Some Considerations on Global Symmetry

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If the recently discovered Y^* state is related to the $T=\frac{3}{2}$, $J=\frac{3}{2}$ resonance in πp scattering, global symmetry considerations should become relevant. In this paper, global symmetry is discussed with a view to understanding its group structure. Also discussed is a possibility of reconciling the conflict, pointed out by Pais, between certain experimental results and global symmetry. The partial widths of the Y^* state are calculated and also those of the companion excited states Z^* and Ξ^*. A generalization of the quantum number G is discussed.

1. INTRODUCTION

RECENT experiments[1,2] have established the existence of an excited state $Y^{*\pm}$ in the $\Lambda+\pi^\pm$ system. The spin and parity of the state are not yet measured. As discussed in reference 1, the state shows certain resemblance to the $J=\frac{3}{2}$, $T=\frac{3}{2}$, p-state resonance N^* of the $p+\pi$ system, and the resemblance is reminiscent of the concept of global symmetry.

In this paper we proceed along this line of thinking and assume that indeed the $Y^*=\Lambda+\pi^\pm$ resonance is in the $J=\frac{3}{2}$, p state, and that the resonance is related to the $J=\frac{3}{2}$, $T=\frac{3}{2}$, p-state resonance N^* of the $p+\pi$ system by global symmetry. To analyze this relation it is necessary to know the quantum numbers of various states with respect to the global symmetry operations. It is therefore important to know the structure of the global symmetry group. Now global symmetry[3–6] means some symmetry, larger than isotopic spin invariance, that describes an approximate analogy between the various baryons. But in the literature its group property has not been fully discussed. We shall in this paper formulate in mathematical terms the requirements that global symmetry must satisfy. It appears that the simplest group $\mathcal{G}_0$ satisfying these requirements can be generated by three independent 2-dimensional unitary unimodular transformations together with a discrete transformation. Adopting this group as the global symmetry group we then try to assign quantum numbers to the various particles and the resonance states Y^*, N^*.

Certain approximate relations are then written down between the widths of Y^* and N^*, and also for the various partial widths of Y^*. Companion resonance states Z^* and Ξ^* are also discussed.

It has been pointed out by Pais[5] that any kind of global symmetry is in conflict with certain experimental facts. We suggest in Sec. 6 that if global symmetry is needed to understand the resonant state Y^*, a way to resolve Pais' conflict is to have the K mesons as a mixture of states which have different quantum numbers under the global symmetry transformations. The interactions between each of these states and other particles could still predominantly satisfy global symmetry. Such a picture, while not completely satisfactory, does offer a possible consistent scheme incorporating global symmetry that leads to useful experimental information.

It should be emphasized that much of our results about widths have already been discussed in the literature from the viewpoint of symmetry considerations.[7,8] Furthermore a detailed calculation using a specific dynamical model has been performed by Amati, Stanghellini, and Vitale[7] for the states Y^* and Z^*. Various discussions on the global symmetry group properties have also existed in the literature.[3–6] The present paper is written not in the spirit of presenting something entirely new, nor even in that of presenting something which we believe to be necessarily relevant[9] to physical facts. But if a similarity between Y^* and N^* exists, an analysis along the present line would be useful.

For completeness we include in Sec. 8 a discussion of charge conjugation invariance, together with a generalization of the quantum number G. Also included are some remarks in Sec. 9 concerning a global symmetry that does not put Ξ and nucleons in the same multiplet.

2. REQUIREMENTS ON THE GLOBAL SYMMETRY GROUP

The global symmetry group must by definition contain the isotopic spin group and the strangeness group [defined by the operators $\exp(iS\theta)$ or $\exp(i(S+N)\theta)$, where S=strangeness and N=baryon number; the strangeness group commutes with the isotopic spin group]. It has an 8×8 unitary representation to which the 8 baryons belong. [Cf. Sec. 9 for the case of a symmetry between N, Σ, and Λ only.] In order that the

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[9] See the discussion of R. H. Dalitz and S. Tuan, Phys. Rev. Letters **2**, 425 (1959), Ann. Phys. **10**, 307 (1960). See also M. Ross and G. Shaw, Phys. Rev. Letters **5**, 578 (1960).

8 baryons be analogous to each other under the global symmetry group, the representation must be irreducible. For the isotopic spin rotation subgroup the 8×8 representation breaks up into two doublets (N and Ξ), one triplet (Σ), and one singlet (Λ). For the strangeness subgroup this breakup must conform with the usual assignments of $S+N=+1, -1, 0, 0$ for N, Ξ, Σ, and Λ, respectively.

To state the above requirements explicitly we introduce as usual the states

$$\begin{aligned} Y^+&=\Sigma^+,\\ Y^0&=\frac{1}{\sqrt{2}}(\Sigma^0+\Lambda^0),\\ Z^0&=\frac{1}{\sqrt{2}}(\Sigma^0-\Lambda^0),\\ Z^-&=\Sigma^-, \end{aligned} \tag{1}$$

and write

$$N\equiv\left\|\begin{matrix}p\\n\end{matrix}\right\|,\quad Y\equiv\left\|\begin{matrix}Y^+\\Y^0\end{matrix}\right\|,\quad Z\equiv\left\|\begin{matrix}Z^0\\Z^-\end{matrix}\right\|,\quad \Xi\equiv\left\|\begin{matrix}\Xi^0\\\Xi^-\end{matrix}\right\|, \tag{2}$$

$$B\equiv\left\|\begin{matrix}N\\\Xi\\Y\\Z\end{matrix}\right\|\equiv\left\|\begin{matrix}p\\n\\\Xi^0\\\Xi^-\\Y^+\\Y^0\\Z^0\\Z^-\end{matrix}\right\|. \tag{3}$$

