon scattering:

$$\begin{aligned} &K^+ + p \to K^+ + p, \quad (A_p) \\ &K^+ + n \to K^+ + n, \quad (A_n) \\ &K^+ + n \to K^0 + p. \quad (A_{\text{exch}}) \end{aligned} \tag{39}$$

According to isotopic spin invariance, the scattering amplitudes may be expressed in terms of $T=1$ and 0 amplitudes as

$$\begin{aligned} A_p &= M_3, \\ A_n &= \tfrac{1}{2}[M_3 + M_1], \\ A_{\text{exch}} &= \tfrac{1}{2}[M_3 - M_1]. \end{aligned} \tag{40}$$

Now in the absence of the $\mathbf{I}\cdot\mathbf{K}$ term, which is equivalent to neglecting the pion interactions, e.g., in the G_+, $F_-(N_1)$ scheme,[22] one obtains $A_p = -A_n$, $A_{\text{exch}}=0$, as a consequence of conservation of $\mathcal{I}^2$, $\mathcal{K}^2$, $\mathcal{I}_3$, and $\mathcal{K}_3$. With the introduction of the $\mathbf{I}\cdot\mathbf{K}$ term of (33), one obtains

$$\begin{aligned} A_p &= A + \tfrac{1}{4}B, \\ A_n &= A - \tfrac{1}{4}B, \\ A_{\text{exch}} &= \tfrac{1}{2}B, \end{aligned} \tag{41}$$

where A and B denote reduced amplitudes appropriate to the I_0K_0 and $(\mathbf{I}\cdot\mathbf{K})$ term of (33), respectively. As a consequence of 4-symmetry, one also obtains $|A_p|^2 + |A_n|^2 > \frac{1}{2}|A_{\text{exch}}|^2$ or $\sigma_p + \sigma_n > \frac{3}{2}\sigma_{\text{exch}}$, and $|A_{\text{exch}}|^2 + |A_n|^2 > \frac{1}{2}|A_p|^2$ from (40).

(b) $\pi + p \to \Sigma(\Lambda) + K$

Consider the hyperon production in π-proton collisions:

$$\pi^+ + p \to \Sigma^+ + K^+, \quad (A_+) \tag{42a}$$

$$\pi^- + p \to \Sigma^0 + K^0, \quad (A_0) \tag{42b}$$

$$\pi^- + p \to \Sigma^- + K^+, \quad (A_-) \tag{42c}$$

$$\pi^- + p \to \Lambda + K^+. \quad (A_\Lambda) \tag{42d}$$

It is well known that if the full 4-symmetry holds, one obtains $A_+=0$, and $A_\Lambda = \pm A_0$, where the $\pm$ depends on whether or not one uses the first or second representation. With the introduction, e.g., of the symmetry-reducing $F_\pm(N_1)$ and the resulting selection rule $\Delta i=0$, ±1, $\Delta k=0$, ±1, $0 \leftrightarrow 0$ which follows from $\mathbf{I}\cdot\mathbf{K}$, reaction (42a) becomes allowed. By application of standard procedure one obtains, apart from a common numerical factor,

$$A_+ = A, \tag{43a}$$

$$A_0 = \tfrac{1}{3}\sqrt{2}\{A + \tfrac{1}{4}B + \tfrac{1}{2}C\}, \tag{43b}$$

$$A_- = \tfrac{1}{3}\{A - \tfrac{1}{2}B - C\}, \tag{43c}$$

$$A_\Lambda = \tfrac{1}{2}\sqrt{2}\{\tfrac{1}{2}B - \tfrac{1}{3}C\}, \tag{43d}$$

where A and B represent reduced amplitudes appropriate to the transition $i=\frac{3}{2}\to\frac{1}{2}$, $k=0\to1$, and $i=\frac{1}{2}\to\frac{1}{2}$, $k=0\to1$, respectively, while C denotes contributions from the scalar-scale term of (33).

