K^--Meson Scattering in Nuclear Emulsion*

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Elastic single scatterings of K^- mesons by nuclei of a dilute $G5$ photographic emulsion have been experimentally measured for energies between 30 and 80 Mev. Accurate numerical calculations of the differential scattering cross sections have been carried out by solving a Klein-Gordon wave equation for an optical-model potential of the Woods-Saxon form. Experimental and theoretical results are in good agreement both on an absolute and on a relative-variation basis if the real part of the K^--nuclear potential is attractive. The connection with low-energy K^--nucleon scattering is discussed.

1. INTRODUCTION

The scattering of K^- mesons by the nuclei in a photographic emulsion, and the light that it throws on the fundamental K^--nucleon interaction, has been the subject of considerable activity in recent years. The problem may be considered to consist of two parts: (a) a determination of the sign of V, the real part of the optical potential for K^- mesons in nuclear matter; and (b) the relating of this quantity to f, the observed K^--nucleon scattering amplitude. Such a description presupposes the validity of a simple optical-model analysis of the scattering by nuclei, of course.[1]

Melkanoff *et al.*[2] have summarized the development of problem (a). The inelastic scattering experiments of Ceccarelli *et al.*[3] at energies around 50 Mev, when analyzed assuming classical K-meson trajectories, strongly favor an attractive nuclear potential. The elastic scattering results, however, whose implications for K-nucleon scattering are much better understood theoretically, were inconclusive.[4] The recent results on elastic scattering by Jones[5,6] and Melkanoff *et al.*[2] at higher energies (around 125 Mev) indicate that the optical potential at these energies is attractive. In this paper we present experimental and theoretical results on elastic scattering in an energy range around 50 Mev. We also conclude that the optical potential is attractive.

In problem (b), one wishes to relate the optical potential to the K^--nucleon scattering amplitude. For the coherent elastic scattering process the multiple-scattering theory of Watson,[7] or equivalent formulations, allow this to be done in some approximation. Of particular interest is the K^--nucleon scattering at very low energies ($\sim$30 Mev) where a relatively unambiguous zero-range analysis may safely be made.[8] In the choice of the most likely set of zero-range parameters, the information obtained about V may thus be of possible use as supplementary evidence. Energy considerations are the dominating limitation in problem (b): The Watson theory provides an approximation useful at *high* energies, in the sense that the relationship between V and Re f which we wish to use, Eq. (7), neglects corrections whose order of magnitude is the ratio of the nucleon Fermi energy to T_K, the kinetic energy of the K meson. On the other hand, the linking up with the Dalitz-Tuan zero-range solutions requires that T_K be small enough that the zero-range hypothesis, with neglect of higher partial waves, is still valid. Karplus *et al.*[9] have given intuitive arguments for the added importance of this last point. Since the $(a-)$ and $(b-)$ solutions probably correspond to an attractive interaction strong enough to form a bound state, it is to be expected that the actual scattering amplitudes to which these are the zero-energy asymptotes will at some quite low value of T_K ($\sim$90 Mev, corresponding to 60 Mev in the K-nucleon center-of-mass system) suffer a reversal in sign of the real part, and acquire a character similar to that of the $(a+)$ and $(b+)$ solutions. While the optical-potential results of Jones[5,6] and Melkanoff *et al.*,[2] at energies greater than 100 Mev, are almost certainly amenable to the Watson analysis, and therefore provide useful information on K^--nucleon scattering at this energy, they are thus of doubtful use to the K^--nucleon zero-range analysis. The present results, with $T_K\sim 50$ Mev, pass muster on this last point, but it may be argued that the corrections to Eq. (7), the zero-order approximation of Watson, can be rather large. We have no information to present on this point, although preliminary calculations indicate that the correction terms are not large enough to change the sign of the relationship.

The experimental work which is reported on in the present paper was commenced several years ago when only the low-energy K^--meson beam of 300 Mev/c

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[1] A discussion of nonlocality and other complicating features of the multiple-scattering process, as applied to K^- mesons, is given by P. B. Jones, Proc. Roy. Soc. (London) **A255**, 253 (1960).