It is now useful to introduce (operating on the column matrix B) the following three sets of operators, each set satisfying the commutation relations for angular momenta:

$$(\mathfrak{L}_1,\mathfrak{L}_2,\mathfrak{L}_3):\quad \mathfrak{L}_i\equiv\tfrac{1}{2}\left\|\begin{matrix}\sigma_i&0&0&0\\0&\sigma_i&0&0\\0&0&\sigma_i&0\\0&0&0&\sigma_i\end{matrix}\right\|,\quad (i=1,2,3), \tag{4}$$

$$(\mathfrak{M}_1,\mathfrak{M}_2,\mathfrak{M}_3):\quad \mathfrak{M}_1\equiv\tfrac{1}{2}\left\|\begin{matrix}0&0&0&0\\0&0&0&0\\0&0&0&I\\0&0&I&0\end{matrix}\right\|,\quad \mathfrak{M}_2\equiv\tfrac{1}{2}i\left\|\begin{matrix}0&0&0&0\\0&0&0&0\\0&0&0&-I\\0&0&I&0\end{matrix}\right\|,\quad \mathfrak{M}_3\equiv\tfrac{1}{2}\left\|\begin{matrix}0&0&0&0\\0&0&0&0\\0&0&I&0\\0&0&0&-I\end{matrix}\right\|, \tag{5}$$

$$(\mathfrak{N}_1,\mathfrak{N}_2,\mathfrak{N}_3):\quad \mathfrak{N}_1\equiv\tfrac{1}{2}\left\|\begin{matrix}0&I&0&0\\I&0&0&0\\0&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \mathfrak{N}_2\equiv\tfrac{1}{2}i\left\|\begin{matrix}0&-I&0&0\\I&0&0&0\\0&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \mathfrak{N}_3\equiv\tfrac{1}{2}\left\|\begin{matrix}I&0&0&0\\0&-I&0&0\\0&0&0&0\\0&0&0&0\end{matrix}\right\|, \tag{6}$$

where

$$\sigma_1\equiv\left\|\begin{matrix}0&1\\1&0\end{matrix}\right\|,\quad \sigma_2\equiv\left\|\begin{matrix}0&-i\\i&0\end{matrix}\right\|,\quad \sigma_3\equiv\left\|\begin{matrix}1&0\\0&-1\end{matrix}\right\|,$$

and

$$I=\left\|\begin{matrix}1&0\\0&1\end{matrix}\right\|.$$

Clearly $\mathfrak{L}_i$, $\mathfrak{M}_j$, $\mathfrak{N}_k$ all commute for any i, j, k.

The physical observables, Q=charge, S=strangeness, N=baryon number, T_1, T_2, T_3=isotopic spin are related to $\mathfrak{L}$, $\mathfrak{M}$, and $\mathfrak{N}$ by

$$T_i=\mathfrak{L}_i+\mathfrak{M}_i, \tag{7}$$

$$\tfrac{1}{2}(S+N)=\mathfrak{N}_3, \tag{8}$$

$$Q=T_3+\mathfrak{N}_3=\mathfrak{L}_3+\mathfrak{M}_3+\mathfrak{N}_3. \tag{9}$$

The 8×8 representation must (i) contain $\exp(iT_1\theta_1)$, $\exp(iT_2\theta_2)$, $\exp(iT_3\theta_3)$, $\exp(i\mathfrak{N}_3\theta_4)$. In other words, T_1, T_2, T_3, $\mathfrak{N}_3$ are among the infinitesimal generators of the representation. Furthermore (ii) the 8×8 representation is irreducible.

Many possible 8×8 representations of groups can be found that satisfy (i) and (ii). Since we are not interested in unnecessary additional symmetries, we take the group to be *isomorphic* with the representation, and shall identify a group element with its 8×8 representation. The simplest such possibility, $\mathcal{G}_0$, is discussed in the following sections. Other possibilities are discussed in Appendix A.

3. THE GROUP $\mathcal{G}_0$

The group contains arbitrary $\mathfrak{L}$ transformations [i.e., $U_{\mathfrak{L}}\equiv\exp i(l_1\mathfrak{L}_1+l_2\mathfrak{L}_2+l_3\mathfrak{L}_3)$, where $l_1l_2l_3$ are real numbers], arbitrary $\mathfrak{M}$ transformations [$\exp i(m_1\mathfrak{M}_1+m_2\mathfrak{M}_2+m_3\mathfrak{M}_3)$, which mixes Y and Z], and arbitrary $\mathfrak{N}$ transformations [$\exp i(n_1\mathfrak{N}_1+n_2\mathfrak{N}_2+n_3\mathfrak{N}_3)$, which mixes N and Ξ]. The product of all these is reducible since no mixing of N, Ξ with Y, Z has been introduced. To effect such a mixing we introduce the discrete element[10]

$$R\equiv\left\|\begin{matrix}0&0&I&0\\0&0&0&I\\I&0&0&0\\0&I&0&0\end{matrix}\right\|, \tag{10}$$

[10] This discrete operator has been discussed by various authors. See reference 5.

TABLE I. The irreducible representations of the group $\mathcal{G}_0$. The quantity $\chi_{\mathcal{M}}(U)$ is the character of the 2×2 matrix U for the irreducible representations of dimension $2\mathcal{M}+1$.

Representation	Dimension	Character for elements (11)	Character for elements (12)
$(\mathcal{L}\mathcal{M}\mathcal{M}+1)$	$(2\mathcal{L}+1)(2\mathcal{M}+1)^2$	$\chi_{\mathcal{L}}(U_{\mathcal{L}})\chi_{\mathcal{M}}(U)\chi_{\mathcal{M}}(U')$	$\chi_{\mathcal{L}}(U_{\mathcal{L}})\chi_{\mathcal{M}}(UU')$
$(\mathcal{L}\mathcal{M}\mathcal{M}-1)$	$(2\mathcal{L}+1)(2\mathcal{M}+1)^2$	$\chi_{\mathcal{L}}(U_{\mathcal{L}})\chi_{\mathcal{M}}(U)\chi_{\mathcal{M}}(U')$	$-\chi_{\mathcal{L}}(U_{\mathcal{L}})\chi_{\mathcal{M}}(UU')$
$(\mathcal{L}\mathcal{M}\mathcal{N})$ $\mathcal{M}>\mathcal{N}$	$2(2\mathcal{L}+1)(2\mathcal{M}+1)(2\mathcal{N}+1)$	$\chi_{\mathcal{L}}(U_{\mathcal{L}})[\chi_{\mathcal{M}}(U)\chi_{\mathcal{N}}(U')+\chi_{\mathcal{M}}(U')\chi_{\mathcal{N}}(U)]$	0

and other necessary elements to form a group. The elements of the group are then the 8×8 matrices of the form

$$U_{\mathcal{L}}\left\|\begin{matrix} aI & bI & 0 & 0 \\ -b^*I & a^*I & 0 & 0 \\ 0 & 0 & a'I & b'I \\ 0 & 0 & -b'^*I & a'^*I \end{matrix}\right\|, \tag{11}$$

and those of the form

$$U_{\mathcal{L}}\left\|\begin{matrix} 0 & 0 & aI & bI \\ 0 & 0 & -b^*I & a^*I \\ a'I & b'I & 0 & 0 \\ -b'^*I & a'^*I & 0 & 0 \end{matrix}\right\|. \tag{12}$$

Here

$$\left\|\begin{matrix} a & b \\ -b^* & a^* \end{matrix}\right\| \equiv U = \text{arbitrary } 2\times 2 \text{ unimodular unitary matrix,}$$

and (13)

$$\left\|\begin{matrix} a' & b' \\ -b'^* & a'^* \end{matrix}\right\| \equiv U' = \text{arbitrary } 2\times 2 \text{ unimodular unitary matrix.}$$

This group will be called $\mathcal{G}_0$. It has an invariant subgroup (11) which is the direct product[11] of three SU_2, and the quotient of the group by this invariant subgroup is the two-element group. [It is, however, not the direct product of the two-element group with the invariant subgroup.]