As expected from the work of Pais, one has $A_0 + \sqrt{2}A_- - A_\Lambda = 0$ in the absence of the symmetry-reducing interaction, while in the limit $C\to0$, one obtains

$$A_0 - \sqrt{2}A_- - A_\Lambda = 0, \quad C=0. \tag{44}$$

Thus if it turns out that $C=0$, the desirable feature of the triangle relation which follows from G_+, $F_+(N_1)$ is maintained in (44) and the undesirable relation $A_{\Sigma^-} = -\sqrt{2}A_{\Sigma^0}$ no longer holds.

The example given here therefore suggests that amplitudes arising from both the $\sum I_0K_0$ and $\sum(\mathbf{I}\cdot\mathbf{K})$ terms of the interaction matrix individually, but not collectively, satisfy the triangular relations due no doubt to properties of Racah and related coefficients. Thus should it turn out that a given set of amplitudes involving Σ^0, Σ^-, and Λ hyperons indeed satisfy the triangular inequality, it seems that one may attribute the fact to vanishing of either the $\sum I_0K_0$ or the $\sum \mathbf{I}\cdot\mathbf{K}$ contribution of (33). The triangular relations which one would expect from isotopic spin invariance, however, remain unchanged.

(c) $K^- + p \to \Sigma(\Lambda) + \pi$

The introduction of the full 4-symmetry is again known to be very bad for the reactions

$$K^- + p \to \Sigma^+ + \pi^-, \quad (B_+) \tag{45a}$$

$$K^- + p \to \Sigma^0 + \pi^0, \quad (B_0) \tag{45b}$$

$$K^- + p \to \Sigma^- + \pi, \quad (B_-) \tag{45c}$$

$$K^- + p \to \Lambda^0 + \pi^0, \quad (B_\Lambda) \tag{45d}$$

as have already been reported in the literature. With the use of (33), however, one now obtains

$$B_+ = -(\tfrac{1}{6})^{\frac{1}{2}}[A+B], \tag{46a}$$

$$B_0 = -(\tfrac{1}{6})^{\frac{1}{2}}[\tfrac{3}{4}B - C], \tag{46b}$$

$$B_- = -(\tfrac{1}{6})^{\frac{1}{2}}[A - \tfrac{1}{2}B + 2C], \tag{46c}$$

$$B_\Lambda = (\tfrac{1}{6})^{\frac{1}{2}}[2A - \tfrac{1}{4}B - C], \tag{46d}$$

where C denotes the I_0K_0 amplitude, and A and B denote reduced amplitudes appropriate to the transi-

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[22] In lowest order one obtains $A_{\text{exch}}\neq0$ if one works in the first representation. The result, however, must be consistent with the result obtained from the second representation. Considerations of the associated Feynman diagram indeed shows that $A_{\text{exch}}=0$ in the lowest order.

tions $i=\frac{1}{2}\rightarrow\frac{3}{2}$, $k=\frac{1}{2}\rightarrow\frac{1}{2}$, and $i=\frac{1}{2}\rightarrow\frac{1}{2}$, $k=\frac{1}{2}\rightarrow\frac{1}{2}$, respectively. In the absence of the symmetry-reducing interaction, one has $B_0=-B_\Lambda$, $B_-=-2B_0$ as has been shown by Pais. On the other hand, when $C=0$, one has $B_0+B_\Lambda=-2B_-$ which is in violent disagreement with experiment.

It is of interest to note, however, that if one has $A=\frac{1}{4}B+C$, one obtains $B_+-B_0+2B_\Lambda=0$, $B_++B_-+4B_\Lambda=0$, and $B_0+B_-+2B_\Lambda=0$. The triangular inequalities which one obtains from these amplitude relations indeed are satisfied empirically. This is another example of how amplitude relations may be obtained from the 4-symmetry, and serves as a counterexample of the consistency relation obtained by Amati and Vitale.[23]

(d) $K^-+p\rightarrow\Xi^0+K^0$

We now consider the amplitudes of the reactions

$$K^-+p\rightarrow\Xi^0+K^0, \tag{47a}$$

$$K^-+p\rightarrow\Xi^-+K^+, \tag{47b}$$

which were discussed briefly in Sec. III. According to the general prescription of this section one has

$$A_{\Xi^0K^0}=A-\tfrac{1}{4}B,\quad A_{\Xi^-K^+}=\tfrac{1}{2}B, \tag{48}$$

where A and B denote reduced amplitudes corresponding to the $\sum I_0K_0$ and $\sum \mathbf{I}\cdot\mathbf{K}$ terms of (33), respectively.

