Investigation of Bremsstrahlung and Pair Production at Energies $>10^{11}$ ev*

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Pair production and bremsstrahlung at energies >100 Bev is investigated in nuclear emulsions by studying 91 primary electron-positron pairs starting high-energy cascades and the first secondary pair. The average energy of the showers is 320 Bev. The experimental results on the total number and the energy spectrum of photons radiated by electrons $\gtrsim 100$ Bev show a lack of soft photons, which is in disagreement with the Bethe-Heitler theory. The experiment agrees well with the new theory given by Landau, Pomeranchuk, Migdal, and Ter-Mikaeljan. A method for obtaining the mean free path L for direct pair production is presented which avoids the use of a correction for spurious tridents. A value $L=12_{-4}^{+7}$ cm is obtained for an average electron energy of about 160 Bev. The conversion length of photons of average energy 320 Bev in nuclear emulsion is 34 ± 5 mm, in agreement with the theoretical value of 37 mm. The distribution of the separation between the electron and the positron of the original high-energy pair is also in agreement with the theoretical distribution. This indicates that no appreciable discrepancy can exist between experiment and the theoretical cross section for the energy partition between an electron and a positron, and the probability of large energy losses by radiation. Several high-energy showers presumably produced by μ mesons and one possible case of a double pair production are described.

1. INTRODUCTION

ELECTROMAGNETIC interactions of electrons and photons with energy $>10^{11}$ ev can at present only be studied in the cosmic radiation. In the past few years a considerable amount of work has been carried out on this problem with the help of stacks of nuclear emulsion exposed to the cosmic radiation at balloon altitudes.[1–12] The approach most frequently used consists in analyzing electron-photon cascades initiated by a single high-energy photon or electron incident on the emulsion stack from the outside. Many aspects of the development of electron-photon cascades can be studied in detail and can be compared with cascade theory. However, the interpretation of data obtained in this way meets with several difficulties. One of these is the well-known problem of dealing with the large fluctuations in the cascade development. Furthermore, the information obtained about the fundamental cross sections for bremsstrahlung and pair production are of an indirect nature. This is especially true for the behavior of the shower at greater depths, which is determined by the cross section at energies much lower than the primary energy, therefore yielding no information about the interesting energy region above the capability of present day accelerators.

The large number of electromagnetic showers collected in this laboratory in the past few years makes possible a more direct approach to this problem. This approach is designed to give direct information on the cross section for bremsstrahlung and pair production above 10^{11} ev. This is done by confining the analysis to the interaction of the original electron-positron pair, which is produced by the conversion of a γ ray of high-energy incident on the stack from outside. The electron-positron pair constitutes a line source of bremsstrahlung γ rays. The distribution of the distance between the primary pair origin and the first bremsstrahlung pair, and the energy of the first bremsstrahlung pair (BSP) are directly related to the cross section for bremsstrahlung of the original high-energy electron-positron pair.

The conversion length of the primary photons can be obtained from the distribution of the pair origins starting showers inside the stack. The partition of the total energy between the two members of the original pair can be studied by relative scattering measurements.

2. EXPERIMENTAL PROCEDURE

Table I lists the stacks of nuclear emulsion used for this investigation. These stacks were scanned for showers of parallel minimum tracks along lines 7.5 cm and 15 cm apart. The showers were traced back to their origins. In this way 87 events starting with high-energy electron-positron pairs were located. They were not apparently connected with any high-energy nuclear interaction inside the stack. This means that in an area of about 500 μ radius around the pair, no parallel minimum tracks were seen. In addition, four electromagnetic showers were

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* Supported in part by a joint program of the U. S. Atomic Energy Commission and the Office of Naval Research, and by the National Science Foundation.

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TABLE I. Stacks used in this investigation

Stack	Dimensions (cm)	Exposure (hrs)	Exposure	Altitude (ft)
G	40×20×6	7.5	Guam—1957	108 000
M	15×15×12	15	Minnesota—1958	120 000
T	30×60×12	13	Texas—1958	116 000
S	15×20×18	9	Sioux Falls—1959	>140 000

used which started from a γ ray produced inside the stack. As will be explained in Sec. 9, these showers can be regarded as purely electromagnetic showers for the purpose of this investigation. For the investigation of the energy spectrum of the first BSP only, nine showers initiated by a single electron entering the stack from the outside were included.