In Appendix A we shall give a few other possible groups satisfying conditions (i) and (ii).

[11] We use SU_n to denote the group of unitary $n\times n$ matrices with determinant unity. Strictly speaking, the invariant subgroup (11) is only locally identical with $SU_2\times SU_2\times SU_2$. If one changes the signs of $U_{\mathcal{L}}$ and a, b, a', b' at the same time, (11) is unchanged. Hence (11) has a 1 to 2 homomorphism with $SU_2\times SU_2\times SU_2$. In other words, $SU_2\times SU_2\times SU_2$ is the covering group of (11). Similarly $\mathcal{G}_0$, which is defined to be the group of matrices of the form (11) and (12), has a covering group which we shall call $\mathcal{G}_0'$. $\mathcal{G}_0'$ has a 2 to 1 homomorphism with $\mathcal{G}_0$. It has an invariant subgroup $SU_2\times SU_2\times SU_2$. The relationship between $\mathcal{G}_0'$ and $\mathcal{G}_0$ is entirely similar to the familiar relationship between SU_2 and O_3, the group of 3×3 rotations. To simplify matters we shall not dwell on the difference between $\mathcal{G}_0$ and $\mathcal{G}_0'$ in the text. The irreducible representations in Table I are actually representations of $\mathcal{G}_0'$. Those representations $(\mathcal{L}\mathcal{M}\mathcal{M}\lambda)$ with $\mathcal{L}+2\mathcal{M}=$integer, and those representations $(\mathcal{L}\mathcal{M}\mathcal{N})$, $\mathcal{M}>\mathcal{N}$, with $\mathcal{L}+\mathcal{M}+\mathcal{N}=$integer are also representations of $\mathcal{G}_0$. (The others are not single-valued representations of $\mathcal{G}_0$.) Since the baryons belong to a representation of this type, and all known particles have transition elements to a collection of baryons and antibaryons, only representations of the same type (i.e., single-valued representations of $\mathcal{G}_0$) enter into the discussion of known particles.

4. IRREDUCIBLE REPRESENTATION OF $\mathcal{G}_0$

For any representation of $\mathcal{G}_0$, the infinitesimal operators $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$, $\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{N}_3$ form three sets of commuting angular momenta. One can diagonalize $\mathcal{L}^2$, $\mathcal{L}_3$, $\mathcal{M}^2$, $\mathcal{M}_3$, $\mathcal{N}^2$, $\mathcal{N}_3$ simultaneously. Now

$$\begin{aligned} R\mathcal{L}_iR^{-1}&=\mathcal{L}_i,\\ R\mathcal{M}_iR^{-1}&=\mathcal{N}_i,\\ R\mathcal{N}_iR^{-1}&=\mathcal{M}_i.\end{aligned} \tag{14}$$

Hence in any representation the set of eigenvalues of $\mathcal{M}^2$ must be the same as those of $\mathcal{N}^2$. One has therefore two kinds of irreducible representations:

(α) $\mathcal{L}^2$, $\mathcal{M}^2$, $\mathcal{N}^2$ have unique eigenvalues $\mathfrak{L}(\mathfrak{L}+1)$, $\mathfrak{M}(\mathfrak{M}+1)$, $\mathfrak{M}(\mathfrak{M}+1)$, respectively. Since R commutes with $\mathcal{M}_3+\mathcal{N}_3$, the state with $\mathcal{M}_3=\mathcal{N}_3=\mathfrak{M}$ is an eigenstate of R. The eigenvalue λ can be ± 1. We denote this representation by the symbol $(\mathfrak{L}\mathfrak{M}\mathfrak{M}\lambda)$, where $2\mathfrak{L}=\text{integer}\geqq 0$, $2\mathfrak{M}=\text{integer}\geqq 0$, $\lambda=\pm 1$.

The states of this representation are designated by $\mathcal{L}_3$, $\mathcal{M}_3$, $\mathcal{N}_3$, each running in integral steps between and including $\pm\mathfrak{L}$, $\pm\mathfrak{M}$, $\pm\mathfrak{M}$ respectively. The operator R switches the indices $\mathcal{M}_3$ and $\mathcal{N}_3$ for a state:

$$R|\mathcal{M}_3=a,\quad \mathcal{N}_3=b\rangle=\lambda|\mathcal{M}_3=b,\quad \mathcal{N}_3=a\rangle. \tag{15}$$

(β) $\mathcal{L}^2$ has a unique eigenvalue $\mathfrak{L}(\mathfrak{L}+1)$. $\mathcal{M}^2$ and $\mathcal{N}^2$ each has two eigenvalues $\mathfrak{M}(\mathfrak{M}+1)$ and $\mathfrak{N}(\mathfrak{N}+1)$ where $\mathfrak{M}\neq\mathfrak{N}$. We denote this representation by the symbol $(\mathfrak{L}\mathfrak{M}\mathfrak{N})$, where $2\mathfrak{L}$, $2\mathfrak{M}$, and $2\mathfrak{N}$ are integers $\geqq 0$ and $\mathfrak{M}>\mathfrak{N}$.

