$\Sigma^0-\Lambda^0$ Relative Parity from Σ^0 Decay

LOUIS MICHEL AND HOUCHANGUE ROUHANINEJAD*
Laboratoire de Physique Théorique et Hautes Energies, Seine et Oise, France

(Received October 12, 1960)

In order to establish how ϵ, the $\Sigma^0-\Lambda^0$ relative parity, can be measured from actual bubble chamber experiments featuring polarized Σ^0 production and decay, followed by Λ^0 decay and γ-pair production or Dalitz pair in the Σ^0 decay, we constructed a correlation function depending on ϵ, another unknown parameter to be measured in the same experiment, and the energy and momenta of the different particles involved. Our study is Lorentz covariant, but the link with the usual "nonrelativistic" formalism is exhibited. In an Appendix it is shown that the polarization of Σ^0 produced in π^-+p^+ reactions is expected to be large.

INTRODUCTION

SINCE the Σ^0 decay is not due to weak coupling, it very likely conserves parity and it can be a tool to measure the $\Sigma^0-\Lambda^0$ relative parity. This has been proposed by several authors[1–3] who have shown the existence of two different relations between the three particle polarizations depending on the sign of ϵ, the $\Sigma^0-\Lambda^0$ relative parity.

The aim of this work is to show how ϵ can effectively be measured from an actual bubble-chamber experiment. For this we shall construct a correlation function whose variables are: ϵ, another unknown parameter denoted by $\alpha\eta$ (such that $-1\leqslant\alpha\eta\leqslant 1$) which is to be determined by the same experiment, and the energy and momenta of the different involved particles.

Our study will be entirely Lorentz covariant. Indeed this is certainly the simplest way to compute the necessary corrections from a nonrelativistic treatment. However, since such a "nonrelativistic" treatment of polarization seems still to occur more frequently in the published literature, at every step of our computation we shall explicitly exhibit the link between the two formalisms.

1. Type of Required Experiment

In the experiment, the Σ^0 *must be polarized.* Since it is produced by couplings assumed to preserve P and T invariance, the production reaction (on an unpolarized target at rest) must contain at least two linearly independent particle momenta. This excludes, for instance, Σ^0 production by K^- mesons stopped in hydrogen, but admits the *collision*

$$K^-+p^+\rightarrow\Sigma^0+\pi^0. \qquad (1)$$

Other examples of possible reactions for the production of polarized Σ^0 are:

$$\pi^-+p^+\rightarrow\Sigma^0+K^0, \qquad (2)$$

stopped K^-:

$$K^-+d^+\rightarrow\Sigma^0+p^++\pi^- \qquad (3)$$

or

$$K^-+d^+\rightarrow\Sigma^0+n+\pi^0 \quad \text{(difficult to analyze)}.$$

Due to the large asymmetry in $\Sigma^+\rightarrow p^++\pi^0$ decay,[4] it is known that the Σ^+ produced in reactions similar to that of Eq. (3) are unpolarized,[5] but those produced in the reaction $\pi^++p^+\rightarrow\Sigma^++K^+$, which corresponds to Eq. (2) by charge independence, with a one-Gev π^+-beam, have a degree of polarization[4] $|\eta_+|>0.7\pm0.3$. We shall show in the Appendix that the present experimental data on cross sections for $\Sigma^{+,0,-}$ production[6] in reactions similar to that of Eq. (2) imply a similar high degree of polarization for the Σ^0 produced in the reaction of Eq. (2) with a one-Gev π^- beam. This favors the choice of reaction (2) for the proposed experiment. On the other hand, we shall see in Sec. 9 that a measure of a lower bound of $|\eta|$, the degree of polarization of Σ^0, will be a necessary by-product of the measurement of ϵ.

In the decay of a polarized Σ^0 into $\Lambda^0+\gamma$ both final particles are polarized, but only the correlation between the photon transverse polarization and the Λ^0 polarization depends on ϵ. The only possible way to measure such a correlation by present day experimental techniques is to observe in the *same decay*, the products of *the* Λ^0 *disintegration* and an *electron pair* produced by the photon. A schematic diagram of the corresponding bubble-chamber picture is drawn in Fig. 1(a) (for the case of reaction 2). The electron pair can be produced directly by $\Sigma^0\rightarrow\Lambda^0+\epsilon^++\epsilon^-$. It is then called a Dalitz pair; the virtual photon producing it is quasi-real. Although the branching ratio

$$\Sigma^0\rightarrow\Lambda^0+\epsilon^++\epsilon^-,$$

relative to

$$\Sigma^0\rightarrow\Lambda^0+\gamma,$$

is not large (it has been computed[1,7] and found to be

---

* On leave of absence from the University of Teheran, Teheran, Iran.

[1] G. Feldman and T. Fulton, Nuclear Phys. **8**, 106 (1958).

[2] We recently learned of two papers, one by J. Sucher and G. A. Snow [Nuovo cimento (to be published)]; the other by N. Byers and H. Burkhardt [Phys. Rev. (to be published)] on the same subject, mainly based on the study of the Dalitz pair decay of the Σ^0.

[3] See also R. Gatto, Phys. Rev. **109**, 610 (1958).

[4] D. Glaser, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959.

[5] L. Alvarez, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959.

[6] F. S. Crawford, Jr., R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959).

[7] G. Feinberg, Phys. Rev. **109**, 1019 (1958).

1/182 for $\epsilon=1$ and 1/161 for $\epsilon=-1$), the Dalitz pairs are somewhat more convenient for the measurement of the polarization correlation. So we establish the correlation function for both cases: ordinary electron pairs and Dalitz pairs. In the latter case the schematic diagram of the corresponding bubble-chamber picture is drawn in Fig. 1(b).

2. Method of Theoretical Analysis

(See Bernstein and Michel[8] for a somewhat similar analysis.)

The polarization state of the Σ^0 is represented by a 2 by 2 density matrix ρ_Σ. The S matrix for the decay is computed up to a factor. Then $R=S\rho_\Sigma S^\dagger$ is the 4 by 4 density matrix which describes the polarization of the $\gamma-\Lambda^0$ system. The Λ^0 decay as a Λ^0-polarization analyzer, and pair production as analyzer for plane polarization of the photon, are represented by 2 by 2 Hermitian matrices, denoted, respectively, by A_Λ and B_γ. Then $F(\epsilon)=\mathrm{Tr}R(B_\gamma\otimes A_\Lambda)$ is the correlation function we want to compute, where $\otimes$ means the direct product of the two matrices.

More than eight particles are involved in the schemes of Fig. 1(a) or 1(b). In order to avoid for each physical quantity the use of an index indicating to which particle it belongs, we have to use many different letters. Table I is a complete summary of our notation.

3. Covariant Description of Spin $\frac{1}{2}$ Particle Polarization

For a given energy momentum $\mathfrak{p}$ we can choose two orthogonal states of polarization represented by the normed kets $+\rangle$ and $-\rangle$ denoted by $\lambda\rangle$ with $\langle\lambda,\mu\rangle=\delta_{\lambda\mu}$. An arbitrary pure polarization state is represented by the normalized ket $\xi\rangle=\xi_\lambda\lambda\rangle$, where $\langle\xi|\xi\rangle=|\xi_+|^2+|\xi_-|^2=\mathrm{Tr}\xi\rangle\langle\xi=1$. One can also represent it by the projector into $\xi\rangle$, i.e., $\xi\rangle\langle\xi=\frac{1}{2}(1+\boldsymbol{\zeta}\cdot\boldsymbol{\tau})$, where $\tau^{(i)}$ are the three Pauli matrices, $\boldsymbol{\zeta}\cdot\boldsymbol{\tau}$ is a shorthand for $\sum_i\zeta_i\tau^{(i)}$ and $\boldsymbol{\zeta}=\langle\xi\boldsymbol{\tau}\xi\rangle=\mathrm{Tr}\boldsymbol{\tau}\xi\rangle\langle\xi$ is the mean value of $\boldsymbol{\tau}$ for the state $\xi\rangle$. The normalization yields $\sum_i\zeta_i^2=1$. The projector $\xi\rangle\langle\xi$ is called the density matrix of the state.

If we do not consider pure states only but include partially polarized states, the density matrix for the polarization of the particle is still

$$\rho=\tfrac{1}{2}(1+\boldsymbol{\zeta}\cdot\boldsymbol{\tau}), \tag{4}$$

but then

$$0\leqslant|\boldsymbol{\zeta}|=(\textstyle\sum_i\zeta_i^2)^{\frac{1}{2}}\leqslant 1, \tag{5}$$

for $|\boldsymbol{\zeta}|$ is the degree of polarization. The set of three ζ_i, i.e., $\boldsymbol{\zeta}=\mathrm{Tr}\rho\boldsymbol{\tau}$ is called the "Stokes vector." For a particle at rest, $\boldsymbol{\tau}$ represents the spin operator (actually it is twice the infinitesimal rotation operator) and $\boldsymbol{\zeta}$, its mean value, is a genuine pseudovector in the three-dimensional space.