[2] M. Melkanoff, D. J. Prowse, and D. H. Stork, Phys. Rev. Letters **4**, 183 (1960).

[3] W. Alles, N. N. Biswas, M. Ceccarelli, and J. Crussard, Nuovo cimento **6**, 571 (1957).

[4] J. D. Prowse, M. Melkanoff, and D. H. Stork, Bull. Am. Phys. Soc. **3**, 401 (1958); D. F. Davis, N. Kwak, and M. F. Kaplon, Phys. Rev. **117**, 850 (1960).

[5] P. B. Jones, Phys. Rev. Letters **4**, 35 (1960).

[6] P. B. Jones, Proc. Roy. Soc. (London) **A257**, 109 (1960).

[7] See for example K. M. Watson, Revs. Modern Phys. **30**, 565 (1958).

[8] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).

[9] R. Karplus, L. Kerth, and T. Kycia, Phys. Rev. Letters **2**, 510 (1959).

($T_K=85$ Mev) was available. The presentation of these results has been delayed pending a numerical optical-model analysis for the particular constants of the diluted photographic emulsion used. It has been commonly expected[10] that, because of the strong nuclear absorption of K^- mesons at such low energies, the influence of the real part of the nuclear potential on the elastic scattering would not be apparent. It turns out, however, that the theoretical predictions for attractive and for repulsive potentials are different enough that the present experiments allow a reliable choice to be made between them. Indeed, because the lower energy allows detection of scatterings for q, the recoil momentum, small enough that the nuclear potential has little effect on the differential cross section, we find that this point can be settled more definitely at the energies used here than at higher energies.

Details of the experiment and observations are given in Sec. 2, and a discussion of the data processing in Sec. 3. An important check on the reliability of the results is the agreement with the Rutherford scattering at small q values. It is for this reason we have included in Sec. 3 a rather full description of the corrections applied to the data, which are essential in obtaining this agreement. It is on this account, also, that we have been unable to utilize other published data in this energy region, because of the lack of complete specification of the individual events. The theoretical optical-model analysis is described briefly in Sec. 4, and a discussion of the results obtained is given in Sec. 5.

2. EXPERIMENTAL DETAILS

Two stacks of water-soaked $G5$ emulsions were exposed at the University of California to the Barkas beam of (300 ± 10) Mev/c K^- mesons. The preparation and processing of these emulsions has already been described.[11] Whereas normal $G5$ emulsion contains the following composition of atoms: Ag:Br:C:O:N:H $=1.01:1.01:1.39:0.94:0.32:3.25$ (unit, 10^{22} atoms/cm^3); water soaked emulsion contained the following modified composition:

$$\text{Ag:Br:C:O:N:H}=0.32:0.32:0.45:2.57:0.10:5.55.$$

The emulsions were scanned for K^- mesons near the entering edge of the pellicles. Tracks lying within the angular spread of the beam and having grain densities within the required statistical spread were followed to where definite identification was made on the basis of having all of the following: (1) the correct range of a 300-Mev/c K^- meson, (2) the characteristic interaction of a K^- meson, (3) the appropriate multiple scattering of an ending K^- meson.

[10] For example, D. H. Stork, *Proceedings of the International Conference on the Nuclear Optical Model, 1958*, edited by A. E. S. Green, C. E. Porter, and D. S. Saxon (Florida State University, Tallahassee, Florida, 1959), p. 225.

[11] G. Ascoli, R. D. Hill, and T. S. Yoon, Nuovo cimento **7**, 565 (1958); **9**, 813 (1958).

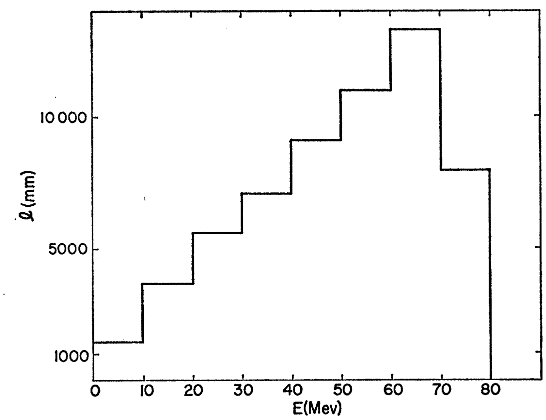


FIG. 1. Histogram of total K^--meson track lengths followed in dilute emulsion, versus kinetic energy.