The states of this representation are states for which

$$\{\mathcal{M}^2=\mathfrak{M}(\mathfrak{M}+1),\quad \mathcal{M}_3=-\mathfrak{M},-\mathfrak{M}+1,\cdots+\mathfrak{M},$$
$$\text{while } \mathcal{N}^2=\mathfrak{N}(\mathfrak{N}+1),\qquad \mathcal{N}_3=-\mathfrak{N},-\mathfrak{N}+1,\cdots+\mathfrak{N}\};$$

and

$$\{\mathcal{M}^2=\mathfrak{N}(\mathfrak{N}+1),\qquad \mathcal{M}_3=-\mathfrak{N},-\mathfrak{N}+1,\cdots+\mathfrak{N},$$
$$\text{while } \mathcal{N}^2=\mathfrak{M}(\mathfrak{M}+1),\quad \mathcal{N}_3=-\mathfrak{M},-\mathfrak{M}+1,\cdots+\mathfrak{M}\}.$$

The operator R switches the states between these two sets. In a suitable representation, R satisfies

$$R|\,\mathfrak{M}\mathfrak{M}_3\mathfrak{N}\mathfrak{N}_3\rangle=|\,\mathfrak{N}\mathfrak{N}_3\mathfrak{M}\mathfrak{M}_3\rangle.$$

The dimensions of the irreducible representations are tabulated in Table I. Also given are the characters of the representations. From the characters the decomposition of the direct product of two representations can be easily

TABLE II. Quantum number assignments for particles and excited states. $T=\mathcal{L}+\mathfrak{M}$=isotopic spin. P_K=parity of K meson. The quantum number G_1 is explained in Sec. 8. In this table only the $(0,\frac{1}{2},\frac{1}{2},\lambda_K)$ part is listed for the K mesons.

Particle	Representation	$\mathcal{L}$	$\mathcal{L}_3$	$\mathfrak{M}$	$\mathfrak{M}_3$	$\mathfrak{N}$	$\mathfrak{N}_3$	G_1	T	R
p, n	$(\frac{1}{2},\frac{1}{2},0)$	$\frac{1}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	
Ξ^0, Ξ^-		$\frac{1}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$	
Y^+, Y^0		$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0		1, 0	
Z^0, Z^-		$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0		1, 0	
π^+	$(1,0,0,\lambda_\pi)$	1	1	0	0	0	0	-1	1	λ_π
π^0	$\lambda_\pi=\pm1$	1	0	0	0	0	0	-1	1	λ_π
π^-		1	-1	0	0	0	0	-1	1	λ_π
K^+	$(0,\frac{1}{2},\frac{1}{2},\lambda_K)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\lambda_K$	$\frac{1}{2}$	λ_K
K^0	$\lambda_K=\pm1$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\lambda_K$	$\frac{1}{2}$	$\{K_1^0: \lambda_K P_K$
$\bar{K}^0$		0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\lambda_K$	$\frac{1}{2}$	$K_2^0: -\lambda_K P_K$
K^-		0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\lambda_K$	$\frac{1}{2}$	λ_K
N^*	$(\frac{3}{2},\frac{1}{2},0)$	$\frac{3}{2}$	$\cdots$	0	0	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{3}{2}$	
Ξ^*		$\frac{3}{2}$	$\cdots$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{3}{2}$	
Y^*		$\frac{3}{2}$	$\cdots$	$\frac{1}{2}$	$\cdots$	0	0		1	
Z^*		$\frac{3}{2}$	$\cdots$	$\frac{1}{2}$	$\cdots$	0	0		2	

found in the standard way. (Except for the quantum number λ, it can also be found by the usual vector sum rule for $\mathcal{L}$, $\mathfrak{M}$, and $\mathfrak{N}$ separately.)

5. QUANTUM NUMBERS

To assign quantum numbers to the states we first notice that Eqs. (7)–(9) give the isotopic spin, the strangeness, and the charge in terms of these quantum numbers.

The 8 baryons clearly belong to the representation $(\frac{1}{2},\frac{1}{2},0)$. It seems natural that the pions should be assigned to the 3×3 representation $(1,0,0,\lambda_\pi)$. The two possibilities $\lambda_\pi=\pm1$ are, of course, physically different, and differentiable. It seems natural to assign the K mesons to the representation $(0,\frac{1}{2},\frac{1}{2},\lambda_K)$ with again the two possibilities $\lambda_K=\pm1$. These assignments are tabulated in Table II.

For the state $N^*=\pi+p$ we notice that $\pi+p$ always belongs to either $(\frac{3}{2},\frac{1}{2},0)$ $T=\frac{3}{2}$, or $(\frac{1}{2},\frac{1}{2},0)$ $T=\frac{1}{2}$. But N^* has a total $T=\frac{3}{2}$. Hence it belongs to $(\frac{3}{2},\frac{1}{2},0)$. The natural assumption is therefore that Y^* is in the same multiplet structure $(\frac{3}{2},\frac{1}{2},0)$, as indicated in Table II. Since Y^* can go into $\Lambda+\pi$, its isotopic spin $T=1$. The multiplet $(\frac{3}{2},\frac{1}{2},0)$ also contains a $T=2$ state[1,4,7] which will be called Z^*. In addition to the 3 Y^* states and 5 Z^* states there should also be 4 Ξ^* states with $T=\frac{3}{2}$.

6. BREAKDOWN OF GLOBAL SYMMETRY

Even if global symmetry has any valid basis, there must be relatively strong interactions that violate it. One manifestation of this violation lies in the mass difference between the hyperons. Another manifestation was first pointed out by Pais,[5] who showed that the following reactions

$$\pi^++p\rightarrow\Sigma^++K^+, \tag{16}$$

$$K^++n\rightarrow K^0+p, \tag{17}$$

and many others violate global symmetry. For the group $\mathcal{G}_0$ discussed above, the conservation of $\mathcal{L}_3$ is violated by both (16) and (17).

In face of these difficulties, does global symmetry have any validity at all? And if it has, does it ever produce useful physical information?

It would be difficult to answer these questions. But if the answers to the above questions are affirmative, presumably the baryons and the states N^*, Y^* allow more directly the application of global symmetry than reactions (16) and (17). For example, if the global-symmetry-destroying interactions produce relatively little mixing for the baryons, pions and Y^*, N^*, but produce large mixing for the K mesons, then apparent violation of global symmetry is not unnatural for (16) and (17). While the mixing may be the multiplet $(0,\frac{1}{2},\frac{1}{2})$ with any multiplet possessing a $T=\frac{1}{2}$, $\mathfrak{N}=\frac{1}{2}$ component, it seems that the mixing of $(0,\frac{1}{2},\frac{1}{2})$ with $(1,\frac{1}{2},\frac{1}{2})$ is the simplest possibility.