In order to minimize any bias resulting from finding and selecting the showers, and to make reliable measurements possible, the following criteria were imposed:

1. Total length l_s of the shower in the stack, starting from the pair origin, $l_s > 7$ cm.
2. Total potential length l_p of the shower in the stack, $l_p > 10$ cm. l_p is the sum of l_s and the distance l traveled by the primary photon in the stack.
3. Primary energy E_0 of the shower >70 Bev.
4. Dip angle $<45°$.

The number of showers given above is already the one accepted under these criteria.

It is an important advantage of this type of investigation that it is not necessary to determine the primary energy E_0 of the showers with precision, since the cross sections depend very little on primary energy. E_0 was determined from the lateral distribution of the shower tracks according to the method developed by Pinkau.[13] With the help of the three-dimensional shower theory of Nishimura and Kamata,[14] a reliable estimate of the primary energy can be obtained. Since the measurements are made at depths exceeding several cascade units, the result depends very little on the high-energy cross sections. For this investigation the values of E_0 were taken from the work of reference 15 on the γ-ray energy spectrum. A more detailed description of the method will be given in this reference.

A careful scan was made along the tracks of the primary electron-positron pair up to the point when the first BSP started. The scan was conducted three times using different magnifications in a cylinder of about 50-μ radius. If the separation between the two pair tracks increased, the scanning volume was increased accordingly. The lateral separation between the first BSP and the nearest member of the original pair, projected into the plane of the emulsion, is shown in Fig. 1. Assuming cylindrical symmetry, the actual distribution in space can be calculated from Fig. 1. This approach is preferable to measuring the distribution in space directly, since the projected separation can be measured much more accurately. The distribution is very narrow, since the first BSP occurs characteristically about 1 cm from the pair origin. No pairs were found at distances $>8\ \mu$. It seems, therefore, very unlikely that any pairs lay outside the scanning volume and were missed. The energy of the first BSP was measured by multiple scattering. This was possible in almost all cases if the BSP energy was ≤ 3 Bev. An accurate and reliable measurement of noise and elimination of spurious scattering and distortions was possible because of the presence of the high-energy tracks of the original pair which served as reference tracks. The influence of energy loss by bremsstrahlung on the multiple scattering was taken into account in a statistical way. It constitutes a very small correction. For higher BSP energies, the further development of the cascade frequently prevents identification of the tracks over a sufficiently long distance. In this case upper and lower limits were established for the energy. A lower limit was established from the failure to observe multiple scattering in the available track length and from the opening angle of the pair. It is easy to see that Borsellino's formula[16,17] for the opening angle of pairs can be used only up to distances of several hundred μ to obtain an estimate of the pair energy. Then the influence of multiple scattering becomes predominant, irrespective of the pair energy.[18] If an observation of the pair separation can be made only after a much longer distance, one will generally obtain only a lower limit for the true

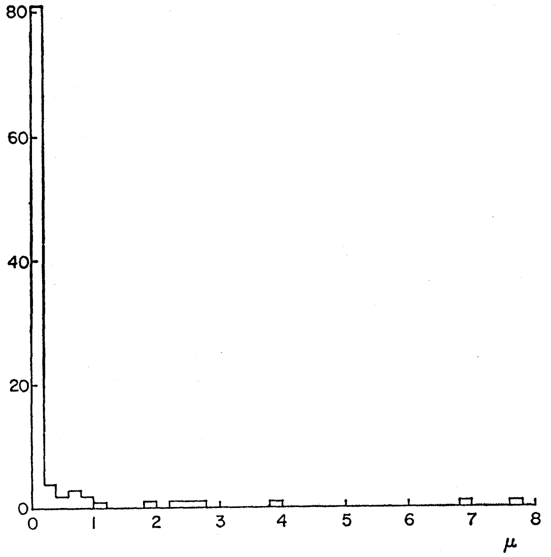

FIG. 1. Distribution of the projected lateral separation between the first secondary pair and the nearest member of the original pair. Ordinate: number of events in 0.2-μ interval.