Now since both the initial and final states constitute pure $\mathcal{I}^2$, $\mathcal{K}^2$ states, and hence (47) is a relatively simple system, and furthermore since both the initial and final states may be coupled to the M baryon with different structures, (47) provides an excellent example for illustrating how the amplitudes obtained from the general prescription will differ for different interactions. We therefore consider the lowest order contributions to the reaction amplitudes under the assumption that the coupling scheme $F_-(N_1)$ holds for the nucleon–K-meson interaction.

In the lowest order process, two ($\mathcal{I}$ scalar)($\mathcal{K}$ scalar) terms α_1 and α_2 can contribute to the $\sum I_0K_0$ part of (33) and three terms β_1, β_2, and β_3 contribute to the $\sum \mathbf{I}\cdot\mathbf{K}$ part of (33). By considering the amplitudes in the two representations, however, the relations satisfied by these terms may be established, and the final result may be expressed in terms of only α_1 and β_1 as follows:

$$F_-(N_4):\quad A_{\Xi^0K^0}=\alpha_1,\quad A_{\Xi^-K^+}=0, \tag{49}$$

$$F_+(N_4):\quad B_{\Xi^0K^0}=\tfrac{1}{2}[\alpha_1-\tfrac{1}{4}\beta_1],\quad B_{\Xi^-K^+}=\tfrac{1}{4}\beta_1, \tag{50}$$

$$F_+{}^3(N_4):\quad C_{\Xi^0K^0}=-\tfrac{1}{4}\beta_1,\quad C_{\Xi^-K^+}=\tfrac{1}{2}\beta_1, \tag{51}$$

$$F_-{}^3(N_4):\quad D_{\Xi^0K^0}=\tfrac{1}{2}[3\alpha_1+\tfrac{1}{4}\beta_1],\quad D_{\Xi^-K^+}=-\tfrac{1}{4}\beta_1. \tag{52}$$

Now it is reasonable to expect that for the lowest order process $F_-\leftrightarrow F_+$ implies, $\Xi^0K^0\leftrightarrow\Xi^-K^+$ as may be verified by considering the lowest order Feynman diagram. Thus on taking $\alpha_1=\frac{1}{4}\beta_1$, one obtains the relations given in Sec. III.

(e) $\Sigma(\Lambda)$-Nucleon Scattering

The M-baryon–nucleon scattering amplitudes may be expressed in terms of four reduced amplitudes; m_1 and m_0 which correspond to the $i=1$ and 0 contributions from $\sum I_0K_0$, and a and b which denote the contributions from $\sum \mathbf{I}\cdot\mathbf{K}$ and which correspond to the transitions $i=1\leftrightarrow 1$, and $i=1\leftrightarrow 0$, respectively. By virtue of charge symmetry and time reversal invariance, only six linear combinations of the four reduced amplitudes are relevant. One has for scattering involving a proton in the initial or final state the following amplitudes:

$$\langle\Sigma^+p|S|\Sigma^+p\rangle=m_1+\tfrac{1}{2}a \equiv A, \tag{53}$$

$$\langle\Sigma^0p|S|\Sigma^0p\rangle=\tfrac{1}{4}[3m_1+m_0+a-b] \equiv B, \tag{54}$$

$$\langle\Sigma^0p|S|\Sigma^+n\rangle=\tfrac{1}{4}\sqrt{2}[m_0-m_1-a-b] \equiv C, \tag{55}$$

$$\langle\Sigma^0p|S|\Lambda^0p\rangle=\tfrac{1}{4}[m_1-m_0-a-b] \equiv D, \tag{56}$$

$$\langle\Sigma^-p|S|\Sigma^-p\rangle=\tfrac{1}{2}[m_1+m_0-b] \equiv E, \tag{57}$$

$$\langle\Sigma^-p|S|\Sigma^0n\rangle=\tfrac{1}{4}\sqrt{2}[m_1-m_0+a+b] =-C, \tag{58}$$