In this view, then, the usual interactions (baryons, pions, and K mesons) are regarded as predominantly globally symmetrical. The globally unsymmetrical interactions give rise to, among others, two effects: (a) mass splitting of the states within each multiplet. (b) a strong mixing for the K meson of a $(1,\frac{1}{2},\frac{1}{2})$ $T=\frac{1}{2}$ component with the $(0,\frac{1}{2},\frac{1}{2})$ state. The globally unsymmetrical interactions may, for example, have a very small range, so that the two effects (a) and (b) are the only ones that one need consider as causing global unsymmetry in the zeroth approximation. The influence of global unsymmetry is then quite limited in scope, though not in magnitude, and one can derive consequences that can be checked with experimental information. [This is true only insofar as one does not probe into the *very small* range where the strong unsymmetric force is assumed to be effective. It may be instructive to recall the well-known symmetry between $e^\pm$ and $\mu^\pm$. In that case, the asymmetry between these two particles

TABLE III. Phase-space factor Ω, projection weight w, and relative partial widths of resonance levels. The weights w are calculated from the quantum number assignments. The phase space factor Ω is computed from experimental resonance energies for N^* and Y^*, and from an assumed energy spectrum for Z^* and Ξ^*. The partial widths of other disintegration processes, such as $Z^{*+}\rightarrow\pi^+ +\Sigma^0$ etc., can be inferred from the table through a simple isotopic spin rotation.

Particle	Total energy (Mev)	Disintegration products	Ω (Mev^3)	w	Computed relative partial width
$(N^*)^{++}$	1237	π^++p	9.7×10^6	1	1
Y^{*+}	1385	$\pi^++\Lambda$	7.3×10^6	$\frac{2}{3}$	0.5
		$\Sigma^++\pi^0$	1.6×10^6	$\frac{1}{6}$	0.03
		$\Sigma^0+\pi^+$	1.6×10^6	$\frac{1}{6}$	0.03
Z^{*++}	$\sim$1539(?)	$\pi^++\Sigma^+$	$\sim18(?)\times10^6$	1	$\sim$1.9(?)
Ξ^{*+}	$\sim$1637(?)	$\pi^++\Xi^0$	$\sim15(?)\times10^6$	1	$\sim$1.5(?)

seems to be completely characterized by their large mass difference which, presumably, is also generated by some unsymmetrical forces, strong in magnitude but remarkably limited in its symmetry destroying effects.]

If one asks how does it happen that only the K particles have a strong mixing, a possible answer could be that without the globally unsymmetrical interaction two multiplets $(1,\frac{1}{2},\frac{1}{2})$ and $(0,\frac{1}{2},\frac{1}{2})$ happen to lie relatively close together and, therefore, result in large mixings for the states with $T=\frac{1}{2}$. It would then be reasonable to expect the existence of other excited states K^*.

In such a picture the K meson is a mixture of $(0,\frac{1}{2},\frac{1}{2})$ and $(1,\frac{1}{2},\frac{1}{2})$, each of which, in interacting with the other particles, still predominantly *satisfies* global symmetry. Thus, e.g., in reactions such as K^-+baryon with multiple pion productions, one can apply the global symmetry arguments and obtain equalities and inequalities between the various related processes.

7. POSITIONS AND WIDTHS OF N^*, Y^*, Z^*, AND Ξ^*

The discussion of Sec. 6 suggests a zeroth order calculation of the partial widths of $N^*Y^*Z^*$ and Ξ^*, for the processes tabulated in Table III. These processes represent a transition from a $(\frac{3}{2},\frac{1}{2},0)$ multiplet to a product of a $(\frac{1}{2},\frac{1}{2},0)$ multiplet and a $(1,0,0,\lambda_\pi)$ multiplet. The decomposition therefore yields unique weights w which are related to the squares of the appropriate transition amplitude. The calculation of these weights from the usual tables of Clebsch-Gordan coefficients is straightforward and the result is tabulated in Table III. Besides these weights due to the projection of the initial state on final states, there is also a phase-space-potential-barrier factor Ω for the p-wave state. We take it to be given by

$$\Omega=q^3E_B/(E_B+E_\pi), \tag{18}$$

where $q=$momentum of pion in the rest system of the resonance state, $E_B=$total energy of the final baryon, and $E_\pi=$total energy of the final pion. In the approximation that other effects due to global asymmetrical interactions are neglected, the partial widths of each resonance level are proportional to the appropriate products of w and Ω. To calculate Ω one needs the excitation energy of the resonance states. For N^* and Y^* we take the values in reference 1. To guess at the energies of Z^* and Ξ^* we write the total energy E of the excited state in the multiplet $(\frac{3}{2},\frac{1}{2},0)$ in the form

$$E(\equiv E_B+E_\pi)$$
$$=E_{N^*}+\alpha'\mathfrak{L}\cdot\mathfrak{M}+\beta'(\mathfrak{N}^2-\tfrac{3}{4})+\gamma'(\mathfrak{N}_3-\tfrac{1}{2}), \tag{19}$$

where α', β' and γ' are constants. Similarly, for the 8 baryons in the multiplet $(\frac{1}{2},\frac{1}{2},0)$ we have an analogous expression

$$E=E_N+\alpha\mathfrak{L}\cdot\mathfrak{M}+\beta(\mathfrak{N}^2-\tfrac{3}{4})+\gamma(\mathfrak{N}_3-\tfrac{1}{2}), \tag{20}$$

where

$$\alpha=E_\Sigma-E_\Lambda\cong77\text{ Mev},$$
$$\beta=\tfrac{4}{3}[\tfrac{1}{2}(E_\Xi+E_N)-\tfrac{1}{4}(3E_\Sigma+E_\Lambda)]\cong-59\text{ Mev}, \tag{21}$$
$$\gamma=-(E_\Xi-E_N)\cong-380\text{ Mev}.$$

One sees that by taking[12]

$$\alpha'=\alpha,\quad\beta'=\beta,\quad\gamma'\cong-400\text{ Mev}, \tag{22}$$

one obtains the experimental resonance energy for Y^*. With this choice the resonance energies for Z^* and Ξ^* can be computed and are tabulated in Table III, with the corresponding phase space factors Ω.

The last column of Table III shows a smaller total width for Y^* than N^* [in the ratio of approximately 0.56:1], and shows a very small branching ratio of $Y^*\rightarrow\Sigma+\pi$. Both of these are in general agreement with experimental information.[1]

8. CHARGE CONJUGATION INVARIANCE

With the inclusion of the unitary operator C, representing charge conjugation, the symmetry group is enlarged. The irreducible representations become larger in general, corresponding to, e.g., the fact that a particle and its antiparticle have the same mass. To study the combined group generated from $\mathcal{G}_0$, C and the baryon number gauge transformation $\exp(i\theta N)$, we start from their commutation relations. For the sake of clarity we shall formulate this discussion in theorems. We shall also only deal with particles and states that have transition matrix elements into n baryons and antibaryons, $n=1, 2, 3, \cdots$.