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energy. At distances exceeding several thousand μ the influence of the original divergence of the pair can be neglected and the observed spread attributed to multiple scattering only. From this one obtains an upper limit for the primary energy by noting that for a given electron energy the probability of a deflection exceeding the expected rms value by a factor of 4 is very small (under the conditions of this experiment $<2\%$). The rms value of the projected displacement y of a track of energy E from its original direction is given (Sec. 8) by an equation of the form:

$$\langle y^2\rangle = k^2 Z^3 E^{-2}, \tag{1}$$

where Z is the distance traversed by the track and k is related to the scattering constant. If the separation y_0 is observed, we obtain from this an upper limit for the energy:

$$E < 4kZ^{\frac{3}{2}}/y_0. \tag{2}$$

If E is expressed in Bev, and y_0 and Z both in μ, $k \approx 4.3 \times 10^{-5}$ Bev $\mu^{-\frac{1}{2}}$.

3. THEORETICAL REMARKS

The Bethe-Heitler (BH) cross-sections[19] for bremsstrahlung and pair production are not valid at very high energies in condensed media. This has been first pointed out by Landau and Pomeranchuk.[20] Multiple scattering of the radiating electron produces a reduction of the probability of emitting low-energy photons, compared with the BH theory.[21]

For example, in nuclear emulsion, for an electron energy of 10^{12} ev, the reduction becomes noticeable for photon energies $\lesssim 10^9$ ev. Detailed expressions for the cross sections for bremsstrahlung and pair production, taking into account this effect, have been calculated by Migdal.[22] For very low photon energies, the deviation of the dielectric constant from unity leads to a further reduction of the probability for emission of soft quanta.[23] In the following the expressions for bremsstrahlung and pair production, according to Landau, Pomeranchuk, Migdal, and Ter-Mikaeljan will be referred to as LPM cross sections.

LPM cross sections for bremsstrahlung in nuclear emulsion have been plotted in the work of reference 24. For pair production in emulsion there is no noticeable deviation from the BH expressions for photon energies $<10^{13}$ ev. Above 10^{13} ev, the conversion length increases, i.e., the photons become more penetrating with increasing energy. Studies of several electromagnetic showers of $>10^{11}$ ev in nuclear emulsions have indicated that the beginning of the cascade development can be better described by using the LPM rather than the BH cross sections.[1,6,8,10] Fowler, Perkins, and Pinkau[9] were able to show that the total number of photons emitted by electrons of energy $>10^{12}$ ev is smaller than given by the BH theory, and is in agreement with the LPM expressions.

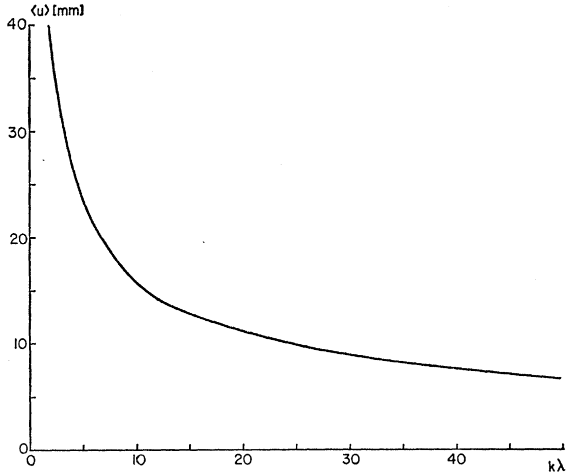

FIG. 2. Average longitudinal distance $\langle u\rangle$ from the origin of the original pair to the origin of the first secondary pair as a function of the effective number $k\lambda$ of photons radiated per conversion length.