$$\langle\Sigma^-p|S|\Lambda^0p\rangle=\tfrac{1}{4}\sqrt{2}[m_1-m_0-a-b] =\sqrt{2}D, \tag{59}$$

$$\langle\Lambda^0p|S|\Lambda^0p\rangle=\tfrac{1}{4}[3m_1+m_0-3a+3b]\equiv F, \tag{60}$$

$$\langle\Lambda^0p|S|\Sigma^+n\rangle=\tfrac{1}{4}\sqrt{2}[m_1-m_0-a-b] =\sqrt{2}D. \tag{61}$$

The scattering amplitudes involving neutrons which are not listed above may be obtained by charge symmetry. Thus, for example, we have

$$\langle\Sigma^+n|S|\Sigma^+n\rangle=\langle\Sigma^-p|S|\Sigma^-p\rangle,$$
$$\langle\Sigma^0n|S|\Lambda^0n\rangle=-\langle\Sigma^0p|S|\Lambda^0p\rangle,\ \text{etc.}$$

Clearly, the six amplitudes A, B, $\cdots F$ are not linearly independent, and we have, e.g., from charge independence,

$$\sqrt{2}A-\sqrt{2}B+C=0,\quad \sqrt{2}B+C-\sqrt{2}E=0,\quad A-E+\sqrt{2}C=0,\quad E+A-2B=0,\ \text{etc.} \tag{62}$$

If the amplitudes D and F can be expressed in terms of A, B, C, and E, the existence of the 4-symmetry may be verified empirically. Unfortunately, such linear relations do not exist. Furthermore, nontrivial inequalities involving the modulus squares of D and F also do not exist due to the relation

$$|A|^2+|E|^2=2|B|^2+|C|^2, \tag{63}$$

[23] D. Amati and B. Vitale, Nuovo cimento **9**, 895 (1958). Amplitudes for reactions Eqs. (45a)–(45d) based on Gell-Mann's assumption in which the k^-+p interaction is treated in perturbation theory have also been obtained by these authors and the reduced amplitudes of Eqs. (46) can be expressed in terms of the amplitudes used by them. The consistency relation obtained by these authors is not in agreement with experiment.

satisfied by the non-Λ-hyperon amplitudes. On the other hand, if the full 4-symmetry holds, i.e., if $a=b=0$, one has $C=-\sqrt{2}D$, which is incompatible with experiment,[9] and $B=F$.

V. CONCLUDING REMARKS

It has been shown that 4-space symmetry may be introduced in the strong interactions in a rather systematic manner which in no way gives rise to additional restrictions on the interactions. As is well known, an immediate advantage to be gained by the introduction of the 4-symmetry lies in the resulting economy of coupling constants. In the past it was known,[3] e.g., that if the $M-\pi$ coupling indeed manifests the symmetry characterized by the coupling constant relation G_+, the symmetry may be broken by the K-meson coupling F_-. It has also been remarked[24] that in this case the two doublet sets (N_2,N_3), (M_1,M_2) must be introduced to express the G_+ and F_- couplings, respectively. Equations (1) and (10) on one hand and Eqs. (13) and (15) on the other show, however, that this is not the case.

The absence of additional directly verifiable selection rules obviously makes the introduction of the 4-symmetry less attractive, although the 4-symmetry enables one to impose additional restrictions on the reduced amplitudes in a simple manner to obtain amplitude relations not inconsistent with experiment. In this respect 4-symmetry is no worse than charge independence.

The presence of two representations which ascribe two different symmetry properties to the various coupling schemes discussed can lead to some confusion, and it is perhaps more desirable to have a single unifying description of the symmetries involved. We have not been able to discover such a scheme. It should be emphasized, however, that recognition of the two representations makes possible a more detailed study of the amplitudes under specific assumptions of the symmetry of the couplings involved. It is of interest therefore to see whether or not a more detailed use of the dual representations will indeed give rise to the various conditions which were imposed on the reduced amplitudes in Sec. IV.

[24] See, e.g., J. J. Sakurai, reference 19.

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