For a spin $\frac{1}{2}$ particle of energy-momentum $\mathfrak{p}$, it is known[9–11] that the polarization can be described by a pseudo four-vector $\mathfrak{s}$ such that

$$\mathfrak{p}\cdot\mathfrak{s}=0 \quad\text{and}\quad (-\mathfrak{s}^2)^{\frac{1}{2}}=|\mathfrak{s}|=\text{degree of polarization.} \tag{6}$$

The relation between $\mathfrak{s}$ and $\boldsymbol{\zeta}$ is the following. A right-handed orthonormal basis in space-time is a set of 4 vectors $\mathfrak{n}^{(\alpha)}$ such that

$$\mathfrak{n}^{(\alpha)}\cdot\mathfrak{n}^{(\beta)}=g^{\alpha\beta}, \tag{7}$$

(here we choose $+\,-\,-\,-$ for the space-time metric), and

$$\frac{1}{4!}\epsilon^{\lambda\mu\nu\rho}n_\lambda{}^{(\alpha)}n_\mu{}^{(\beta)}n_\nu{}^{(\gamma)}n_\rho{}^{(\delta)}=-\epsilon^{\alpha\beta\gamma\delta}, \tag{8}$$

where $\epsilon^{\lambda\mu\nu\rho}$ is the completely antisymmetrical tensor

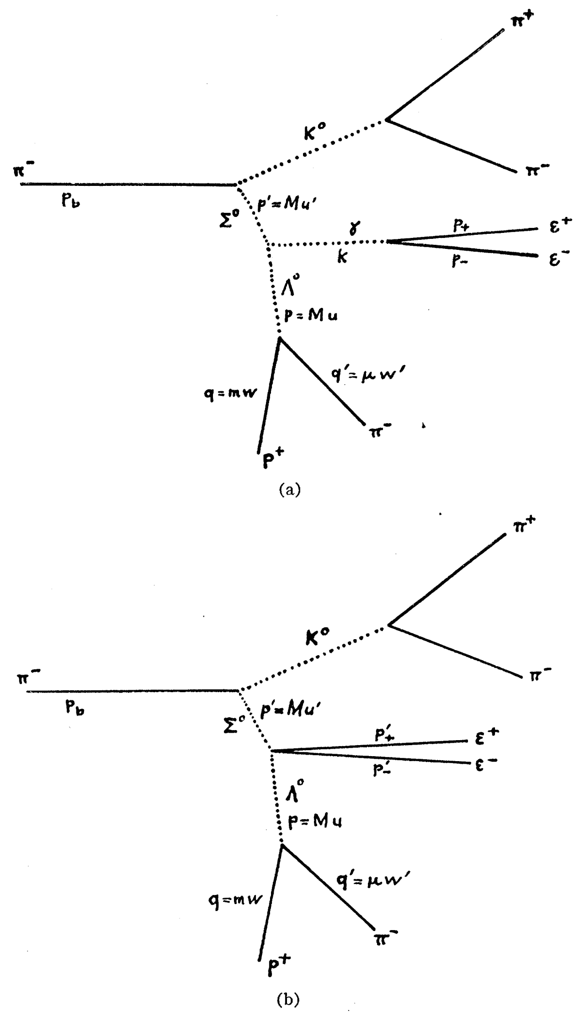

FIG. 1. (a) Schematic diagram of bubble chamber picture of $\Sigma^0\rightarrow\Lambda^0+\gamma$ decay. The notation for energy-momenta is indicated. (b) Schematic diagram of bubble chamber picture of $\Sigma^0\rightarrow\Lambda^0+\epsilon^++\epsilon^-$.

---

[8] J. Bernstein and L. Michel, Phys. Rev. **118**, 871 (1960).

[9] L. Michel and A. S. Wightman, Phys. Rev. **98**, 1190 (1955).

[10] C. Bouchiat and L. Michel, Nuclear Phys. **5**, 416 (1958).

[11] L. Michel, Suppl. Nuovo cimento **14**, 95 (1959).

TABLE I. Complete summary of notations.

					Decay product of Λ^0		Ordinary pair		Dalitz pair	
Particle	Beam	Σ^0	Λ^0	γ	p^+	π^-	ϵ^+	ϵ^-	ϵ^+	ϵ^-
Mass		M'	M	0	m	μ	m_e	m_e	m_e	m_e
Momentum	$\mathbf{p}_b$	$\mathbf{p}'$	$\mathbf{p}$	$\mathbf{k}$	$\mathbf{q}$	$\mathbf{q}'$	$\mathbf{p}_+$	$\mathbf{p}_-$	$\mathbf{p}_+'$	$\mathbf{p}_-'$
Energy-momentum four-vectors	$\mathfrak{p}_b$	$\mathfrak{p}'=M'\mathfrak{u}'$	$\mathfrak{p}=M\mathfrak{u}$	$\mathfrak{k}$	$\mathfrak{q}=m\mathfrak{w}$	$\mathfrak{q}'=\mu\mathfrak{w}'$	$\mathfrak{p}_+$	$\mathfrak{p}_-$	$\mathfrak{p}_+'$	$\mathfrak{p}_-'$
Covariant polarization		$\mathfrak{s}'=\eta\mathfrak{d}$	$\mathfrak{s}$	$\mathfrak{e}$						
Polarization Stokes vector		$\boldsymbol{\eta}=\eta\boldsymbol{\delta}$	$\boldsymbol{\zeta}$	$\boldsymbol{\gamma}$						

Units time-like vectors used:

$\mathfrak{t}=(1,0)$, $\mathfrak{u}'$, $\mathfrak{u}$, $\mathfrak{w}$, $\mathfrak{w}'$ defined above.

Units space-like vectors used:

$\mathfrak{n}^{(1)}$, $\mathfrak{n}^{(2)}$, $\mathfrak{n}^{(3)}$, $\mathfrak{n}'^{(3)}$ [defined in Sec. 5, mainly Eq. (33)];

$\mathfrak{d}$, $\mathfrak{e}$, $\mathfrak{a}=K_3\mathfrak{w}''$ [see Eq. (43)];

$\mathfrak{b}$ defined in (50).

In lab, $\mathfrak{d}=(0,\boldsymbol{\delta})$ with $\boldsymbol{\delta}=(\mathbf{p}_b\times\mathbf{p}')/(|\mathbf{p}_b\times\mathbf{p}'|)$.

Numerical constants introduced:

$$K_1=(M'^2+M^2)/(M'^2-M^2)=15.326;$$
$$K_2=2MM'/(M'^2-M^2)=15.294;$$
$$K_1^2-K_2^2=1.$$
$$K_3=2M\mu[\Delta(M,m,\mu)]^{-\frac{1}{2}}=1.397;$$
$$K_4=(M^2-m^2+\mu^2)/2M\mu=1.230;$$

where

$$\Delta(M,m,\mu)=(M+m+\mu)(M+m-\mu)(M-m+\mu)(M-m-\mu).$$

Parameters η, α, β satisfy $-1\leqslant\eta,\ \alpha,\ \beta\leqslant 1$; $\epsilon=\pm 1$.

Orthogonality relation between four-vectors:

$$\mathfrak{n}^{(1)}\cdot\mathfrak{n}^{(2)}=\mathfrak{n}^{(1)}\cdot\mathfrak{n}^{(3)}=\mathfrak{n}^{(2)}\cdot\mathfrak{n}^{(3)}=\mathfrak{n}^{(1)}\cdot\mathfrak{n}'^{(3)}=\mathfrak{n}^{(2)}\cdot\mathfrak{n}'^{(3)}=0;$$
$$\mathfrak{n}^{(1)}\cdot\mathfrak{p}'=\mathfrak{n}^{(1)}\cdot\mathfrak{p}=\mathfrak{n}^{(1)}\cdot\mathfrak{k}=\mathfrak{n}^{(2)}\cdot\mathfrak{p}'=\mathfrak{n}^{(2)}\cdot\mathfrak{p}=\mathfrak{n}^{(2)}\cdot\mathfrak{k}=0;$$
$$\mathfrak{e}\cdot\mathfrak{k}=0=\mathfrak{b}\cdot\mathfrak{k},\quad \mathfrak{d}\cdot\mathfrak{t}=\mathfrak{d}\cdot\mathfrak{p}_b=\mathfrak{d}\cdot\mathfrak{u}'=0\quad \mathfrak{a}\cdot\mathfrak{u}=0.$$

Relation between vectors:

Definition:

$$\mathfrak{n}'^{(3)}=K_1\mathfrak{u}'-K_2\mathfrak{u};\quad \mathfrak{n}^{(3)}=-K_1\mathfrak{u}+K_2\mathfrak{u}';$$
$$\mathfrak{a}=K_3\mathfrak{w}''=K_3(\mathfrak{w}'-K_4\mathfrak{u}).$$

Conservation of energy and momentum.

In $\Sigma^0\rightarrow\Lambda^0+\gamma$:

$$M'\mathfrak{u}'=M\mathfrak{u}+\mathfrak{k}\quad\text{implies}\quad \mathfrak{u}\cdot\mathfrak{u}'=K_1/K_2;$$

in $\Lambda^0\rightarrow p^++\pi^-$:

$$M\mathfrak{u}=m\mathfrak{w}+\mu\mathfrak{w}'\quad\text{implies}\quad \mathfrak{w}'\cdot\mathfrak{u}=K_4;$$

in $\Sigma^0\rightarrow\Lambda^0+\epsilon^++\epsilon^-$:

$$M\mathfrak{u}=M\mathfrak{u}+\mathfrak{x}\quad\text{with}\quad \mathfrak{x}=\mathfrak{p}_+'+\mathfrak{p}_-'.$$

Furthermore, for that decay

$$\mathfrak{n}'^{(3)}=K_1'\mathfrak{u}'-K_2'\mathfrak{u};\quad \mathfrak{n}^{(3)}=-K_1'\mathfrak{u}+K_2'\mathfrak{u}',$$

with

$$K_1'=(M'^2+M^2-x^2)[\Delta(M',M,x)]^{-\frac{1}{2}};\quad x=|\mathfrak{x}|=(\mathfrak{x}^2)^{\frac{1}{2}};$$
$$K_2'=2MM'(\Delta(M',M,x))^{-\frac{1}{2}};\quad \Delta(M',M,x)\text{ defined above};$$
$$\mathfrak{u}\cdot\mathfrak{u}'=K_1'/K_2';\quad K_1'^2-K_2'^2=1;$$
$$\mathfrak{b}\cdot\mathfrak{u}'=\mathfrak{b}\cdot\mathfrak{u}=\mathfrak{b}\cdot\mathfrak{p}_+'=\mathfrak{b}\cdot\mathfrak{p}_-'=0.$$

with $\epsilon^{0\,1\,2\,3}=1$. The completeness relation yields

$$g_{\alpha\beta}n_\lambda{}^{(\alpha)}n_\mu{}^{(\beta)}=g_{\lambda\mu}. \tag{9}$$

For the particle with energy momentum $\mathfrak{p}$ and mass m, let $\mathfrak{u}=\mathfrak{p}/m$, and $\mathfrak{n}^{(i)}$ with $i=1, 2, 3$, be such a right-handed orthonormal base that we shall call shortly a "tetrad." Then the ζ_i are the components of $\mathfrak{s}$ in this tetrad:

$$\mathfrak{s}=\sum_i\zeta_i\mathfrak{n}^{(i)}, \tag{10}$$

hence, from Eq. (7)

$$\zeta_i=-\mathfrak{s}\cdot\mathfrak{n}^{(i)}. \tag{11}$$

Using the square dot $\centerdot$ notation for $\sum_i$, $\boldsymbol{\zeta}$ for the set of ζ_i, and $\boldsymbol{\mathfrak{n}}$ for the set of $\mathfrak{n}^{(i)}$, Eqs. (10) and (11) can be written:

$$\mathfrak{s}=\boldsymbol{\zeta}\centerdot\boldsymbol{\mathfrak{n}}\quad\text{and}\quad\boldsymbol{\zeta}=-\mathfrak{s}\cdot\boldsymbol{\mathfrak{n}}. \tag{12)–(13}$$