Approximately 1500 tracks, identified in the above way, were followed from their endings back along the tracks to their entering points. The tracks were carefully searched for single scatters. A histogram of the total track lengths followed in the various energy intervals is shown in Fig. 1.

For energies between 30 and 80 Mev, sharp single scattering events having projected angles of 3° or larger were recorded. Those scatterings in which there was any evidence of a grain density change of abnormally shorter range were rejected as not being elastic. The energy of the meson at the point of scattering was obtained from a measurement of its residual range at this point. Dip angles of the tracks before and after scattering were measured, and space angles at the point of scattering were then computed. A histogram of the number of scatterings as a function of space scattering angle is shown in Fig. 2.

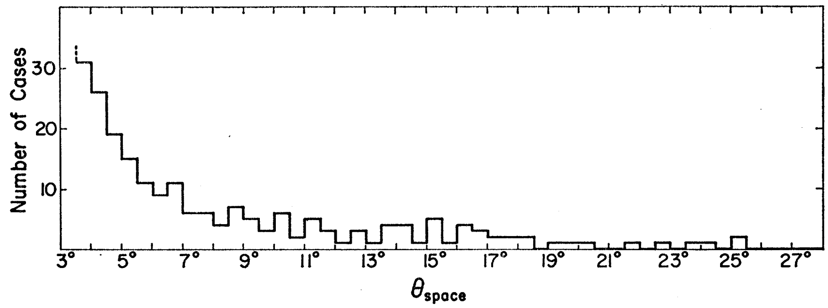
3. PROCESSING OF EXPERIMENTAL RESULTS

The recorded events must be corrected for two observational biases. The first bias is the missing of scatterings having projected angles less than 3° but having, because of significant scattering in dip, space scattering angles greater than 3°.

It can be shown, following the discussion of Lim and Bosgra,[12] that the ratio of the probability of making an observation when the projected angle of cutoff is δ, to the probability of not making an observation, is $(\pi/2\phi)-1$, where $\sin\phi=\cot\theta\tan\delta\cos\alpha$; θ being the space scattering angle and α being the dip angle of the incoming meson before scattering. Each observation was multiplied by a weighting factor equal to $1/(1-2\phi/\pi)$. In order that the correction factor would not be excessive, analyzable events were limited to those which had space angles of 3.5° or more. For zero dip angle and a space angle of 3.0°, the correction factor would be, of course, infinite.

[12] T. G. Lim and S. J. Bosgra, Nuovo cimento **8**, 340 (1958).

FIG. 2. Histogram of number of K^--meson single scattering events versus space scattering angle. The angles for six other such events, outside the range of the plotted histogram, were: 52.1°, 30.8° 40.9°, 34.1°, 63.3°, and 63.9°.



The second bias arises because the scatterings are taken, unavoidably, over a wide range of energies. This effect will now be discussed in relation to the way in which the results of the present experiments are to be presented.

One way of presenting scattering results is to plot the elastic differential scattering cross section versus scattering angle and compare with theory at one average energy. The theoretical differential scattering cross section, however, is strongly dependent on energy and the effect of averaging over energy is to obscure the details of the differential cross section.

In the present experiments, the elastic differential scattering cross section was plotted as a function of momentum transfer to the nucleus. The advantage of this method of comparison with theory, as shown by Igo *et al.*,[13] is that the differential scattering cross section is only very slightly dependent on energy. However, the observational bias lies in the fact that not all of the momentum transfer bins may have the same probabilities for collecting scattering events over the intervals of the energy. The evaluation of these probabilities will now be discussed.