4. TOTAL NUMBER OF RADIATED PHOTONS

The total number of photons of energy $>2m_ec^2$ radiated per unit path length by a high-energy electron will be denoted by k'. The probability that the first BSP does not materialize along a particular track within a longitudinal distance u from the origin of the primary pair is[9]

$$P(>u) = \exp\{-k'[u-\lambda+\lambda\exp(-u/\lambda)]\}; \tag{3}$$

λ is the conversion length, 37 mm in emulsion.

The average distance $\langle u\rangle$, where the first BSP materializes, is given by

$$\langle u\rangle = \lambda e^b b^{-b}(b-1)!\,I(b^{\frac{1}{2}}, b-1), \tag{4}$$

where $b = k'\lambda$ and $I(v,p)$ is the incomplete Γ function in the notation of Pearson.[25] $\langle u\rangle$ is plotted as a function of $k\lambda$ in Fig. 2.

An approximate expression for $\langle u\rangle$ for the values of interest here is

$$\langle u\rangle \approx 1.07(\pi/2)^{\frac{1}{2}}(\lambda/k')^{\frac{1}{2}}. \tag{5}$$

Below about 1 Bev the conversion length λ increases with decreasing energy. This variation with energy can be taken into account with the help of Eq. (5) which shows that $\langle u\rangle$ depends only on the combination λ/k'. A

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TABLE II. List of showers starting with a pair.

Group	No. of showers	Primary energy E (Bev)	Average energy $\langle E\rangle$ (Bev)
I	46	70 to 250	150
II	45	250 to 2000	500
I and II	91	70 to 2000	320
III	19	400 to 2000	700

good approximation can be obtained by defining an "effective number" k of photons assuming a constant conversion length λ and compensating this by introducing a cutoff in the bremsstrahlung spectrum at 25 Mev.[9] The "effective number" of photons k defined in this way will produce the correct value of $\langle u\rangle$ if inserted in Eq. (4) instead of k' together with a value $\lambda=37$ mm. For the BH cross sections an approximate expression for an electron energy $E\gg 1$ Bev is

$$\lambda k = 1.45\ \ln(E/25\text{ Mev}). \tag{6}$$

The numerical factor includes a 2% correction for the average energy loss of the electron by bremsstrahlung. More accurate values of λk for the BH and LPM cross section are plotted in Fig. 3. In the case of an electron-positron pair, one has to average over the energy distribution of the pair members. The value of $k\lambda$ for this case is also shown in Fig. 3. A good approximation is to calculate $k\lambda$ using an electron-positron pair with the value 3.5 of the ratio of the two energies.[9]

For comparison with the theory, the showers were divided into groups, as shown in Table II. E is the total energy of the primary pair.

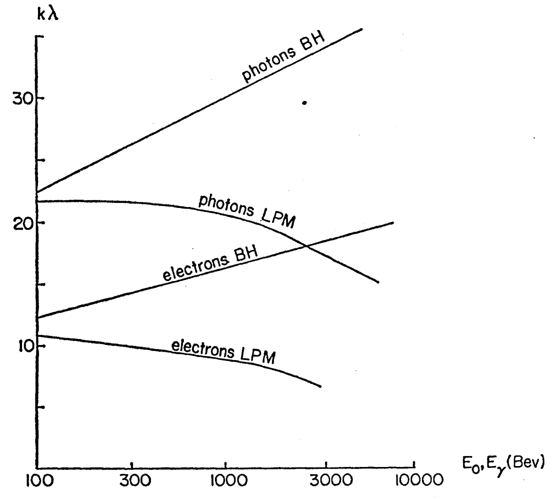

FIG. 3. Effective number $k\lambda$ of photons radiated per conversion length by: (a) an electron-positron pair (curve: "photons," total energy E_γ); (b) a single electron (curve: "electrons," energy E_0) according to the BH and LPM theory.