Theorem 1.

$$G_1\equiv C\exp[i\pi(\mathfrak{L}_2+\mathfrak{M}_2+\mathfrak{N}_2)] \tag{23}$$

commutes with all elements of the group $\mathcal{G}_0$.

Proof: For a single baryon-antibaryon the explicit representations of C, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, R are given in Appendix B. The theorem follows from a straightforward explicit

[12] A similar guess on the masses of excited levels has been made by A. Pais (private communication). See also reference 7.

computation of the commutators. For other states the theorem follows because G_1, C, $\exp(i\pi\mathfrak{L}_2)$, $\exp(i\pi\mathfrak{M}_2)$, and $\exp(i\pi\mathfrak{N}_2)$ are all multiplicative for a collection of particles.

Theorem 2.

$$G_1N+NG_1=0,\quad [N,\mathcal{G}_0]_-=0,\quad G_1^2=1. \tag{24}$$

Proof: This theorem follows directly from the explicit representation of Appendix B.

Theorem 3. The full group generated by $\mathcal{G}_0$, G_1 and $\exp(iN\theta)$ is the direct product group $\mathcal{G}_0\times O_2^\pm$, where $O_2^\pm$ is the group of all 2×2 real orthogonal matrices with determinant $=\pm1$.

The proof of this theorem is again straightforward.

The irreducible representations of $O_2^\pm$ are either (a) 2×2 in size, in which $N=+\alpha$ and $N=-\alpha$ [α=integer] each occurs once, representing physically a pair of particles, and G_1 switches the two states; or (b) the representation is of dimension 1×1 in which $N=0$ and either $G_1=+1$ or $G_1=-1$.

For a state with $N=0$, the operator G_1 is *one and the same numerical constant* $(=\pm1)$ *for all states in a multiplet of* $\mathcal{G}_0$. By (23), C brings one state into another in the same multiplet of $\mathcal{G}_0$.

Theorem 4. For the pions, $G_1=-1$.

Proof: The pions are eigenfunctions of $\mathfrak{N}$ with eigenvalue $\mathfrak{N}=0$. Hence[13] $G_1=G=-1$.

Theorem 5. For the $(0,\frac{1}{2},\frac{1}{2},\lambda)$, $N=0$ representation, if the total angular momentum $J=0$, and the system has transition matrix elements into a baryon-antibaryon pair, then

$$G_1\lambda=-1. \tag{25}$$

For the $(1,\frac{1}{2},\frac{1}{2},\lambda)$, $N=0$ representation under the same assumption,

$$G_1\lambda=1. \tag{26}$$

Proof: In the notation of Appendix B, we have four possible states for the baryon-antibaryon pair that belong to $(0,\frac{1}{2},\frac{1}{2})$, with $\mathfrak{M}_3=\frac{1}{2}$, $\mathfrak{N}_3=\frac{1}{2}$:

$$\{p\bar{Z}^0\}+\{n\bar{Z}^+\}, \tag{27}$$

$$\{\bar{Z}^0p\}+\{\bar{Z}^+n\}, \tag{28}$$

$$\{Y^+\bar{\Xi}^0\}+\{Y^0\bar{\Xi}^+\}, \tag{29}$$

and

$$\{\bar{\Xi}^0Y^+\}+\{\bar{\Xi}^+Y^0\}, \tag{30}$$

where each curly bracket represents a state, for which the first symbol inside specifies the state of particle a and the second symbol that of particle b. Consider a [c number] product wave function of an orbital part, a spin part, and a charge part [depending on the other quantum numbers, $\mathfrak{L}$, $\mathfrak{L}_3$, $\mathfrak{M}$, $\mathfrak{M}_3$, etc.] of the two particles. For a state $J=0$, the product of the first two parts is antisymmetry in the interchange $a\leftrightarrow b$. Hence the charge part is symmetric since the entire wave function must be antisymmetric. Now under $a\leftrightarrow b$,

$$(27)\leftrightarrow(28),\quad (29)\leftrightarrow(30).$$

Hence the states are either

$$(27)+(28)\quad\text{or}\quad(29)+(30), \tag{31}$$

or superpositions of these two. By using the explicit matrices listed in Appendix B it can be directly verified that under

$$R:\quad (27)\leftrightarrow(29),\qquad (28)\leftrightarrow(30),$$
$$G_1:\quad (27)\leftrightarrow-(30),\quad (28)\leftrightarrow-(29).$$

Hence under RG_1 both wave functions in (31) remain themselves but change sign. Thus (25) is proved. A similar proof holds for (26).

Applied to the K mesons, if the K meson admixture $(1,\frac{1}{2},\frac{1}{2})$ can be neglected, Theorem 5 states that

$$(G_1)_K=-\lambda_K.$$

While all the four states $\mathfrak{M}_3=\pm\frac{1}{2}$ and $\mathfrak{N}_3=\pm\frac{1}{2}$ are eigenstates of G_1 with eigenvalues $-\lambda_K$, only two: $\mathfrak{M}_3=\mathfrak{N}_3=\pm\frac{1}{2}$ are eigenstates of R with eigenvalues λ_K.

If, further, one assumes that time reversal invariance holds, then the two states K_1^0 and K_2^0 have simple behaviors under R. To see this, we notice that $(G_1)_K$ is a numerical constant. Hence (23) shows that

$$C|\mathfrak{M}_2=\tfrac{1}{2},\,\mathfrak{N}_2=-\tfrac{1}{2}\rangle=-G_1|\mathfrak{M}_2=-\tfrac{1}{2},\,\mathfrak{N}_2=\tfrac{1}{2}\rangle.$$

If P is the parity operator, we have

$$CP|\mathfrak{M}_2=\tfrac{1}{2},\,\mathfrak{N}_2=-\tfrac{1}{2}\rangle=-G_1P|\mathfrak{M}_2=-\tfrac{1}{2},\,\mathfrak{N}_2=\tfrac{1}{2}\rangle.$$

Now K_1^0, (K_2^0) is an eigenstate of CP with eigenvalue $+1$, (-1). Hence

$$|K_{1,2}^0\rangle=|\mathfrak{M}_2=\tfrac{1}{2},\,\mathfrak{N}_2=-\tfrac{1}{2}\rangle\mp(G_1P)|\mathfrak{M}_2=-\tfrac{1}{2},\,\mathfrak{N}_2=\tfrac{1}{2}\rangle.$$

One obtains with the use of (15):

$$R|K_{1,2}^0\rangle=\mp(G_1P)|K_{1,2}^0\rangle=\pm(\lambda_KP_K)|K_{1,2}^0\rangle,$$

where P_K is the parity of K^0 [with respect to, say, $\bar{\Lambda}n$]. Thus we obtain the entries in Table II for K_1^0, K_2^0 under R.