We obtain for the square of the degree of polarization, the equivalent expressions:

$$|\mathfrak{s}|^2=-\mathfrak{s}^2=-\mathfrak{s}\cdot\mathfrak{s}=\sum_i\zeta_i^2=\boldsymbol{\zeta}\centerdot\boldsymbol{\zeta}=\boldsymbol{\zeta}^2=|\boldsymbol{\zeta}|^2. \tag{14}$$

With these notations, the covariant expression for the 2 by 2 density matrix written in Eq. (4), is

$$\rho=\tfrac{1}{2}(1-\mathfrak{s}\cdot\boldsymbol{\mathfrak{n}}\centerdot\boldsymbol{\tau})=\tfrac{1}{2}(1-\textstyle\sum_i\mathfrak{s}\cdot\mathfrak{n}^{(i)}\tau^{(i)}). \tag{15}$$

This is the expression we shall use for the density matrix of the Λ^0-particle (we assume that both Σ and Λ hyperons have spin $\frac{1}{2}$). As we already pointed out, from P and T invariance for the production reaction, only the direction of the Σ polarization is known, but its sign and its degree are not. In the "nonrelativistic" notation, the direction of the Σ^0 polarization is given by the unit vector $\boldsymbol{\delta}=\mathbf{p}_b\times\mathbf{p}'/\mathbf{p}_b|\times\mathbf{p}'|$ (see Table I for notations) and the polarization Stokes vector is $\boldsymbol{\eta}=\eta\boldsymbol{\delta}$, where $-1\leqslant\eta\leqslant 1$, η standing for the unknown polarization degree and sign. The corresponding vector covariant to $\boldsymbol{\delta}$ is the unit space-like vector orthogonal to $\mathfrak{p}$ beam, $\mathfrak{p}$ target, $\mathfrak{p}_\Sigma=\mathfrak{p}'$ [and right-handed with them; i.e., the unit vector obtained by normalizing $\epsilon^{\lambda\mu\nu\rho}(\mathfrak{p}_b)_\mu(\mathfrak{p}_t)_\nu\mathfrak{p}'_\rho$]. In the laboratory system, the target is at rest, the Σ^0 polarization is transverse, and $\mathfrak{d}=(0,\boldsymbol{\delta})$ as written in Table I. So the density matrix for the polarization of the Σ^0 is

$$\rho_\Sigma=\tfrac{1}{2}(1+\eta\boldsymbol{\delta}\centerdot\boldsymbol{\tau})=\tfrac{1}{2}(1-\eta\mathfrak{d}\cdot\boldsymbol{\mathfrak{n}}'\centerdot\boldsymbol{\tau})=\tfrac{1}{2}(1-\mathfrak{s}'\cdot\boldsymbol{\mathfrak{n}}'\centerdot\boldsymbol{\tau}), \tag{16}$$

if we denote by $\mathfrak{u}'$, $\boldsymbol{\mathfrak{n}}'$ a tetrad associated with the Σ^0.

4. Covariant Description of Photon Polarization

Since a photon of given momentum has two linearly independent states of polarization, its 2 by 2 density matrix for polarization is similar to that of spin $\frac{1}{2}$ particles. Let us choose for basic states $\lambda\rangle$, with $\lambda=\pm$, the right and left circular polarization. An arbitrary pure state of polarization is $\xi\rangle=\xi_\lambda\lambda\rangle$ and the density matrix is $\xi\rangle\langle\xi=\rho_\gamma=\frac{1}{2}(1+\boldsymbol{\gamma}\cdot\boldsymbol{\tau})$ with $\boldsymbol{\gamma}=\langle\xi\boldsymbol{\tau}\xi\rangle$ and $\boldsymbol{\gamma}^2=1$. Since τ_3 is diagonal, $\gamma_3=+1$ corresponds to pure right circular polarization, $\gamma_3=-1$ to pure left circular polarization, and $\gamma_3=0$ to plane polarization.

For partial polarization, the density matrix has the same form:

$$\rho_\gamma=\tfrac{1}{2}(1+\boldsymbol{\gamma}\cdot\boldsymbol{\tau}), \tag{17}$$

where

$$0\leqslant|\boldsymbol{\gamma}|=(\boldsymbol{\gamma}^2)^{\frac{1}{2}}=\text{degree of polarization}\leqslant 1. \tag{18}$$

Although the three numbers γ_i can never be the components of a vector, their set $\boldsymbol{\gamma}$ is often called the "Stokes vector" since (three linear combinations of) the γ_i were introduced by Stokes[12] in 1852. The use of γ_i is well spread nowadays.[13,14]

We recall here how the Stokes vector is related to the covariant formalism,[11,14] since the situation is now radically different from that of spin $\frac{1}{2}$ particles.

One shows that γ_3 is a pseudoscalar and $\gamma_p=(\gamma_1^2+\gamma_2^2)^{\frac{1}{2}}$ is a scalar for the Lorentz group, so instead, to use the vocabulary of elliptical polarization we shall call $\gamma_c=|\gamma_3|$ and γ_p the degrees of circular and plane polarization.

Since $\mathfrak{k}^2=0$ there are only two other linearly independent four-vectors orthogonal to $\mathfrak{k}$; we denote them by $\mathfrak{n}^{(1)}$ and $\mathfrak{n}^{(2)}$; they satisfy

$i=1, 2$:

$$\mathfrak{k}^2=\mathfrak{k}\cdot\mathfrak{n}^{(i)}=0,\quad -\mathfrak{n}^{(i)}\cdot\mathfrak{n}^{(j)}=\delta_{ij},\quad \mathbf{k}\cdot\mathbf{n}^{(1)}\times\mathbf{n}^{(2)}>0. \tag{19}$$

Note that the $\mathfrak{n}^{(i)}$ are defined up to an arbitrary component along $\mathfrak{k}$. The photon polarization vector

$$\mathfrak{e}=\xi_1\mathfrak{n}^{(1)}+\xi_2\mathfrak{n}^{(2)}, \tag{20}$$

which describes pure states of polarization, is a genuine vector orthogonal to $\mathfrak{k}$ and defined up to a component along $\mathfrak{k}$ [in the choice $\mathfrak{e}=(0,\mathbf{e})$, $\mathbf{e}$ is proportional to the photon electric vector] but its length is defined in an Hermitian metric:

$$(\mathfrak{e},\mathfrak{e})=1=|\xi_1|^2+|\xi_2|^2. \tag{21}$$

In a real base, such as (19), we can define the complex conjugated vector: $\mathfrak{e}^*=\xi_1^*\mathfrak{n}^{(1)}+\xi_2^*\mathfrak{n}^{(2)}$, and we have the following identity for vectors orthogonal to $\mathfrak{k}$:

$$(\mathfrak{a},\mathfrak{b})=-\mathfrak{a}^*\cdot\mathfrak{b}. \tag{22}$$

[12] G. G. Stokes, Proc. Cambridge Phil. Soc. **9**, 399 (1852).

[13] U. Fano, J. Opt. Soc. Am. **39**, 859 (1949) and Revs. Modern Phys. **29**, 74 (1957).

[14] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), p. 42.

The circular polarization vector,

$$\mathfrak{e}_\pm=\frac{1}{\sqrt{2}}(\mathfrak{n}^{(1)}\pm i\mathfrak{n}^{(2)}), \tag{23}$$

satisfies

$$(\mathfrak{e}_\lambda,\mathfrak{e}_\mu)=\delta_{\lambda\mu}=-\mathfrak{e}_\lambda^*\cdot\mathfrak{e}_\mu \quad\text{and}\quad \mathfrak{e}_\lambda^*=\mathfrak{e}_{-\lambda}. \tag{24}$$

Then, the polarization vector can be expanded:

$$\mathfrak{e}=\xi_\lambda\mathfrak{e}_\lambda, \tag{25}$$

where

$$\xi_\lambda=(\mathfrak{e}_\lambda,\mathfrak{e})=\frac{1}{\sqrt{2}}(\xi_1-i\lambda\xi_2)=-\mathfrak{e}_\lambda^*\cdot\mathfrak{e}. \tag{26}$$

The ξ_λ are also the component of the representative ket introduced in the beginning of this section. Using this isomorphism between $\xi\rangle$ and $\mathfrak{e}$, we can construct the density matrix in terms of a tensor orthogonal to $\mathfrak{k}$. For a pure state,

$$\rho=-\mathfrak{e}\otimes\mathfrak{e}^*=-\tfrac{1}{2}(1+\boldsymbol{\gamma}\cdot\boldsymbol{\tau})_{\lambda\mu}\mathfrak{e}_\lambda\otimes\mathfrak{e}_\mu^*. \tag{27}$$

The right-hand side represents also partial polarization when $0\leqslant|\boldsymbol{\gamma}|\leqslant 1$.

Indeed, in this isomorphism the unit matrix represents the tensor

$$I=-\textstyle\sum_{\lambda\mu}\delta_{\lambda\mu}\mathfrak{e}_\lambda\otimes\mathfrak{e}_\mu^*=-\sum_{ij}\delta_{ij}\mathfrak{n}^{(i)}\otimes\mathfrak{n}^{(j)}, \tag{28}$$

and the Pauli matrices represent the tensors

$$P_k=-\textstyle\sum_{\lambda\mu}(\tau^{(k)})_{\lambda\mu}\mathfrak{e}_\lambda\otimes\mathfrak{e}_\mu^* \quad\text{or}\quad P_k'=-\sum_{ij}(\tau_{ij}{}^{(k)})\mathfrak{n}^{(i)}\otimes\mathfrak{n}^{(j)}, \tag{29}$$

depending on which basis ($\mathfrak{e}_\lambda$ or $\mathfrak{n}^{(i)}$) is chosen for polarization vectors. We leave it to the reader to prove that

$$P_1=P_3',\quad P_2=P_1',\quad P_3=P_2', \tag{30}$$

and that a photon density matrix can always be written covariantly as

$$\rho=\tfrac{1}{2}[I(1-\gamma)-2\gamma\mathfrak{e}\otimes\mathfrak{e}^*], \tag{31}$$

where γ is the degree of polarization and $\mathfrak{e}$ a unit complex vector (i.e., $-\mathfrak{e}\cdot\mathfrak{e}^*=1$) orthogonal to $\mathfrak{k}$ (it is defined up to a component along $\mathfrak{k}$).