The number of scatters, $N(\bar{q})$, in the momentum transfer interval q to $q+\Delta q$ and having a mean value $\bar{q}$ is

$$N(\bar{q})=nl(d\sigma/dq)_{\bar{q}}\Delta q, \quad (1)$$

where n is the number of scattering centers per unit volume, l is the track length in a particular interval of kinetic energy E, and $(d\sigma/dq)_{\bar{q}}$ is the differential scattering cross section per momentum transfer interval. It is to be pointed out that $N(\bar{q})$ has been assumed to be a function only of q and has no dependence on E. There may in fact be a weak dependence on E but, because experimentally it is necessary to accept events over a wide range of E and because the theoretical dependence, for constant V and W, is much less than the statistical variation of the experimental points, this dependence has been ignored.

From the equation for q,

$$q^2=2(E^2+2mc^2E)(1-\cos\theta), \quad (2)$$

where m is the rest mass of the K^- meson and θ is the scattering angle, Eq. (1) becomes

$$N(\bar{q})=nl(d\sigma/d\Omega)_{\bar{q}}2\pi q\Delta q/(E^2+2mc^2E), \quad (3)$$

where $d\sigma/d\Omega$ is the elastic differential scattering cross section in barns/sr. Thus

$$\left(\frac{d\sigma}{d\Omega}\right)_{\bar{q}}=\frac{(\bar{E}^2+2mc^2\bar{E})N(\bar{q})}{2\pi\bar{q}(nl\Delta q)}. \quad (4)$$

Mean values of the energy, $\bar{E}$, and of momentum transfer, $\bar{q}$, were obtained by averaging the actual values of the experimentally observed scatters. In the low momentum-transfer bins, values of $\bar{E}$ and $\bar{q}$ did not differ very much from the median values of the intervals. However, in some of the high E and q bins, where only a few scatters were observed, $\bar{E}$ and $\bar{q}$ were somewhat different from the median values.

The factor $(nl\Delta q)$ in Eq. (4) must be carefully evaluated in order not to produce a bias in the differential scattering cross section. The product of n and l is the number of atoms per unit area and can be directly obtained for each kinetic energy interval, ΔE, from the histogram of Fig. 1. The width, Δq, of some of the momentum transfer bins, however, may be restricted by dynamics of the scattering, even though the nominal widths of the bins might be the same. This point can be explained easily by reference to Fig. 3. Curves of energy, E, are here plotted versus the angle of scattering, θ, for different values of q. It is clear that for high q values, scattering events for all incident energies between 30 and 80 Mev can be observed; but for the two lowest q bins, 10–15 Mev/c and 15–20 Mev/c, a correction must be applied for scattering events which are lost because of the 3.5° space angle cutoff. If the factor $(nl\Delta q)$ is written $(nl\Delta q\Delta E)/\Delta E$, it is seen that the correction can be obtained, relative to those bins for which no correction is required, by comparing areas bounded by the limiting values of E, q and θ in the particular bins.

Data were assembled for the five 10-Mev energy intervals between 30 and 80 Mev and for ten variable-width momentum transfer intervals between 10 Mev/c and 185 Mev/c. These data are shown plotted in Fig. 4. The curve drawn in Fig. 4 is a $1/q^4$ line, normalized so as to pass through the two points at lowest q value. This

[13] G. Igo, D. G. Ravenhall, J. J. Tiemann, W. W. Chupp, G. Goldhaber, S. Goldhaber, J. E. Lannutti, and R. M. Thaler, Phys. Rev. **109**, 2133 (1958).

