Theory of Conservation of Isotopic Spin from $d+d$ Reactions

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The theoretical cross section for the reaction $d+d \rightarrow He^4+\pi^0$ has been calculated in the impulse approximation. It is assumed that the matrix element contains no isospin dependence which, due to conservation laws, would ordinarily prohibit pion production. The comparison of the theoretical and experimental cross sections for the reaction yields an upper limit of 6.5% for the amount of isospin nonconservation in strong interactions. A discussion of the effects of different He^4 wave functions in the calculation is also included.

A TEST of the validity of the conservation of isotopic spin in strong interactions can be obtained by measuring the cross section of a reaction in which the isospin quantum number changes. Such an experiment, involving the reaction $d+d\rightarrow He^4+\pi^0$, has been described in the previous article.[1] The measured cross section for $(d+d\rightarrow He^4+X)=\sigma^{\exp}$ can be related to the theoretical cross sections for the production of particles X whose mass≈pion mass,[1] in terms of the change in isospin number I, from initial to final state by

$$\sigma^{\exp}(d+d\rightarrow He^4+X)=P_{\Delta I=0}\sigma(d+d\rightarrow He^4+\pi_0{}^0)+P_{\Delta I\neq 0}\sigma(d+d\rightarrow He^4+\pi^0), \quad (1)$$

where $\pi_0{}^0$ represents a possible isosinglet meson, and π^0 is the ordinary neutral pion. In Eq. (1), we have explicitly factored the isospin dependence of the cross sections, i.e., $P_{\Delta I=0}$ and $P_{\Delta I\neq 0}$ are the probabilities that isospin is conserved and not conserved, respectively. If we calculate $\sigma(d+d\rightarrow He^4+\pi^0)=\sigma^T$, and assume it gives the only contribution to the $I\neq 0$ cross sections, then we may write $\sigma^{\exp}/\sigma^T \geqslant P_{\Delta I\neq 0}$, which sets an upper limit for $P_{\Delta I\neq 0}$, the probability for nonconservation of isospin. The purpose of this note then, is to outline the calculation of the theoretical cross section for the reaction if isospin were not involved in the matrix element.

The calculation of the differential cross section σ^T in the framework of the impulse approximation follows the treatment of Ruderman[2] and Bludman[3] for the production of pions in $p+d$ reactions. In our case, the $d+d$ reaction is related to the experimental cross sections, $\sigma(p+d\rightarrow He^3+\pi^0)$ and $\sigma(n+d\rightarrow H^3+\pi^0)$, which we assume here to be equal. The differential cross section is

$$\sigma(d+d\rightarrow He^4+\pi^0)=4(\tfrac{1}{9})v_{dd}^{-1}[E_{He^4}/(E_{He^4}+E_\pi)]\times\left|\frac{f(\theta)g(\theta)+f(\pi-\theta)g(\pi-\theta)}{2}\right|^2\left[3v_{pd}\frac{E_{He^3}+E_\pi}{E_{He^3}}\right], \quad (2)$$

where $|g(\theta)|^2=\sigma(p+d\rightarrow He^3+\pi^0)$, and $f(\theta)$ is defined in Eq. (3). The term in brackets, $g(\theta)$, and $g(\pi-\theta)$ are to be evaluated at the energy corresponding to the production of a meson of the same momentum as observed in the $d+d$ reaction. Aside from the factor of $\frac{1}{9}$ due to spin statistics, our expression (2) differs from that of Ruderman and Bludman in two respects. First, we must use an average of the angular functions at θ and $\pi-\theta$ since the process is symmetric in the $d-d$ center-of-mass system (either deuteron may be considered as the incident particle). Second, since the term in brackets should be spin independent, a factor of 3 appears in order to compensate for the spin-statistical factor of $\frac{1}{3}$ implicit in the observed $p+d$ and $n+d$ cross sections. [In the previous calculations[2,3] of $\sigma(p+d\rightarrow H^3+\pi^+)$ from $\sigma(p+p\rightarrow d+\pi^+)$, a factor of $\frac{4}{3}$ should have been included in a similar way.]

The angle-dependent term $f(\theta)$ is defined as

$$f(\theta)=N\int d\mathbf{x}\exp(-i\mathbf{\Delta}\cdot\mathbf{x})\chi_\alpha(\mathbf{x})\varphi_d(\mathbf{x}), \quad (3)$$

where $\chi_\alpha(\mathbf{x})$ is an appropriate normalized single-particle wave function for the spectator particle in He^4, $\varphi_d(\mathbf{x})$ is the deuteron wave function, and we define the momentum transfer: $\mathbf{\Delta}=\frac{1}{2}\mathbf{k}-\frac{1}{4}\mathbf{q}$; $\mathbf{k}$ and $\mathbf{q}$ being the momenta of the incident deuteron and outgoing pion, respectively. The "sticking factor" N is just the overlap integral of the three (interacting) particles of He^4 with the ground state of He^3. If we neglect the rather weak angular dependence of N, then it is just a number, $N\leqslant 1$, to be determined below. In the calculations that follow, we assume $N\chi_\alpha$, and therefore, $f(\theta)$ to be equal for both the $n+d$ and $p+d$ processes.