5. The (up to a Factor) S Matrix for Σ^0 Decay

To express S explicitly in a chosen basis, we have first to choose the tetrads associated with Σ^0 and Λ^0. Let us consider the $\Sigma^0\rightarrow\Lambda^0+\gamma$ decay with given energy-momenta which satisfy

$$\mathfrak{p}'=M'\mathfrak{u}'=\mathfrak{p}+\mathfrak{k}=M\mathfrak{u}+\mathfrak{k}. \tag{32}$$

We shall choose $\mathfrak{n}'^{(1)}=\mathfrak{n}^{(1)}$ and $\mathfrak{n}'^{(2)}=\mathfrak{n}^{(2)}$ in the 2-plane (i.e., the two-dimensional plane) orthogonal to $\mathfrak{u}'$, $\mathfrak{u}$, $\mathfrak{k}$ and such that they satisfy (19). Then $\mathfrak{n}_3'$ and $\mathfrak{n}_3$ are completely determined; indeed,

$$\mathfrak{n}_3'=K_1\mathfrak{u}'-K_2\mathfrak{u} \quad \text{and} \quad \mathfrak{n}_3=-K_1\mathfrak{u}+K_2\mathfrak{u}', \qquad (33)$$

where K_1 and K_2 are constants defined in Table I. We denote by $\sigma\rangle$, $\lambda\rangle$, $\gamma\rangle$ (with $\sigma=\lambda=\gamma=\pm1$) the kets representing the states of Σ^0, Λ^0, γ with energy-momentum and polarization: $\mathfrak{p}'$, $\sigma\mathfrak{n}'^{(3)}$ for Σ^0; $\mathfrak{p}$, $\lambda\mathfrak{n}^{(3)}$ for Λ^0; $\mathfrak{k}$, circular polarization $\mathfrak{e}_\gamma$ for γ.

These form a complete base for the polarization states of the considered decay. Furthermore they are proper states of all the transformations of the one-parameter group R_2' defined as the set of all transformations of the connected Lorentz group which leave invariant every four-vector of the 2-plane which contains $\mathfrak{p}'$, $\mathfrak{p}$, and $\mathfrak{k}$. The group R_2' is isomorphic to the two-dimensional rotation group R_2. Invariance under this group implies conservation of one component of angular momentum and it requires for the S-matrix elements defined by

$$\gamma\rangle\otimes\lambda\rangle=S_{\gamma\lambda,\sigma}\sigma\rangle, \qquad (34)$$

that all be equal to 0 except $S_{+-,+}=\varphi$ and $S_{-+,-}=\varphi'$. A mirroring which leaves invariant the 2-plane $\mathfrak{p}'$, $\mathfrak{p}$, $\mathfrak{k}$, exchanges these two S-matrix elements and parity conservation requires

$$\varphi'=-\epsilon\varphi.$$

Therefore, up to a factor, S is of the form

$$\begin{matrix}\gamma & \lambda \\ + & + \\ + & - \\ - & + \\ - & -\end{matrix} \quad S=\overset{\sigma\; + \quad -}{\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & -\epsilon\\ 0 & 0\end{bmatrix}}. \qquad (35)$$

6. The Density Matrix R for the Polarization of the $\gamma-\Lambda^0$ System

The density matrix for the polarization of the $\gamma-\Lambda^0$ system is

$$R=S\rho_\Sigma S^\dagger, \qquad (36)$$

where ρ_Σ is written in (16). Explicitly

$$R(\epsilon)=\tfrac{1}{2}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1+\eta\delta_3 & -\epsilon\eta(\delta_1-i\delta_2) & 0\\ 0 & -\epsilon\eta(\delta_1+i\delta_2) & 1-\lambda\eta\delta_3 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \qquad (37)$$

or

$$R(\epsilon)=\tfrac{1}{4}[1\otimes1-\tau_3\otimes\tau_3+\eta\delta_3(\tau_3\otimes1-1\otimes\tau_3) -\epsilon\eta\{\delta_1(\tau_1\otimes\tau_1+\tau_2\otimes\tau_2)-\delta_2(\tau_1\otimes\tau_2-\tau_2\otimes\tau_1)\}]. \qquad (38)$$

The symbol $\otimes$ indicates the tensor (or Kronecker) product. If ρ_i and σ_j are the usual Dirac matrices (see for instance Dirac's book), then $\rho_i=\tau_i\otimes1$, $\sigma_j=1\otimes\tau_j$, and therefore $\rho_i\sigma_j=\tau_i\otimes\tau_j$.

The unknown quantities in R are ϵ and η, the sign and degree of Σ^0 polarization.

7. The Λ^0 Decay as Λ^0-Polarization Analyzer

An observable is described by an Hermitian operator. For a polarization analyzer the part of the Hermitian operator corresponding to the polarization of a spin $\frac{1}{2}$ matrix with well-defined momentum is a 2 by 2 Hermitian matrix which can be written

$$A=\chi(1+\boldsymbol{\alpha}\cdot\boldsymbol{\tau}), \qquad (39)$$

and the probability of "counting" the particle whose polarization matrix is ρ [written in (4)] is

$$\mathrm{Tr}A\rho=\chi(1+\boldsymbol{\alpha}\cdot\boldsymbol{\xi}). \qquad (40)$$

This shows that χ is the probability (or counting rate or cross section, etc.) for the observation of unpolarized particles and that $0\leqslant\boldsymbol{\alpha}^2\leqslant1$; indeed $|\boldsymbol{\alpha}|$ is the efficiency of the polarization analyzer and the direction of $\boldsymbol{\alpha}$ corresponds to the "setting" of this apparatus. In the following we shall use a unit "Stokes vector" $\boldsymbol{\lambda}$ for the description of the setting of the apparatus; then

$$\boldsymbol{\alpha}=\alpha\boldsymbol{\lambda} \quad \text{with} \quad \boldsymbol{\lambda}^2=1. \qquad (41)$$

This is more adapted to the physical situation, because the sign of α is not known in Λ^0 decay. The covariant corresponding to $\boldsymbol{\lambda}$ is the unit four-vector defined [compare to Eq. (12)] by

$$\mathfrak{a}=\boldsymbol{\lambda}\cdot\boldsymbol{\mathfrak{n}}; \qquad (42)$$

so the covariant form of the matrix A defined in Eq. (39) is

$$A=(1-\alpha\mathfrak{a}\cdot\boldsymbol{\mathfrak{n}}\cdot\boldsymbol{\tau}), \qquad (43)$$

and, with ρ given in (15):

$$\mathrm{Tr}A\rho=(1-\alpha\mathfrak{a}\cdot\mathfrak{s}). \qquad (44)$$

Let us consider a Λ^0 with energy momentum $\mathfrak{p}=M\mathfrak{u}$ and polarization $\mathfrak{s}$ decaying into a proton and a π^- meson with energy-momenta $\mathfrak{q}=m\mathfrak{w}$ and $\mathfrak{q}'=\mu\mathfrak{w}'$. Spin $\frac{1}{2}$ for the Λ^0 implies that the transition probability is linear in $\mathfrak{s}$ and nonconservation of parity implies that it is the sum of a scalar and of a pseudoscalar; its most general form is therefore $(1-\alpha K_3\mathfrak{w}'\cdot\mathfrak{s})$. The choice of $\mathfrak{w}'$ and not of $\mathfrak{w}$ is in agreement with general use.[4] Only $\mathfrak{w}''=\mathfrak{w}'-(\mathfrak{u}\cdot\mathfrak{w}')\mathfrak{u}$, the component of $\mathfrak{w}'$ orthogonal to $\mathfrak{u}$, is significant. The constant K_3 is such that $K_3\mathfrak{w}''$ is a unit vector and so $-1\leqslant\alpha\leqslant1$. The value of K_3 is given in Table I.

The comparison of the Λ^0 decay rate with (44) shows that the matrix A_Λ representing Λ^0-decay as Λ^0-polarization analyzer is proportional to

$$A=1-\alpha K_3\mathfrak{w}''\cdot\boldsymbol{\mathfrak{n}}\cdot\boldsymbol{\tau}=1-\alpha K_3\mathfrak{w}'\cdot\boldsymbol{\mathfrak{n}}\cdot\boldsymbol{\tau}. \qquad (45)$$

Let η_Λ be the transverse polarization of the Λ-hyperon particle produced in a given reaction ($\eta_\Lambda>0$ if the polarization $\boldsymbol{\zeta}$ is along the direction $+\mathbf{p}_b\times\mathbf{p}_\Lambda$, and $\eta_\Lambda<0$ if $\boldsymbol{\zeta}$ is along the direction $-\mathbf{p}_b\times\mathbf{p}_\Lambda$). Some asymmetry measurements[4] for Λ^0 decay have yielded $\alpha\eta_\Lambda=0.73\pm0.14$; this value is a lower limit on $|\alpha|$

(and most physicists believe that $|\alpha|=1$). As we shall see, we do not need the sign of α, but the product $\alpha\eta$ (we recall that η is the sign and degree of Σ^0 polarization) will have to be measured in the same experiment.

8. Pair Production as γ-Plane Polarization Analyzer

As Stokes pointed out more than a century ago, a light polarization analyzer is to be described by a 2 by 2 Hermitian matrix. We can write it as in (39): $B=\chi(1+\boldsymbol{\beta}\cdot\boldsymbol{\tau})$ and then, with (17), we find for the "counting" rate:

$$\mathrm{Tr}B\rho_\gamma=\chi(1+\boldsymbol{\beta}\cdot\boldsymbol{\gamma});\qquad(46)$$

or we can write B as in Eq. (31), namely:

$$B=\chi[I(1-\beta)-2\beta\mathfrak{b}\otimes\mathfrak{b}^*],\qquad(47)$$

where β is the efficiency of the process as polarization analyzer and the unit vector $\mathfrak{b}$ is the "setting." The transition rate is then

$$\mathrm{Tr}B\rho_\gamma=\chi(1-\beta\gamma+2\beta\gamma|\mathfrak{b}\cdot\mathfrak{e}^*|^2).\qquad(48)$$

If $\mathfrak{b}$ and/or $\mathfrak{e}$ are real (as it is in the case of plane polarization) this can be written:

$$\mathrm{Tr}B\rho_\gamma=(1+\beta\gamma\cos2\phi),\qquad(49)$$

where $\cos\phi=-\mathfrak{e}\cdot\mathfrak{b}$.