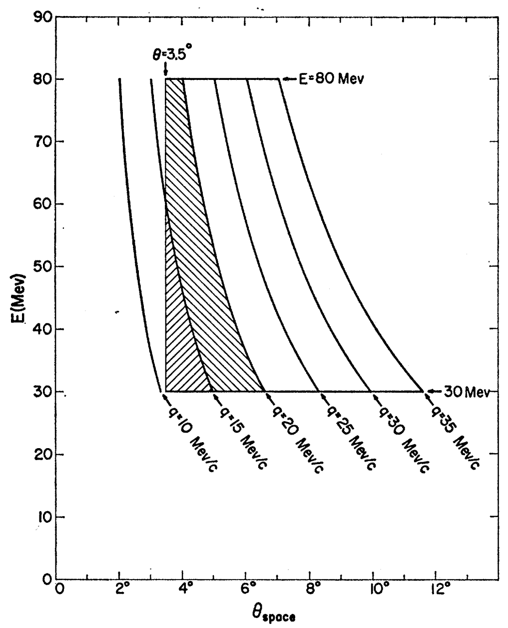


FIG. 3. Correction factor applied for different observational efficiencies of momentum transfer bins. The hatched areas give the relative efficiencies for the bins 10–15 Mev/c and 15–20 Mev/c. In these cases, the bins are restricted by the choice of the arbitrary cutoff angle of 3.5°.

plot shows how small the departures are from a pure Rutherford scattering law.

4. THEORETICAL ANALYSIS

Differential cross sections were calculated from the Klein-Gordon equation for an optical-model potential. An exact numerical phase-shift calculation was programmed for the ILLIAC digital computer.

The Klein-Gordon equation is

$$\hbar^2c^2\nabla^2\psi+(E_{\text{tot}}-\bar{V}_c)^2\psi-(mc^2+\bar{V}_s)^2\psi=0, \qquad (5)$$

where $\bar{V}_c$ is the Coulomb potential for a finite size charge distribution and $\bar{V}_s$ is a scalar optical model potential. The charge density $\rho(r)$ for the nucleus and the optical model potential were assumed to have identical distributions of the Woods-Saxon[14] form:

$$\rho(r)=\rho_0\{1+\exp[(r-R)/a]\}^{-1},$$
$$\bar{V}_s(r)=(V+iW)\{1+\exp[(r-R)/a]\}^{-1}, \qquad (6)$$

where $R=r_0A^{\frac{1}{3}}$.

In Figs. 5 and 6, the quantity $q^4d\sigma/d\Omega$, denoted by k, is plotted against q. The curves were calculated for the two radii, $r_0=1.07$ and $r_0=1.33$, and for several different values of V, with W adjusted so that the total reaction cross section was 640 mb. They were calculated for an emulsion consisting of two kinds of nuclei: $Z=8$, $A=16$ and $Z=41$, $A=94$ in the nuclear proportions: 0.817:0.183, and for an incident energy of 52.5 Mev. The parameter a in Eq. (6) was taken to be 0.57×10^{-13} cm.

A preliminary calculation, based on a three-component emulsion, $Z=8$, $A=16$; $Z=35$, $A=80$; $Z=47$, $A=108$, in the proportions 0.817:0.0915:0.0915, was made for two different incident energies, 65 Mev and 40 Mev. The difference between the results of these two calculations was no larger than 20% of the standard error at any of the experimental points shown in Figs. 5 and 6. All further calculations were therefore made at the one mean energy of 52.5 Mev. The three-component emulsion calculations were then compared with those of the two-component emulsion, and these results differed from one another by less than 3% in the region of the interference effects. All subsequent calculations were based on a two-component emulsion for an energy of 52.5 Mev.

Calculations of reaction cross sections for a large number of values from 500 to 800 mb showed that meaningful values of W could be obtained by equating the calculated reaction cross section to its experimental value. These values of W are attached to the curves shown in Figs. 5 and 6.

As an accurate experimental value of the reaction cross section for the dilute emulsion was not readily