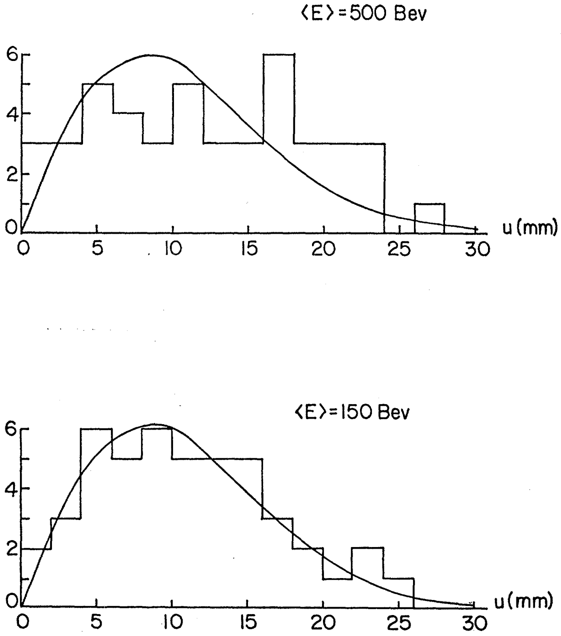

FIG. 4. Distribution of u for showers of group II (average energy $E=500$ Bev) and group I (average energy $E=150$ Bev). Smooth curve: theoretical distribution according to Eq. (3) and LPM theory.

The comparison between the experimentally found value of $\langle u\rangle$ and the expected value according to the BH and the LPM theory is shown in Table III for the three groups. Also included are the results of the Bristol group[9] from a very similar experiment. There is good agreement between the experimental results of both laboratories and the predictions of the LPM theory, and a significant disagreement with the BH theory for the high energy groups II and III.

Our experimental value of $\langle u\rangle$, as given in Table III, includes two corrections. One correction (−3.6%) was applied to account for the path traveled by the shower in the air gap between emulsions. It was determined experimentally by observing the skip of very flat showers. The other correction (+4.8%) was applied to account for direct pair production. This will be described in Sec. 6.

The distribution of u is shown for groups I and II in Fig. 4. It agrees with the theoretically expected distribution from Eq. (3) and the LPM theory, which is also included in Fig. 4.

TABLE III. Average distance $\langle u\rangle$ of first secondary pair.

Group	Average energy (Bev)	$\langle u\rangle$ experiment (mm)	$\langle u\rangle$ from BH theory (mm)	$\langle u\rangle$ from LPM theory (mm)	$\langle u\rangle$ from ref. 9 (mm)
I	150	11.0±1.0	10.0	10.8	10.0
II	500	12.2±1.0	9.3	11.0	
III	700	13.1±1.7	9.0	11.0	
Ref. 9	2000		8.6	11.5	13.5

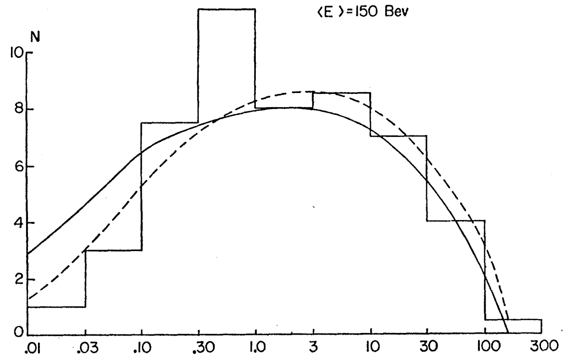

FIG. 5. Distribution of the energy E_k of the first secondary pair for showers of group I. Solid curve: distribution expected according to BH theory. Dashed curve: distribution expected according to LPM theory.

5. ENERGY DISTRIBUTION OF BREMSSTRAHLUNG PAIRS

In the preceding section it has been shown that the total number of radiated photons is smaller than predicted by the BH theory. It remains to be shown that this is produced by the lack of BSP of low energy. Figures 5 and 6 show the energy distribution of the first BSP for groups I and II of showers described in Table II. Five showers of group I and four showers of group II, starting with a single electron from outside, are included in Figs. 5 and 6. Also drawn in are the energy spectra expected from the BH and the LPM theory normalized to the same number of events. In calculating the theoretical curves, the known variation of λ with energy was taken into account and an integration over the energy distribution of the primary electron-positron pair was carried out.