In the proposed experiment we are not interested in the photon circular polarization since in (38) the coefficients of τ_3 matrices, which correspond to γ-circular polarization, do not contain ϵ. Note, however, that if this circular polarization measurement can be performed, it would give the value of both α and η separately (including their sign). This would be a very important result. However, this experiment cannot be performed, with present experimental techniques, in a bubble chamber. For instance, pair production is a poor analyzer of the circular polarization of high-energy photons (efficiency β of the order of m_e/E_γ). On the other hand, Dalitz pairs do not analyze circular polarization.

The most efficient phenomenon for analyzing high-energy photon plane polarization seems to be electron pair production. (Compton scattering has a too low β, nuclear photoeffects a too low χ.) The corresponding β and $\mathfrak{b}$ are complicated functions of $\mathfrak{k}$, $\mathfrak{p}^+$, $\mathfrak{p}^-$. We shall not give them explicitly. However, the angles between $\mathbf{k}$, $\mathbf{p}_+$, $\mathbf{p}_-$ are difficult to measure (they are small and there is multiple scattering). If the only measured angle is ϕ_a, the azimuth around $\mathbf{k}$ of the normal $(\mathbf{p}_+\times\mathbf{p}_-)$ of the plane of the pair, then $\mathfrak{b}=(0,\mathbf{b})$, with $\mathbf{b}$ the unit vector of

$$(\mathbf{k}\cdot\mathbf{p}_+)(\mathbf{k}\times\mathbf{p}_-)-(\mathbf{k}\cdot\mathbf{p}_-)(\mathbf{k}\times\mathbf{p}_+),\qquad(50)$$

and the corresponding β has been computed by Karlson.[15] Figure 2 gives β for 66-Mev photons, as a function

[15] E. Karlson, Arkiv Fysik 13, 1 (1957).

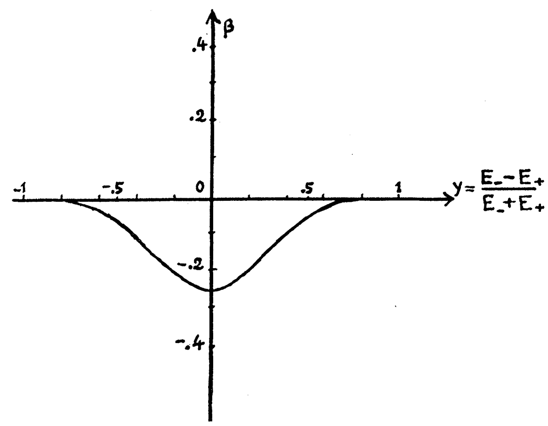

FIG. 2. The efficiency $\beta(y)$ as plane polarization analyzer for ordinary electron pair; from Karlson, reference 15.

of $(E_+-E_-)/(E_++E_-)$, the repartition of energy in the pair.[16]

9. Correlation Functions for $\Sigma^0\to\Lambda^0+\gamma$ Decay

We have to define a notation for "partial traces." Consider a 4 by 4 matrix $Z=\sum_i c_i\Gamma_i\otimes\Lambda_i$, where Γ_i and Λ_i are 2 by 2 matrices. We define

$$\mathrm{Tr}_\Gamma Z=\textstyle\sum_i c_i(\mathrm{Tr}\Gamma_i)\Lambda_i,\quad \mathrm{Tr}_\Lambda Z=\sum_i c_i(\mathrm{Tr}\Lambda_i)\Gamma_i.\qquad(51)$$

We verify that

$$\mathrm{Tr}Z=\mathrm{Tr}_\Gamma(\mathrm{Tr}_\Lambda Z)=\mathrm{Tr}_\Lambda(\mathrm{Tr}_\Gamma Z)=\textstyle\sum_i c_i(\mathrm{Tr}\Gamma_i)(\mathrm{Tr}\Lambda_i).\qquad(52)$$

The matrix R, Eq. (37) or (38), contains all possible information concerning the particle polarizations. For instance, if we observe the Λ^0 by the analyzer represented by A_Λ, the photon is in the state $\mathrm{Tr}_\Lambda R(1\otimes A_\Lambda)$.

Conversely, let us suppose that we do not observe the γ-polarization ($B_\gamma=1$). Then the Λ^0 polarization is described by

$$\mathrm{Tr}_\gamma R=\tfrac{1}{2}(1-\eta\delta_3\tau_3)=\tfrac{1}{2}(1+\eta\mathfrak{d}\cdot\mathfrak{n}'^{(3)}\tau_3)$$
$$=\tfrac{1}{2}(1-K_2\mathfrak{d}\cdot\mathfrak{u}:\tau_3),\qquad(53)$$

and the correlation function yielded by Λ^0 decay only (nonobservation of the γ) is then

$$H=\mathrm{Tr}R(1\otimes A_\Lambda)=\mathrm{Tr}(A_\Lambda\,\mathrm{Tr}_\gamma R),$$
$$H=1+\alpha\eta K_2K_3\mathfrak{d}\cdot\mathfrak{u}\mathfrak{w}'\cdot\mathfrak{n}^{(3)},\qquad(54)$$
$$H=1+\alpha\eta K_2K_3\mathfrak{d}\cdot\mathfrak{u}(K_2\mathfrak{w}'\cdot\mathfrak{u}'-K_1K_4).$$

This correlation function shows how $\alpha\eta$ can be measured. While α is a universal constant characterizing Λ^0 decay, η is expected to be a function of the beam energy and the angle of production of Σ^0 or, in terms of four-vectors, a function of $\mathfrak{p}_b$ and $\mathfrak{u}'$. (See also the Appendix.)

[16] The sign of β has been the subject of some controversy; see, e.g., T. H. Berlin and L. Madansky, Phys. Rev. 78, 623 (1950), G. C. Wick, Phys. Rev. 81, 467 (1951), and reference 17. It is true, indeed, that the sign of β is opposite for ordinary pairs ($\beta<0$) and for Dalitz pairs ($\beta>0$) as we shall see in Sec. 10.

Instead, to compute directly the final correlation function $F(\epsilon)$, let us proceed by steps. If the Λ^0 decay is observed, what is the polarization state of the γ?

$$\rho_\gamma = \mathrm{Tr}_\Lambda R(1\otimes A_\Lambda) = \tfrac{1}{2}\{1-\alpha\eta\lambda_3\delta_3 + (\eta\delta_3-\alpha\lambda_3)\tau_3 - \epsilon\alpha\eta[(\delta_1\lambda_1-\delta_2\lambda_2)\tau_1 + (\delta_1\lambda_2+\delta_2\lambda_1)\tau_3]\}. \quad (55)$$

The terms $(\eta\delta_3-\alpha\lambda_3)\tau_3$ are for partial circular polarization. In this section we are interested only in the plane polarization and we shall drop these τ_3 terms.

Then Eq. (55) can be written, with I defined in Eq. (28) and H in Eq. (54):

$$\rho_\gamma = \tfrac{1}{2}[H+\epsilon\alpha\eta(-\mathfrak{d}\cdot I\cdot\mathfrak{a}+\mathfrak{d}\otimes\mathfrak{a})], \quad (56)$$

where

$$\mathfrak{d}\cdot I\cdot\mathfrak{a} = \mathfrak{d}\cdot\mathfrak{a} + (\mathfrak{d}\cdot\mathfrak{n}'^{(3)})(\mathfrak{a}\cdot\mathfrak{n}'^{(3)}) = K_3[\mathfrak{d}\cdot\mathfrak{w}' - K_2\mathfrak{d}\cdot\mathfrak{u}(K_1\mathfrak{w}'\cdot\mathfrak{u}'-K_2K_4)].$$

The use of Eq. (47) for B gives us the correlation function F as

$$F(\epsilon) = 1-K_3\alpha\eta\left[K_2G+\epsilon\frac{1+\beta}{2}(\mathfrak{d}\cdot\mathfrak{w}'+K_1G) + \beta\mathfrak{d}\cdot\mathfrak{b}\mathfrak{b}\cdot\mathfrak{w}'\right], \quad (57)$$

where

$$G = \mathfrak{d}\cdot\mathfrak{u}(K_1K_4-K_2\mathfrak{w}'\cdot\mathfrak{u}'), \quad (58)$$

and $\mathfrak{b}$ is defined in (50) and β drawn in Fig. 2.

For the sake of completeness we also give F as a function of $\mathfrak{p}_+$ and $\mathfrak{p}_-$ although it has no practical value for the discussed experiment. In the literature, the cross section for electron pair production by a totally plane polarized photon is given by (see May,[17] also reference 14, p. 374) (up to a factor)

$$D-(\mathfrak{r}\cdot\mathfrak{e})^2+(\mathfrak{r}'\cdot\mathfrak{e})^2, \quad (59)$$

where $\mathfrak{t}=(1,\mathbf{0})$,

$$D = \frac{|\mathbf{k}\times(\mathbf{p}_++\mathbf{p}_-)|^2}{(\mathfrak{k}\cdot\mathfrak{p}_+)(\mathfrak{k}\cdot\mathfrak{p}_-)}, \quad \mathfrak{r} = 2\left[\frac{\mathfrak{t}\cdot\mathfrak{p}_+}{\mathfrak{k}\cdot\mathfrak{p}_-}\mathfrak{p}_- + \frac{\mathfrak{t}\cdot\mathfrak{p}_-}{\mathfrak{k}\cdot\mathfrak{p}_+}\mathfrak{p}_+ - (\mathfrak{k}\cdot\mathfrak{t})\mathfrak{t}\right],$$

$$\mathfrak{r}' = |\mathbf{k}-\mathbf{p}_+-\mathbf{p}_-|\left(\frac{1}{\mathfrak{k}\cdot\mathfrak{p}_-}\mathfrak{p}_- - \frac{1}{\mathfrak{k}\cdot\mathfrak{p}_+}\mathfrak{p}_+\right).$$

The corresponding 2 by 2 matrix is

$$B'_\gamma = DI+\mathfrak{r}\otimes\mathfrak{r}-\mathfrak{r}'\otimes\mathfrak{r}'. \quad (61)$$

The cross section for unpolarized photons is

$$\tfrac{1}{2}\,\mathrm{Tr}B'_\gamma = \tfrac{1}{2}(2D+\mathfrak{r}^2-\mathfrak{r}'^2), \quad (62)$$

and the corresponding correlation function F' is

$$F'(\epsilon) = 1-\alpha\eta K_3\left[K_2G+\epsilon\frac{(\mathfrak{d}\cdot\mathfrak{w}'+K_1G)(D+\mathfrak{r}^2-\mathfrak{r}'^2)-(\mathfrak{d}\cdot\mathfrak{r})(\mathfrak{w}'\cdot\mathfrak{r})+(\mathfrak{d}\cdot\mathfrak{r}')(\mathfrak{w}'\cdot\mathfrak{r}')}{2D+\mathfrak{r}^2-\mathfrak{r}'^2}\right], \quad (63)$$

where G is given in (58).