The calculation of Eq. (2) then depends on the choice of single-particle wave functions $\varphi_d(\mathbf{x})$ and $\chi_\alpha(\mathbf{x})$ for the deuteron and alpha particle, respectively. Regarding the choice of φ_d, it has been shown[3] that a hard-core modification of the usual Hulthén deuteron function, $(e^{-\eta x}-e^{-\xi x})/x$, significantly alters the amplitude calculated with Eq. (3). However, due to the equal im-

[1] J. A. Poirier and M. Pripstein, preceding paper [Phys. Rev. **122**, 1917 (1961)]. Other $d-d$ experiments have been reported by N. Booth, O. Chamberlain, and E. Rogers, Bull. Am. Phys. Soc. **4**, 446 (1959), and K. Akimov, O. V. Savchenko, and L. M. Soroko, *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester*, (Interscience Publishers, Inc., New York, 1960), p. 49. No analysis of these last data could be made, since one cannot obtain the differential cross section from their information. Their conclusions for the degree of isotopic spin conservation represent only a rough estimate, due to the absence of a theoretical number for the π^0 production cross section.

[2] M. A. Ruderman, Phys. Rev. **87**, 383 (1952).

[3] S. A. Bludman, Phys. Rev. **94**, 1722 (1954).

portance of $\chi_\alpha(\mathbf{x})$ and $\varphi_d(\mathbf{x})$, it seems impractical to further improve on the deuteron wave function in this way until one has a He^4 function whose accuracy is on the same order as the Hulthén expression. It is generally accepted that no single-particle He^3 (or He^4) wave functions which satisfy this requirement exist at present.

We have investigated this problem and will give the details in a subsequent paper. It suffices here to mention that for the very light nuclei, we merely solve an appropriate single-particle wave equation in the same approximation that yields the Hulthén function in the deuteron problem. For this reason, we believe the result to be about as accurate as the Hulthén expression. The effective single-particle potential required in the equation is obtained from a Hartree-type approximation with Yukawa two-body forces. The wave function we obtain has the form:

$$\chi_\alpha(x)=(e^{-\beta x}-e^{-\gamma x})/x+A(e^{-\beta x}-e^{-\delta x})/x+Be^{-\delta x}, \quad (4)$$

where β is related to the separation energy, E, by $\beta=[(A-1)/A]^{\frac{1}{2}}(2ME)^{\frac{1}{2}}/\hbar$, and A, B, γ, δ (γ, $\delta>\beta$) are fixed parameters whose values are given by the same variational procedure that produces the Hulthén function. It can be seen that Eq. (4) has the correct asymptotic form for large x, and yet is quite smooth as $x\rightarrow 0$, similar to the usual symmetric wave functions for very light nuclei.[4] But the use of this wave function in Eq. (3) yields results significantly different from those obtained with either the symmetric-type or asymptotic functions. As a check, we have also applied this function to the $p+d\rightarrow H^3+\pi^+$ reaction[3] and found the experimental results can be fit by a smaller, more reasonable value for the hard-core radius ($\lesssim 0.50$ fermi rather than 0.70 fermi) in the deuteron Hulthén function. Thus, the hard-core explanation of that experiment still appears quite valid, and we have incorporated its effect in our calculation for the $d+d$ reaction.

The product of the sticking factor N and the single-particle momentum distribution for He^4 has been obtained experimentally.[5] By comparing the Fourier transform of Eq. (4) with these results, we can obtain not only a value for N, but also find how well our single-particle function agrees with experiment. From such a comparison, we conclude that N is close to unity, and more important, that the wave function given by Eq. (4) represents the single-particle behavior of He^4 quite well. Therefore, we may proceed with the calculation of Eq. (2) with the confidence that our results will be reasonably accurate.

For this calculation, we have used $\beta=0.830$, $\gamma=3.05$, $\delta=7.25$, and $B=0.395$ in units of $(\text{fermi})^{-1}$, and $A=0.277$, $N=1$. Using the Hulthén deuteron wave function with $\eta=0.232$ $(\text{fermi})^{-1}$ and $\xi=6\eta$, we find at 90° in the center-of-mass, $\sigma^T(d+d\rightarrow He^4+\pi^0)=0.080\sigma(p+d\rightarrow He^3+\pi^0)$, and extrapolating the $p+d$ data[6] (including experimental errors) to a pion energy of 101 Mev, we obtain $\sigma(d+d\rightarrow\pi^0+He^4)=0.038\pm0.005$ μb/sr. Comparing this number with the cross section reported in the preceding article, we conclude that within two standard deviations

$$P_{\Delta I\neq 0}\leqslant 6.5\%.$$

A similar analysis using the data of Booth *et al.*[1] yields $P_{\Delta I\neq 0}\leqslant 8.7\%$. The larger errors in this value occur for two reasons: (1) Since the experiment was carried out at zero degrees, the functions $g(\theta)$ and $g(\pi-\theta)$ of Eq. (2) are not simply related to $\sigma(p+d\rightarrow He^3+\pi^0)$, and (2) the experimental uncertainties[6] for $\sigma(p+d\rightarrow He^3+\pi^0)$ are largest for small angles. Both of these uncertainties are minimized in the work of Poirier and Pripstein at 90°.

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[4] H. Fröhlich, K. Huang, and I. N. Sneddon, Proc. Roy. Soc. (London) **A191**, 61 (1947).

[5] W. Selove, Phys. Rev. **101**, 231 (1956), and W. Selove and J. M. Teem, *ibid.* **112**, 1658 (1958).

[6] W. J. Frank, K. C. Bandtel, R. Madey, and B. J. Moyer, Phys. Rev. **94**, 1716 (1954).