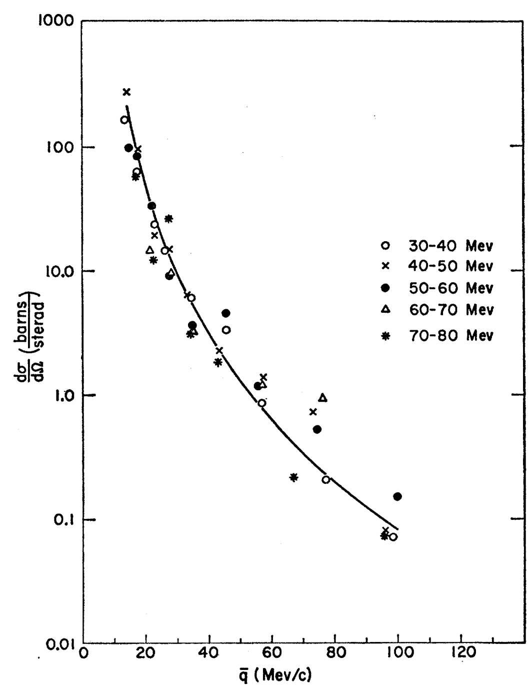


FIG. 4. Variation of observed differential scattering cross section, $d\sigma/d\Omega$, with mean momentum transfer, $\bar{q}$. Each point represents a value for a 10-Mev energy bin and a particular q bin. Five equal energy bins from 30 to 80 Mev were used. The variable-width q bins chosen were: 10–15, 15–20, 20–25, 25–30, 30–40, 40–65, 65–90, 90–125, 125–185 Mev/c. Points are plotted for the mean values of $\bar{q}$ of the observed momentum transfers q within a particular bin. The curve is a $1/q^4$ line normalized so as to pass through the weighted mean of the first two observations at lowest q value.

[14] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).

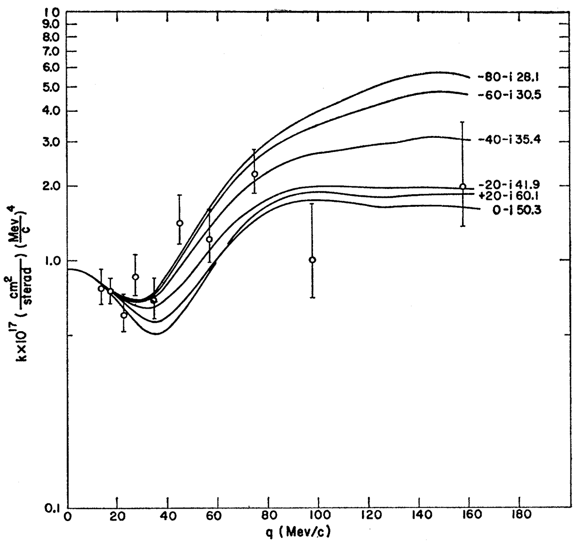


FIG. 5. Plot of $k=[q^4(d\sigma/d\Omega)](\mathrm{Mev}/c)^4(\mathrm{cm}^2/\mathrm{sr})$ versus $q(\mathrm{Mev}/c)$. The theoretical curves were calculated for an r_0 of 1.07 f, and the optical model potentials $V(\mathrm{Mev})+iW(\mathrm{Mev})$ indicated on the individual curves. The experimental points are for energies between 30 and 80 Mev and for 10 variable width q bins, already noted in the legend to Fig. 4.

available, a well-determined value[15] for normal G5 was used to derive a cross section for dilute G5. It was assumed that the cross sections for the individual components were proportional to $A^{\frac{2}{3}}$. This assumption leads to a reaction cross section of 640 mb in the dilute emulsion. Calculations of the reaction cross sections for the larger radius, $r_0=1.33$ f, showed that the cross sections closely obeyed an $A^{\frac{2}{3}}$ law. In the case of the smaller radius $r_0=1.07$ f, there was a small departure from the $A^{\frac{2}{3}}$ law; but the effect on the reaction cross section was only 4% of the 640 mb over the range of values of V and W used. This is approximately the same as the error in the experimental value of the reaction cross section.

At this point we justify the rather limited range of parameters tried in the optical potential, bearing in mind the rather imprecise nature of the experimental data, compared with the corresponding nuclear- physics results. The decision as to the sign of V is made largely on the comparison in the region of the interference minimum, i.e., q values between 20 and 40 Mev/c in Fig. 5. On the basis partly of our own calculations, partly of other published optical-model work,[16] and mainly on physical reasoning about the scattering process, we find that this region of the differential cross section is insensitive to the detailed *shape* of optical potential; to the extent that there is a dependence on the shape which cannot be compensated by changing V, the radius seems to be the most important factor, and consequently it is for a radius variation only that results are presented in this paper. The additional effects of changing the surface thickness, a, or even of altering the analytic form of the surface, show up at larger q values, where "diffraction structure" is seen, and can become very important there.[17] It is thus fortunate that in the present experiment the "interference region," in which the effects of Coulomb and nuclear interaction are comparable, occurs at distinctly smaller q values.