We can also write $F(\epsilon)$ in a form similar to (49), which shows better its structure and also its relation with noncovariant formalism:

$$F(\epsilon) = 1-\alpha\eta[\cos\theta_1\cos\theta_2+\epsilon\beta\sin\theta_1\sin\theta_2\cos2\phi], \quad (64)$$

where

$$2\phi = 2\phi_a-\phi_1-\phi_2, \quad (65)$$

and the quantities θ_1, θ_2, ϕ_a, ϕ_1, and ϕ_2 are defined as follows:

$$\begin{aligned}
-\mathfrak{d}\cdot\mathfrak{n}^{(3)} &= \delta_3 = \cos\theta_1,\\
-\mathfrak{d}\cdot\mathfrak{n}^{(1)} &= \delta_1 = \sin\theta_1\cos\phi_1,\\
-\mathfrak{d}\cdot\mathfrak{n}^{(2)} &= \sin\theta_1\sin\phi_1,\\
-K_3\mathfrak{w}'\cdot\mathfrak{n}^{(3)} &= \lambda_3 = \cos\theta_2,\\
-K_3\mathfrak{w}'\cdot\mathfrak{n}^{(1)} &= \lambda_1 = \sin\theta_2\cos\phi_2,\\
-K_3\mathfrak{w}'\cdot\mathfrak{n}^{(2)} &= \sin\theta_2\sin\phi_2,\\
-\mathfrak{b}\cdot\mathfrak{n}^{(1)} &= \cos\phi_a, \quad -\mathfrak{b}\cdot\mathfrak{n}^{(2)} = \sin\phi_a.
\end{aligned} \quad (66)$$

Except for an arbitrary and immaterial parameter (origin of the azimuth around $\mathbf{k}$) for $\mathfrak{n}^{(1)}$, the vectors $\mathfrak{n}^{(1)}$, $\mathfrak{n}^{(2)}$, $\mathfrak{n}^{(3)}$, and $\mathfrak{n}'^{(3)}$ have been defined in paragraph 5.

The usual "nonrelativistic" treatment proceeds as follows: Let us suppose the photon with a plane polarization $\mathfrak{e}$ corresponding to the azimuth ϕ around $\mathbf{k}$, i.e. [see (26)]

$$\mathfrak{e} = (\mathfrak{n}^{(1)}\cos\phi+\mathfrak{n}^{(2)}\sin\phi) = \frac{1}{\sqrt{2}}(e^{-i\phi}\mathfrak{e}_++e^{i\phi}\mathfrak{e}_-) = \frac{1}{\sqrt{2}}\sum_\gamma e^{-i\gamma\phi}\mathfrak{e}_\gamma, \quad (67)$$

where $\gamma=\pm1$.

The S matrix between Σ^0 and Λ^0 polarization states is then [see (34)] S' such that

$$\lambda\rangle = S'_{\lambda\sigma}\sigma\rangle, \quad (68)$$

with

$$S'_{\lambda\sigma} = \sum_\gamma \frac{1}{\sqrt{2}}e^{i\gamma\phi}\langle\gamma S_{\gamma\lambda,\sigma}, \quad (69)$$

or

$$\sqrt{2}S' = \begin{pmatrix} 0 & -\epsilon e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix}$$

$$= \tau_1\cos\phi+\tau_2\sin\phi \quad \text{when} \quad \epsilon=-1$$

$$= -i\left[\tau_1\cos\left(\phi+\frac{\pi}{2}\right)+\tau_2\sin\left(\phi+\frac{\pi}{2}\right)\right] \quad \text{when} \quad \epsilon=1. \quad (70)$$

[17] M. M. May, Phys. Rev. 84, 265 (1951).

This can be written, up to a factor,

$$S'=(\boldsymbol{\tau}\cdot\mathbf{e}'),\quad \text{when}\quad \epsilon=-1,\quad \mathbf{e}'=\mathbf{e} \tag{71}$$

$$\text{when}\quad \epsilon=1,\quad \mathbf{e}'=\mathbf{k}\times\mathbf{e}/|\mathbf{k}|. \tag{72}$$

Generally, physicists postulate directly these forms of S' on parity conservation and "rotational invariance" grounds. It is very clear how this has to be interpreted.

Using ρ_Σ in (16), we can compute

$$\rho_\Lambda=S'\rho_\Sigma S'^\dagger=\tfrac{1}{2}S'(1+\eta\boldsymbol{\delta}\cdot\boldsymbol{\tau})S'^\dagger=\tfrac{1}{2}(1+\boldsymbol{\zeta}\cdot\boldsymbol{\tau}), \tag{73}$$

where

$$\boldsymbol{\zeta}=-\eta[\boldsymbol{\delta}-2\mathbf{e}'(\mathbf{e}'\cdot\boldsymbol{\delta})]. \tag{74}$$

In plain words, the Λ^0-polarization is obtained from Σ^0-polarization by a rotation of π around $\mathbf{e}'$. We obtain easily $F(\epsilon)$ as in (64) if we take $\mathrm{Tr}_\gamma R1/2(1+\boldsymbol{\gamma}\cdot\boldsymbol{\tau})\otimes A_\Lambda$ with $\gamma_1=\beta\cos 2\phi_a$, $\gamma_2=\beta\sin 2\phi_a$, $\gamma_3=0$. We have also explicitly displayed the relativistic meaning of the three-component Stokes vectors used in the so-called "non-relativistic" formalism.

10. Correlation Functions for $\Sigma^0\to\Lambda^0+\varepsilon^++\varepsilon^-$ Decay

Although they are rare, decays with a Dalitz pair are much more interesting from the experimental point of view. Indeed the angle between $\mathbf{p}'_+$ and $\mathbf{p}_-'$ is larger, on the average, than that for ordinary pairs, so the direction of the normal to the plane of the pair can be determined for Dalitz pairs while multiple scattering make this barely possible for ordinary pairs in a hydrogen bubble chamber. Also, as we shall see, the efficiency β of the Dalitz pair as a plane polarization analyzer is greater.

A refined theoretical treatment of the $\Sigma^0\to\Lambda^0+\epsilon^++\epsilon^-$ decay[18] would require the determination of the two independent form factors (for each value of ϵ) of the $\Sigma^0-\Lambda^0$ current. However, as is seen from Kroll and Wada's[19] study of the general problem of Dalitz pairs, the azimuthal distribution of the plane of the pair is not sensitive to the detailed structure of the form factors.

In this section we shall give the value of β as a function of $\mathfrak{p}_+'$ and $\mathfrak{p}_-'$. For this, we define (as in reference 19)

$$\mathfrak{x}m_e=\mathfrak{p}_+'+\mathfrak{p}', \tag{75}$$

so that $x^2=\mathfrak{x}^2$ is the square of the "virtual photon" mass (in electron mass units)

$$y=(E_-'-E_+')/|\mathbf{p}_+'+\mathbf{p}_-'|, \tag{76}$$

[in practice y is the energy partition $(E_-'-E_+')/(E_-'+E_+')$ of the pair].

Except for a slight modification of the constants K_1 and K_2 in Eq. (33) which defines $\mathfrak{n}_3$ and $\mathfrak{n}_3'$ (see Table I):

$$\begin{aligned}K_1'&=(M'^2+M^2-x^2)[\Delta(M',M,|\mathfrak{x}|)]^{-\frac{1}{2}},\\ K_2'&=2MM'[\Delta(M',M,x)]^{-\frac{1}{2}}.\end{aligned} \tag{77}$$

(This modification can be forgotten in the experimental analysis since $\langle x^2\rangle\ll M^2$.) Equations (57), (58), (64), (65), and (66) are valid for Dalitz pairs. In these equations, $\mathfrak{b}$ is the unit vector (its sign is irrelevant) orthogonal to the decay plane.

$$\mathfrak{b}\cdot\mathfrak{u}'=\mathfrak{b}\cdot\mathfrak{u}=\mathfrak{b}\cdot\mathfrak{p}_+'=\mathfrak{b}\cdot\mathfrak{p}_-'=0. \tag{78}$$

We define:

$$N=(M'-M)/(M'+M)=0.03,$$
$$X=(M'-M)/m_e=147,$$
$$Y=(1-4/x^2)^{\frac{1}{2}}.$$

Then if we do again the computations in reference 19, without integration over ϕ, the azimuth of the normal to the plane of the pair, the transition probability, up to a factor, for the $\Sigma^0\to\Lambda^0+\epsilon^++\epsilon^-$ decay is

$$\begin{aligned}C\propto\int_2^X dx\int_{-Y}^{Y}dy&[1-(x/X)^2(1+N^2)+N^2(x/X)^4]^{\frac{1}{2}}\\ &\times[1-2N^2(1+N^2)^{-1}(x/X)^2]x^{-1}\\ &\times\left\{R_T\left[\left(\frac{4}{x^2}+1+y^2\right)-\left(\frac{4}{x^2}-1+y^2\right)\cos 2\phi\right]\right.\\ &\left.+2R_L(1-y^2)(1+N)^2(x/X)^2[1+N(x/X)^2]^{-2}\right\},\end{aligned} \tag{79}$$

where R_T and R_L are defined in Eq. (7) of reference 19. While the computation of these quantities[18] is in progress, their dependence on x^2 does not greatly affect the value of β and the contribution of R_L is negligible compared to that of R_T. We shall neglect N compared to 1 ($N<1/30$) and the dependence of R_T on ϵ. Then (79) reads [in analogy with Eq. (49)]

$$C\propto\int_2^X dx\int_{-Y}^{Y}C(x,y)(1+\beta\cos 2\phi), \tag{80}$$

where

$$C(x,y)=[1-(x/X)^2]^{\frac{1}{2}}x^{-3}[4+x^2(1+y^2)], \tag{81}$$

and

$$\beta(x,y)=\frac{x^2(1-y^2)-4}{x^2(1+y^2)+4}. \tag{82}$$

This is the value of β to be used in Eq. (57), (58), or (64), (65) and (66).