9. REMARKS

For a symmetry to exist between Y^* and N^* it is no necessary that all 8 baryons be brought into globa symmetry. For example, one could have a symmetry between N, Y, and Z without Ξ. A simple symmetry group in such a case[11] is $SU_2\times SU_3$ which has more parameters than $\mathcal{G}_0$. The irreducible representations in such a case can be written down and an analysis like the above for $\mathcal{G}_0$ can be made. There also would be a companion Z^* $T=2$ state together with N^* and Y^*. The weight factors w for N^*, Y^*, Z^* remain the same as in Table III.

APPENDIX A

We give here several examples other than $\mathcal{G}_0$ satisfying conditions (i) and (ii) of Sec. 2.

(A) The group $\mathcal{G}_1$. The group is isomorphic with $SU_2\times SU_4$ where SU_2 consists of the $\mathfrak{L}$ transformations

[13] T. D. Lee and C. N. Yang, Nuovo cimento 3, 749 (1956).

$U_{\mathfrak{L}}=\exp i(l_1\mathfrak{L}_1+l_2\mathfrak{L}_2+l_3\mathfrak{L}_3)$ and SU_4 consists of all the unimodular unitary transformations between N, Y, Z, Ξ. Clearly R is an element of SU_4. Hence $\mathcal{G}_0$ is a subgroup of $\mathcal{G}_1$. The usual $g_1=g_2=g_3=g_4$ case[4,5] [where g_1, g_2, g_3, g_4 are, respectively, the coupling constants between pion and $\bar{N}N$, $\bar{Y}Y$, $\bar{Z}Z$, $\bar{\Xi}\Xi$] has this larger symmetry $\mathcal{G}_1$ rather than $\mathcal{G}_0$. Notice that if the coupling constants $g_1=-g_2=-g_3=g_4$, the symmetry group is the smaller $\mathcal{G}_0$, with the pion assignment $(1,0,0,\lambda_\pi)$ where $\lambda_\pi=-1$.

(B) The group O_7'. The group is generated by the invariant subgroup $SU_2\times SU_2\times SU_2$ of $\mathcal{G}_0$, together with the elements

$$\exp(i\theta)\begin{Vmatrix}0 & 0 & \sigma_1 & 0\\ 0 & 0 & 0 & \sigma_1\\ \sigma_1 & 0 & 0 & 0\\ 0 & \sigma_1 & 0 & 0\end{Vmatrix}. \tag{32}$$

To find the group we introduce the matrices

$$\rho_1=\begin{Vmatrix}0 & 0 & I & 0\\ 0 & 0 & 0 & I\\ I & 0 & 0 & 0\\ 0 & I & 0 & 0\end{Vmatrix},\quad \rho_2=i\begin{Vmatrix}0 & 0 & -I & 0\\ 0 & 0 & 0 & -I\\ I & 0 & 0 & 0\\ 0 & I & 0 & 0\end{Vmatrix},\quad \rho_3=\begin{Vmatrix}I & 0 & 0 & 0\\ 0 & I & 0 & 0\\ 0 & 0 & -I & 0\\ 0 & 0 & 0 & -I\end{Vmatrix},$$

$$\tau_1=\begin{Vmatrix}0 & I & 0 & 0\\ I & 0 & 0 & 0\\ 0 & 0 & 0 & I\\ 0 & 0 & I & 0\end{Vmatrix},\quad \tau_2=i\begin{Vmatrix}0 & -I & 0 & 0\\ I & 0 & 0 & 0\\ 0 & 0 & 0 & -I\\ 0 & 0 & I & 0\end{Vmatrix},\quad \tau_3=\begin{Vmatrix}I & 0 & 0 & 0\\ 0 & -I & 0 & 0\\ 0 & 0 & I & 0\\ 0 & 0 & 0 & -I\end{Vmatrix}.$$

The infinitesimal generators of the invariant subgroup $SU_2\times SU_2\times SU_2$ of $\mathcal{G}_0$ are

$$\sigma_1,\ \sigma_2,\ \sigma_3,\quad (1+\rho_3)\tau_1,\quad (1+\rho_3)\tau_2,\quad (1+\rho_3)\tau_3,$$
$$(1-\rho_3)\tau_1,\quad (1-\rho_3)\tau_2,\quad (1-\rho_3)\tau_3, \tag{33}$$

or

$$\sigma_i,\ \tau_i,\ \rho_3\tau_i.$$

The infinitesimal generator for (32) is

$$\rho_1\sigma_1.$$

Taking the commutator of this generator with those listed in (33) one obtains additional generators. Altogether by repeatedly taking commutators one obtains the following 21 infinitesimal generators:

$$\sigma_i,\ \tau_i,\ \rho_1\sigma_i,\ \rho_3\tau_i,\ \rho_2\sigma_i\tau_j. \tag{34}$$

Taking further commutators gives rise to no new independent generators. The group obtained from these 21 generators is O_7' which has a 2-to-1 homomorphism with the group of 7×7 proper real orthogonal matrices O_7, as already discussed by various authors[6,3] in the literature. To see this we define seven anticommuting Hermitian matrices

$$\gamma_1=\rho_3\sigma_1,\quad \gamma_2=\rho_3\sigma_2,\quad \gamma_3=\rho_3\sigma_3,\quad \gamma_4=\rho_1\tau_1,$$
$$\gamma_5=\rho_1\tau_2,\quad \gamma_6=\rho_1\tau_3,\quad \gamma_7=\rho_2. \tag{35}$$

Then

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}, \tag{36}$$

$$\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6\gamma_7=-i. \tag{37}$$

The 21 infinitesimal generators (34) are then $i\gamma_\mu\gamma_\nu$ $(\mu\neq\nu)$. The group generated by (34) is therefore of the form

$$\exp(\sum a_{\mu\nu}\gamma_\mu\gamma_\nu),$$

where $a_{\mu\nu}$ are real numbers. Now

$$[\exp(\sum a_{\mu\nu}\gamma_\mu\gamma_\nu)]\gamma_i[\exp(-\sum a_{\mu\nu}\gamma_\mu\gamma_\nu)]=\sum_{j=1}^{7} b_{ij}\gamma_j, \tag{38}$$

where $\|b_{ij}\|$ is a 7×7 real orthogonal matrix with determinant $=1$. It is easy to prove that, conversely, for every such $\|b_{ij}\|$, there exist two sets of real $a_{\mu\nu}$'s satisfying (38). The group has thus a 2-to-1 homomorphism with O_7. [T. A. Tarski has pointed out to us that O_7' is called spin (7) in the standard language.]