5. COMPARISON BETWEEN THEORY AND EXPERIMENT

In order to bring out the departures from Rutherford scattering it was clear that the points plotted in Fig. 4 should be combined into wider energy groups than 10 Mev in order to improve statistical accuracy. The experimental values of the differential cross sections shown in Fig. 4 do not appear to show any clear correlation with energy.

The variation of $d\sigma/d\Omega$ with q, as shown in Fig. 4, follows closely a $1/q^4$ law. The values of $d\sigma/d\Omega$ were therefore weighted according to q^4, within a particular q bin, in order to obtain an average over the whole energy range from 30 to 80 Mev. The weighted mean values, $\langle(d\sigma/d\Omega)_{\bar{q}}\rangle$ were then multiplied by $\bar{q}^4$, where $\bar{q}$ is the mean momentum transfer for all points within a particular q bin. This product, denoted by k, which is valuable in showing departures from purely Rutherford scattering, is plotted for each of the 10 q bins in Figs. 5 and 6. The errors on the points are based on statistical arguments and do not include possible systematic errors.

In Figs. 5 and 6, the curves for attractive nuclear potentials, $V<0$, appear to give a better fit qualitatively

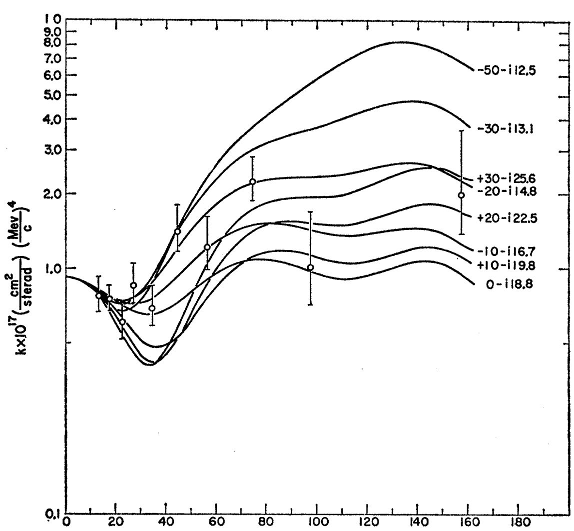


FIG. 6. The same as Fig. 5, except that $r_0=1.33$ f.

[15] Y. Eisenberg, W. Koch, E. Lohrmann, M. Nikolic, M. Schneeberger, and W. Winzeler, Nuovo cimento 9, 745 (1958); G. B. Chadwick, S. A. Durrani, P. B. Jones, J. W. G. Wignall, and D. H. Wilkinson, Phil. Mag. 3, 1193 (1958).

[16] For example, J. S. Nodvik, reference 10, p. 21.

[17] M. Melkanoff, D. J. Prowse, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 5, 108 (1960).

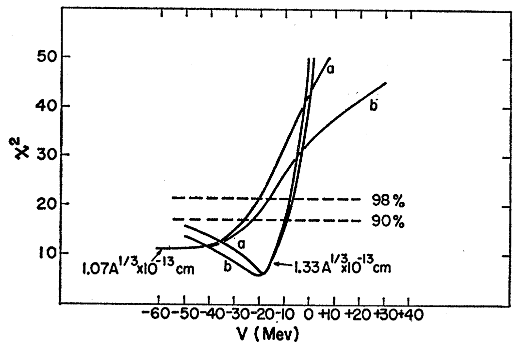


FIG. 7. Curves of χ^2, the mean square deviation of $d\sigma/d\Omega$, versus V, where V is the real part of the optical model potential. Curves labeled 1.07 were derived from Fig. 5; curves labeled 1.33 were derived from Fig. 6. Curves labeled a were obtained from Figs. 5 and 6 by a direct comparison of the points with the curves as they stand. Curves labeled b were obtained from Figs. 5 and 6 after a scale factor adjustment had been made to the individual curves so as to best fit the experimental points. The 90% and 98% levels of confidence are shown.