Integration over y yields

$$\begin{aligned}C\propto\int_2^X dx&[1-(x/X)^2]^{\frac{1}{2}}(1-4/x^2)^{\frac{1}{2}}x^{-1}\\ &\times[1+(2/x^2)-\tfrac{1}{2}(1-4/x^2)\cos 2\phi].\end{aligned} \tag{83}$$

---

[18] This is being done by one of us (H.R.). This paper is part of a work submitted as his thesis to the University of Paris.

[19] N. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955).

If we define

$$\langle\beta^n\rangle=\int_2^X\int_{-Y}^Y dxdy\, C(x,y)\beta^n(x,y) \Big/ \int_2^X\int_{-Y}^Y dxdy\, C(x,y), \quad (84)$$

then we find $\langle\beta\rangle=0.44$ which is much better than that for an ordinary pair (see Fig. 1; the corresponding $\langle\beta\rangle$ is about -0.1). If each event is weighted according to its efficiency, the weighted average of β is then $\langle\beta^3\rangle/\langle\beta^2\rangle=0.8$ which is indeed a very promising value.

CONCLUSION

The Σ^0 is the only hyperon which does not decay via weak couplings. Theoretically, its decay can yield important information of several kinds. For instance, only the assumption of angular momentum conservation implies that the Σ^0 polarization η_0 and the asymmetry parameter α in the subsequent Λ^0 decay could be in principle determined. But only the product $\alpha\eta_0$ can be easily measured. However, this is an important item of information. Let us call α_+, α_+', α_-, α the asymmetry in the decays $\Sigma^+\rightarrow p^++\pi^0$, $\Sigma^+\rightarrow n+\pi^+$, $\Sigma^-\rightarrow n+\pi^-$, $\Lambda\rightarrow p^++\pi^-$, and η_+, η_0, η_-, η_Λ the hyperon polarization, at a given energy, in the reaction $\pi+N\rightarrow Y+K$, where Y is, respectively, Σ^+, Σ^0, Σ^-, Λ^0. The present experimental data[4–6] for the α's and η's are $\alpha_+\eta_+=0.7\pm0.3$, $\alpha_+'\eta_+\approx0$, $\alpha_-\eta_-\approx0$, $\alpha\eta_\Lambda\approx0.73\pm0.17$. The measurement of $\alpha\eta_0$ will yield the ratio η_0/η_Λ and also a lower limit of $|\eta_0|$, which, with that of $|\eta_+|$ and the values of the cross sections σ_+, σ_0, σ_-, for $\pi+p\rightarrow\Sigma+K$, will provide a test for charge independence. If, as is likely, isobaric spin is a good quantum number in Σ, K associated production, then the same data will predict a rather limited domain of values for η_- which can be used to prove that the observed absence of asymmetry in Σ^- decay is genuine and not due to a lack of polarization.

This conclusion seems already pointed out by the present experimental data. Indeed we show in the Appendix that $\eta_+\approx\eta_0\approx\eta_-$ for a one-Gev π^- beam; this will also yield, with the measurement of $\alpha\eta_0$, the relative sign of α_+/α. Furthermore, some experiments are in progress[4] to measure the sign of α (through the measurement of the polarization of protons from Λ^0 decays). The measurement of $\alpha\eta_0$, and the relation (A.13), will determine the sign of the η's and of α_+. Assuming, moreover, parity conservation in Σ^0 decay, we then showed the feasibility of the measurement of ϵ, the $\Sigma^0-\Lambda^0$ relative parity. Should parity be not conserved, then the parameter ϵ in our paper will satisfy $-1<\epsilon<1$ and be a measure of parity nonconservation.

ACKNOWLEDGMENTS

We are grateful to the French "Service des Poudres" for financial support. This work was completed while one of us (L.M.) was participating in the Summer Institute for Theoretical Physics at the University of Wisconsin, sponsored by the National Science Foundation. He wishes to thank Professor M. L. Good and Professor R. G. Sachs for very informative discussions.

APPENDIX

Polarization of the Σ^0 Produced in the Reaction $\pi^-+p^-\rightarrow\Sigma^0+K^0$

If the proton is unpolarized, one can conclude from P and T invariance that the Σ polarizetion is orthogonal to the plane of the reaction. We have represented the polarization by the pseudovector $\eta\mathfrak{d}$, where $\mathfrak{d}=(0,\boldsymbol{\delta}=(\mathbf{p}_\pi\times\mathbf{p}_\Sigma)/|\mathbf{p}_\pi\times\mathbf{p}_\Sigma|)$ and $-1\leqslant\eta\leqslant1$. The parameter η represents the sign and the degree of the polarization. Both are unknown. We shall first study in this Appendix what can be deduced for the value of η from the present experimental data on the reactions:

$$\pi^++p^+\rightarrow\Sigma^++K^+, \quad (A.1)$$

$$\pi^-+p^+\rightarrow\Sigma^0+K^0, \quad (A.2)$$

$$\pi^-+p^+\rightarrow\Sigma^-+K^+, \quad (A.3)$$

and from the hypothesis of charge independence.

Our argument will be based on the following lemma about triangular relations. Let a, b, and c be three positive numbers. The following relations are equivalent:

I. $a\leqslant b+c,\quad b\leqslant a+c,\quad c\leqslant a+b,$

II. $(a-b-c)(a-b+c)(a+b+c)\leqslant0,$

or

II′. $\Delta(a,b,c)=(a+b+c)(a-b+c)\times(a-b-c)(a+b-c)\leqslant0,$

III. $-2ab\leqslant a^2+b^2-c^2\leqslant2ab.$

Note that these relations also imply:

$$|a-b|\leqslant c,\quad |a-c|\leqslant b,\quad |b-c|\leqslant a.$$

These relations are called triangular relations. We note that $\Delta(a,b,c)\leqslant0$ when a, b, and c verify a triangular relation.

Lemma 1. If $\Delta(a_i,b_i,c_i)\leqslant0$ for n sets a_i, b_i, c_i, of positive numbers, then $\Delta((\sum_i a_i^2)^{\frac12}, (\sum_i b_i^2)^{\frac12}, (\sum_i c_i^2)^{\frac12})\leqslant0$.

For the proof, let us write relation III for each i and add them up; we obtain

$$-2\textstyle\sum_i a_ib_i\leqslant\sum_i(a_i^2+b_i^2-c_i^2)\leqslant2\sum_i a_ib_i. \quad (A.4)$$

On the other hand, the following relations are equivalent and always satisfied:

$$0\leqslant\sum_{i<j}(a_ib_j-a_jb_i)^2 \quad\text{or}\quad \sum_{i\neq j}a_ib_ia_jb_j\leqslant\sum_{i\neq j}a_i^2b_j^2,$$

or

$$\textstyle\sum_{ij}a_ib_ia_jb_j\leqslant\sum_{ij}a_i^2b_j^2,$$

or

$$\textstyle\sum_i a_ib_i\leqslant(\sum_i a_i^2)^{\frac12}(\sum_j b_j^2)^{\frac12}. \quad (A.5)$$

By transitivity, (A.4) and (A.5) yield

$$-2(\sum_i a_i^2)^{\frac{1}{2}}(\sum_j b_j^2)^{\frac{1}{2}} \leqslant \sum_i a_i^2+\sum_j b_j^2 -\sum_k c_k^2 \leqslant 2(\sum_i a_i^2)^{\frac{1}{2}}(\sum_j b_j^2)^{\frac{1}{2}}, \quad (A.6)$$

which proves the lemma.

Lemma 2. If

$$\Delta(a_i,b_i,c_i)\leqslant 0 \text{ and } (\sum_i c_i^2)^{\frac{1}{2}}=(\sum_j a_j^2)^{\frac{1}{2}}+(\sum_j b_j^2)^{\frac{1}{2}}, \quad (A.7)$$

then

$$c_i=a_i+b_i \quad \text{and} \quad a_i/a_j=b_i/b_j=c_i/c_j.$$

Proof: By squaring (A.7) we obtain the first equality in (A.6) and from (A.4), (A.5), and (A.6) we obtain

$$-2(\sum_i a_i^2)^{\frac{1}{2}}(\sum_j b_j^2)^{\frac{1}{2}}=-2\sum_i a_ib_i =\sum_i(a_i^2+b_i^2-c_i^2). \quad (A.8)$$

The first equality of (A.8) is equivalent to

$$\sum_{i<j}(a_ib_j-a_jb_i)^2=0 \quad \text{or} \quad \frac{a_i}{a_j}=\frac{b_i}{b_j}=\frac{a_i+b_i}{a_j+b_j}. \quad (A.9)$$

The second equality of (A.8) is equivalent to $\sum_i c_i^2 =\sum_i(a_i+b_i)^2$ which, combined with relation I for each value of i, yields $c_i=a_i+b_i$. This with (A.9) proves the lemma.

If one assumes charge independence, the amplitude for the three reactions (A.1), (A.2), (A.3) for given states of energy-momenta and polarizations satisfy the linear relation[3] $\sqrt{2}f_0=f_+-f_-$; hence a triangular relation $\Delta((2\sigma_0)^{\frac{1}{2}},(\sigma_+)^{\frac{1}{2}},(\sigma_-)^{\frac{1}{2}})$, where σ_α with $\alpha=+,\ 0,\ -$, are the corresponding cross sections.