In terms of the γ's, the element R of $\mathcal{G}_0$ is

$$R=\rho_1=-i\gamma_4\gamma_5\gamma_6=\gamma_1\gamma_2\gamma_3\gamma_7$$
$$=[\exp(\pi\gamma_1\gamma_2/2)][\exp(\pi\gamma_7\gamma_3/2)]. \tag{39}$$

Thus R is an element of O_7', hence $\mathcal{G}_0$ is a subgroup of O_7'.

Both of the above two groups contain $\mathcal{G}_0$ as a subgroup. There exist also groups that satisfy conditions (i) and (ii) but do not contain $\mathcal{G}_0$ as a subgroup.

(C) The group SU_3. It was pointed out to us by Speiser and Tarski[14] that the group[11] SU_3 has an irreducible 8×8 representation which satisfies both conditions (i) and (ii). However, in this case it seems impossible to incorporate π mesons and K mesons without introducing more new bosons. It is clear that $\mathcal{G}_0$ is not a subgroup of SU_3. The full implications and consequences of such possibilities still need to be investigated.

APPENDIX B

We give in this Appendix explicit matrices for G_1, R, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, and N between the 16 states that describe a

[14] D. R. Speiser and J. A. Tarski (private communication).

single baryon or antibaryon:

$$\begin{Vmatrix} B \\ B' \end{Vmatrix},$$

where

$$B=\begin{Vmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \\ Y^+ \\ Y^0 \\ Z^0 \\ Z^- \end{Vmatrix} \quad \text{and} \quad B'=\begin{Vmatrix} \bar{\Xi}^+ \\ -\bar{\Xi}^0 \\ -\bar{n} \\ \bar{p} \\ \bar{Z}^+ \\ -\bar{Z}^0 \\ -\bar{Y}^0 \\ \bar{Y}^- \end{Vmatrix}.$$

The antibaryon states[15] are defined such that under the charge conjugation operation all baryon states p, n, Ξ^0, Ξ^-, Y^+, Y^0, Z^0, Z^- are transformed in an *identical* way into their respective antibaryon states $\bar{p}$, $\bar{n}$, $\bar{\Xi}^0$, $\bar{\Xi}^+$, $\bar{Y}^-$, $\bar{Y}^0$, $\bar{Z}^0$, and $\bar{Z}^+$. The minus signs in B' are so chosen that the matrices G_1, R, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, and N are given by

$$G_1=\begin{Vmatrix} & & & & I & 0 & 0 & 0 \\ & & & & 0 & I & 0 & 0 \\ & & & & 0 & 0 & I & 0 \\ & & & & 0 & 0 & 0 & I \\ I & 0 & 0 & 0 & & & & \\ 0 & I & 0 & 0 & & & & \\ 0 & 0 & I & 0 & & & & \\ 0 & 0 & 0 & I & & & & \end{Vmatrix},$$

$$R=\begin{Vmatrix} 0 & 0 & I & 0 & & & & \\ 0 & 0 & 0 & I & & & & \\ I & 0 & 0 & 0 & & & & \\ 0 & I & 0 & 0 & & & & \\ & & & & 0 & 0 & I & 0 \\ & & & & 0 & 0 & 0 & I \\ & & & & I & 0 & 0 & 0 \\ & & & & 0 & I & 0 & 0 \end{Vmatrix},$$

$$N=\begin{Vmatrix} I & 0 & 0 & 0 & & & & \\ 0 & I & 0 & 0 & & & & \\ 0 & 0 & I & 0 & & & & \\ 0 & 0 & 0 & I & & & & \\ & & & & -I & 0 & 0 & 0 \\ & & & & 0 & -I & 0 & 0 \\ & & & & 0 & 0 & -I & 0 \\ & & & & 0 & 0 & 0 & -I \end{Vmatrix},$$

$$\mathfrak{L}_i=\tfrac{1}{2}\begin{Vmatrix} \sigma_i & 0 & 0 & 0 & & & & \\ 0 & \sigma_i & 0 & 0 & & & & \\ 0 & 0 & \sigma_i & 0 & & & & \\ 0 & 0 & 0 & \sigma_i & & & & \\ & & & & \sigma_i & 0 & 0 & 0 \\ & & & & 0 & \sigma_i & 0 & 0 \\ & & & & 0 & 0 & \sigma_i & 0 \\ & & & & 0 & 0 & 0 & \sigma_i \end{Vmatrix}, \quad (i=1,2,3)$$

$$\mathfrak{M}_1=\tfrac{1}{2}\begin{Vmatrix} 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & I & & & & \\ 0 & 0 & I & 0 & & & & \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & I \\ & & & & 0 & 0 & I & 0 \end{Vmatrix},$$

$$\mathfrak{M}_2=\tfrac{1}{2}i\begin{Vmatrix} 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & -I & & & & \\ 0 & 0 & I & 0 & & & & \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & -I \\ & & & & 0 & 0 & I & 0 \end{Vmatrix},$$

$$\mathfrak{M}_3=\tfrac{1}{2}\begin{Vmatrix} 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & I & 0 & & & & \\ 0 & 0 & 0 & -I & & & & \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & I & 0 \\ & & & & 0 & 0 & 0 & -I \end{Vmatrix},$$

$$\mathfrak{N}_1=\tfrac{1}{2}\begin{Vmatrix} 0 & I & 0 & 0 & & & & \\ I & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ & & & & 0 & I & 0 & 0 \\ & & & & I & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \end{Vmatrix},$$

$$\mathfrak{N}_2=\tfrac{1}{2}i\begin{Vmatrix} 0 & -I & 0 & 0 & & & & \\ I & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ & & & & 0 & -I & 0 & 0 \\ & & & & I & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \end{Vmatrix},$$

and

$$\mathfrak{N}_3=\tfrac{1}{2}\begin{Vmatrix} I & 0 & 0 & 0 & & & & \\ 0 & -I & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & & \\ & & & & I & 0 & 0 & 0 \\ & & & & 0 & -I & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \end{Vmatrix},$$

where $\sigma_i (i=1,2,3)$ are the 2×2 Pauli spin matrices and I is the 2×2 unit matrix. All empty places in the above matrices are zeroes.

[15] We use the notation that, e.g., $\bar{Y}^-$ is the antiparticle of Y^+ and is negatively charged.