to the experimental points than is given by those for repulsive potentials. A more quantitative measurement of the fit was made by a χ^2 evaluation, which is given in Fig. 7. Curves denoted by a are a measure of χ^2 evaluation for the experimental results treated on an absolute basis. Curves denoted by b are for the experimental results treated on a basis relative to each other. In the b comparison, the individual differential cross-section curves were multiplied by a scale factor which was adjusted by least squares to best fit the experimental points. The b curves, therefore, give a measure of the accuracy with which the experimental points conform merely to the shape of a theoretical $d\sigma/d\Omega$ curve. The important conclusion to be drawn from the χ^2 plots of the present experiment is, that no matter how the results are compared with theory, the real part of the K^--meson nucleus potential is attractive.

Our confidence in these results stems mainly from the ability, at this low energy, to measure differential cross sections at the small q values. The cross section there is almost Rutherford, and is independent of V. We are thus able to check the absolute values of our measured cross section. Furthermore, the differential cross sections for attractive and repulsive V have distinctly different shapes only if this region is included. With experimental results at higher energies, which do not include this small q region, the decision as to V must thus be made largely on the basis of an absolute cross-section measurement, on which there is no check.

Before a best value for the magnitude of the real part of the potential can be quoted, the nuclear radius must be decided on. Fortunately, in inferring the K^--nuclear amplitude from V, the dependence on nuclear radius is partially compensated, so that when comparing with other analyses it is only important to quote potentials for the same radii as those used at the higher energy. For $r_0 = 1.07$ f, which is the value obtained from electron scattering and probably too small for K^- mesons, the best value for V at our energy, $E \sim 55$ Mev, is around -60 Mev. At $E \sim 125$ Mev, Jones obtained -30 Mev, with the same radius. At a considerably larger radius, $r_0 = 1.33$ f, we find a best value for $V = -20$ Mev. Jones's value at $r_0 = 1.20$ f is -16 Mev, so that at $r_0 = 1.33$ f his value would be somewhat lower than this. Thus V seems to be attractive, and decreasing with energy somewhat. We remark in addition that over our energy range, 30 to 80 Mev, there is no major variation in character detectable in our data, supporting the view that V does not change rapidly with energy.

As has been mentioned in the Introduction, our ability to contribute useful direct information about the K^--nucleon scattering amplitude depends on the validity, at least as regards sign, of the optical-model relationship between V and the real part of the K^--nucleon scattering amplitude f:

$$V_p = -\frac{2\pi\hbar^2}{m}\rho_0\frac{m+M}{M}\,\mathrm{Re} f_{p_r}(0). \qquad (7)$$

Here ρ_0 is the nucleon density, and $f(0)$ is evaluated at the K^--nucleon center-of-mass momentum:

$$p_r = [M/(m+M)]p.$$

For equal numbers of neutrons and protons, f is related to the scattering amplitudes f_0 and f_1 for the $T=0$ and $T=1$ K^--nucleon states by

$$f = \tfrac{1}{4}f_0 + \tfrac{3}{4}f_1. \qquad (8)$$

Clearly, if (7) is valid at least as regards sign, then our results indicate that at $T_K \sim 55$ Mev, i.e., $p \sim 220$ Mev/c, Re$f(0)$ is positive. On the assumption that the zero-range approximation used by Dalitz and Tuan is still applicable at this energy, our result would rule out their $(a-)$ and $(b-)$ solutions.

If relationship (7) is used to calculate V from the Dalitz-Tuan solutions, the following values are obtained (for the case $r_0 = 1.33$ f): $(a+)$, -25 Mev; $(b+)$, -24 Mev; $(a-)$, $+26$ Mev; $(b-)$, $+15$ Mev.

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