Lemma 1 shows that this relation is also valid for cross sections after summation over the proton polarization and/or the Σ polarization. Let σ_α be the cross sections for unpolarized particles. They are only functions of the energy of the incoming beam (the target is at rest) and of the angle of production (angle between $\mathbf{p}_b$ and $\mathbf{p}_\Sigma$); for each value of these variables, they satisfy

$$\Delta((2\sigma_0)^{\frac{1}{2}},(\sigma_+)^{\frac{1}{2}},(\sigma_-)^{\frac{1}{2}})\leqslant 0. \quad (A.10)$$

The Σ polarizations η_+, η_0, η_- are functions of the same variables. The quantities $\sigma_\alpha(1\pm\eta_\alpha)$ (where $\alpha=+,\ 0,\ -$) are the cross sections for production of totally polarized Σ^α (with polarization $\pm\mathfrak{d}$); they satisfy

$$\Delta([2\sigma_0(1\pm\eta_0)]^{\frac{1}{2}},[\sigma_+(1\pm\eta_+)]^{\frac{1}{2}},[\sigma_-(1\pm\eta_-)]^{\frac{1}{2}}) \leqslant 0. \quad (A.11)$$

The quantities already measured (for a beam energy around 1.1 Bev) are σ_+, σ_0, σ_- and a lower limit for $|\eta_+|$, i.e., $|\eta_+|\geqslant 0.7\pm 0.3$ (see references 4, 5, 6). In these experimental data the relation

$$(2\sigma_0)^{\frac{1}{2}}\leqslant(\sigma_+)^{\frac{1}{2}}+(\sigma_-)^{\frac{1}{2}} \quad (A.12)$$

is barely satisfied and a simplifying hypothesis, suggested in reference 6, is that for all angles (or at least for a large range of θ, the angle of production in the center-of-mass system), the cross sections satisfy the equality in (A.12). Then Lemma 2 tells us that

$$\eta_+=\eta_0=\eta_-. \quad (A.13)$$

We will now outline the principle of the easiest method for obtaining some information on $|\eta_0|$. Indeed the K^0 meson produced in Eq. (A.2) is expected to be visible in only one third of the bubble-chamber pictures (one cannot see θ_2^0 or $\theta_1^0\to 2\pi^0$). Hence in most pictures one can measure only the energy-momentum $\mathfrak{p}_b$ of the beam, that of the target: $m\mathfrak{t}=(m,\mathbf{0})$, and that of the Λ^0: $M\mathfrak{u}$. We define a unit time vector $\mathfrak{u}''$ and a "mass" M'' by

$$\mathfrak{p}_b+m\mathfrak{t}=M''\mathfrak{u}''. \quad (A.14)$$

Energy-momentum conservation in Eq. (A.2) gives

$$M''\mathfrak{u}''=M'\mathfrak{u}'+\mu_K\mathfrak{u}_K, \quad (A.15)$$

where μ_K is the K^0 meson mass. We deduce

$$\mathfrak{u}''\cdot\mathfrak{u}'=(M''^2+M'^2-\mu_K^2)/2M'M''=(1+K^2)^{\frac{1}{2}}, \quad (A.16)$$

where $K=(M''-M'+\mu_K)(M''-M'-\mu_K)/2M'M''$.

Let $\mathfrak{u}''$, $\mathfrak{l}$, $\mathfrak{l}'$, $\mathfrak{l}''$ be a tetrad defined by

$$\mathfrak{l}=(0,\mathbf{l}) \quad \text{with} \quad \mathbf{l}=(\mathbf{p}_b\times\mathbf{u})/|\mathbf{p}_b\times\mathbf{u}|, \quad (A.17)$$

$$\mathfrak{l}'=(0,\mathbf{l}') \quad \text{with} \quad \mathbf{l}'=(\mathbf{p}_b\times\mathbf{l})/|\mathbf{p}_b|, \quad (A.18)$$

and let θ be the angle of production in the rest-system of Σ, and φ the azimuth of $\mathbf{p}'$ around $\mathbf{p}_b$; i.e.,

$$\mathfrak{u}'=(1+K^2)^{\frac{1}{2}}\mathfrak{u}''+K\sin\theta\cos\varphi\mathfrak{l} +K\sin\theta\sin\varphi\mathfrak{l}'+K\cos\theta\mathfrak{l}''$$

and

$$\mathfrak{d}=-\mathfrak{l}\sin\varphi+\mathfrak{l}'\cos\varphi.$$

When the Σ^0 is not observed, θ and φ are unknown, but they must satisfy the following relation:

$$\mathfrak{u}\cdot\mathfrak{u}'=K_1/K_2=\mathfrak{u}\cdot\mathfrak{u}''(1+K^2)^{\frac{1}{2}}+K\mathfrak{u}\cdot\mathfrak{l}\sin\theta\cos\varphi +K\mathfrak{u}\cdot\mathfrak{l}''\cos\theta, \quad (A.19)$$

due to the energy-momentum conservation in $\Sigma^0\to\Lambda^0+\gamma$ decay. Note that θ is an even function of φ. When the Σ^0 is observed, the Λ^0 polarization is given by Eq. (53):

$$\mathfrak{s}=-\eta K_2(\mathfrak{d}\cdot\mathfrak{u})\mathfrak{n}^{(3)}=-\eta(\theta)K_2(\mathfrak{d}\cdot\mathfrak{u})(-K_1\mathfrak{u}+K_2\mathfrak{u}') =\mathfrak{s}(\varphi). \quad (A.20)$$

If the Σ^0 is not observed, the Λ^0 polarization is then:

$$\mathfrak{s}=\langle\mathfrak{s}(\varphi)\rangle=\frac{1}{2\pi}\int^{2\pi}\mathfrak{s}(\varphi)d\varphi. \quad (A.21)$$

We leave to the reader to compute:

$$\langle\mathfrak{s}\rangle=-\mathfrak{l}'KK_2^2\mathfrak{u}\cdot\mathfrak{l}\frac{1}{2\pi}\int_0^{2\pi}\eta(\theta)\sin^2\varphi d\varphi. \quad (A.22)$$

From argument of invariance under P, T we could predict that $\langle \mathfrak{S} \rangle$ is transverse and orthogonal to the 3-plane $\mathfrak{k}$, $\mathfrak{p}_b$, $\mathfrak{p}_\Lambda$ as if the Λ^0 were produced directly by $\pi^- + p^+ \to \Lambda^0 + K^0$. [The proof of Σ^0 production is given by

$$(M''\mathfrak{u}'' - M\mathfrak{u})^2 = M''^2 + m^2 - 2MM''\mathfrak{u}\cdot\mathfrak{u}'' \neq \mu_K^2]. \quad \text{(A.23)}$$

The asymmetry in Λ^0 decay even when the Σ^0 is not observed is therefore a measure of the function,

$$\frac{1}{2\pi}\int_0^{2\pi} \eta(\theta) \sin^2\varphi d\varphi,$$

of the polarization η of the Σ^0.

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

π^- Scattering from Complex Nuclei*

R. M. Edelstein,† W. F. Baker,‡ and J. Rainwater
Columbia University, New York, New York

(Received November 29, 1960)

Differential cross sections were measured for π^--carbon scattering at 69.5 and 87.5 Mev and π^--oxygen scattering at 87.5 Mev from 20° to 125° extending the technique of Baker, Rainwater, and Williams. The energy resolution was sufficient to measure pure elastic as well as 5- and 10-Mev inelastic cross sections. The modified Kisslinger optical-model equation was used to fit the elastic-cross-section data. A χ^2 analysis for the 69.5-Mev carbon data gave a nuclear radius parameter $r_0 = 1.05 \pm 0.02$ fermis and a fall-off parameter $t = 1.16 \pm 0.07$ fermis. These parameters give good fits to the other data as well. An energy dependence in the strength parameters for carbon is observed in qualitative agreement with prediction.

I. INTRODUCTION

MEASUREMENTS have been made, using scintillation counters, of the angular distributions of π^- mesons scattered from carbon at 69.5 and 87.5 Mev and from oxygen at 87.5 Mev. The experimental work is an extension of that of Baker, Rainwater, and Williams[1] (BRW), in which the scattering of 80-Mev π^- mesons from Li, C, Al, and Cu was measured. In their experiment, scattered pion energy was determined from the range of pions stopped in a counter. This technique afforded considerable improvement in energy resolution over that obtained previously with counters[2–4] and cloud chambers.[5–7] The present experiment employed four such counters in succession, the "multicounter," to increase the data-taking rate. The energy resolution in either experiment was sufficient to separate out pure elastic scattering from all inelastic scattering for carbon and oxygen. In the case of lithium, BRW employ the electron scattering data[8] to argue that the contribution of scattering from the first excited state to the measured elastic scattering is small. No other levels contribute.

Recent experiments have been performed by Kane,[9] π^+ scattering from carbon at 31.5 Mev; and Fujii,[10] 150-Mev π^- scattering from C, Al, Cu, and Pb. Kane measured total pion energy by means of pulse height in a scintillation counter with an (absolute) energy resolution comparable to our own. Fujii measured quasi-elastic scattering into a 15-Mev interval by means of total energy determination in a Čerenkov counter but could not separate out pure elastic scattering.

Baker, Byfield, and Rainwater[11] (BBR) have fitted optical model calculations to the data of BRW. The optical potential used was a modification of the one of Kisslinger.[12] It removes a nonphysical divergence in the unmodified form. The potential includes a term in the gradient of the nuclear density which arises from the important p-wave contribution to the elemental π-nucleon scattering in the nucleus. Hence, the predictions are particularly sensitive to the nuclear edge thickness. The model gives good fits to the data at all angles and for nuclear radii consistent with the results

* This research is supported by the U. S. Atomic Energy Commission and the Office of Naval Research.

† Present address: Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

‡ Present address: Brookhaven National Laboratory, Upton, New York.

[1] W. F. Baker, J. Rainwater, and R. E. Williams, Phys. Rev. **112**, 1763 (1958).

[2] A. Pevsner, J. Rainwater, R. E. Williams, and S. J. Lindenbaum, Phys. Rev. **100**, 1419 (1955).

[3] R. E. Williams, J. Rainwater, and A. Pevsner, Phys. Rev. **101**, 412 (1956).

[4] R. E. Williams, W. F. Baker, and J. Rainwater, Phys. Rev. **104**, 1695 (1956).

[5] H. Byfield, J. O. Kessler, and L. M. Lederman, Phys. Rev. **86**, 17 (1952).

[6] J. O. Kessler and L. M. Lederman, Phys. Rev. **94**, 689 (1954).

[7] G. Saphir, Phys. Rev. **104**, 535 (1956).

[8] J. F. Streib, Phys. Rev. **100**, 1797 (1955).

[9] P. P. Kane, Phys. Rev. **112**, 1337 (1958).

[10] T. A. Fujii, Phys. Rev. **113**, 695 (1959).

[11] W. F. Baker, H. Byfield, and J. Rainwater, Phys. Rev. **112**, 1773 (1958). Equations (2), (13a), and (13b) are printed incorrectly in this paper. The sign of the right side of Eq. (2) should be reversed. The expressions for C and C' in (13a, b) should be divided by A. In the present paper $C \to C_1$ and $C' \to C_0$.

[12] L. S. Kisslinger, Phys. Rev. **98**, 761 (1955).