Even for the low-energy group the lack of pairs <1 Bev compared to the BH cross section can be seen. For the high-energy group the observed number of pairs <3 Bev is 16, whereas 27 would be expected from the BH theory. Below 0.3 Bev the numbers are 4 observed vs 14 expected. The experimental distribution is in agreement with the LPM theory.

At energies <2 Bev accurate scattering measurements were possible on all events but two. The corresponding number for energies >3 Bev is 7. Above 3 Bev frequently only upper and lower limits for the pair energy could be established as described in Sec. 2. Characteristically the ratio of upper to lower limit is a factor of 10. The experiment is therefore only able to represent the gross behavior of the spectrum above 3 Bev. Measurements which are sensitive to the behavior of the spectrum at energies comparable to the primary electron energy will be described in Sec. 8. All measurements are consistent with the LPM theory.

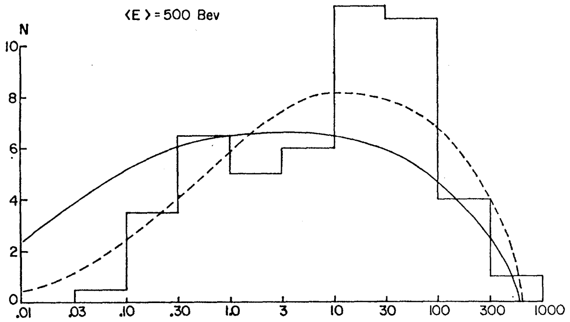

FIG. 6. Distribution of the energy E_k of the first secondary pair for showers of group II. Solid curve: distribution expected according to BH theory. Dashed curve: distribution expected according to LPM theory.

6. DIRECT PAIR PRODUCTION BY ELECTRONS

Straightforward measurements[2,26–33] of the mean free path L for the direct production of an electron-positron pair by a high-energy ($\gtrsim 10$ Bev) electron in emulsion meet with a difficulty which is demonstrated by Fig. 1. It shows that a large fraction (in this experiment $\approx 80\%$) of the BSP have a projected separation $\leq 0.2\ \mu$ from the original electron track and thus cannot be distinguished from genuine cases of direct pair production. A correction for accidental BSP, as introduced by Kaplon and Koshiba,[2] is therefore generally made. If the fraction of accidental BSP is large, the result depends sensitively on this correction. Apart from increasing the statistical error of the result appreciably, the correction requires also an exact knowledge of the primary electron energy which is in many cases not available. This is probably the reason why contradictory results have been reached by different authors.

If a large number of showers is available, it is possible to make a measurement of the direct pair production mean free path L by a method which avoids these difficulties and does not require any precise energy measurements.[34]

The differential distribution of the distance u between the primary pair origin and the first BSP is according to Eq. (3) for $u \ll \langle u \rangle$:

$$P'(u)du = (uk/\lambda)du, \tag{7}$$

i.e., $P'(u)$ is proportional to u. This is true in the absence of direct pair production. The distribution of the pairs originating from direct pair production only is given by

$$P''(u)du = du/L, \tag{8}$$

i.e., $P''(u)$ is independent of u.

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The actual experimental distribution is $[P'(u)+P''(u)]du$ for $u \ll \langle u \rangle$. L can therefore be found by plotting the differential distribution of u and finding its value at $u=0$ by extrapolation. This plot is shown in Fig. 7. In order to increase the statistics, the distribution of this paper was combined with the one of the Bristol group.[9] Figure 7 represents a total number of 196 showers. At the present time the statistics are not good enough to give a reliable result for L. Using the value of the slope calculated from Eq. (7) and Table III, one obtains

$$L = 12_{-4}^{+7} \text{ cm.}$$

The lower limit of 8 cm corresponds to an extrapolated value $N=10$ for $u=0$. This result seems to exclude some of the very small values which have sometimes been quoted for L in the past. Our result is obtained for an average electron energy of 160 Bev. It is compatible with the one expected from theory[27,35,36] (16 cm).

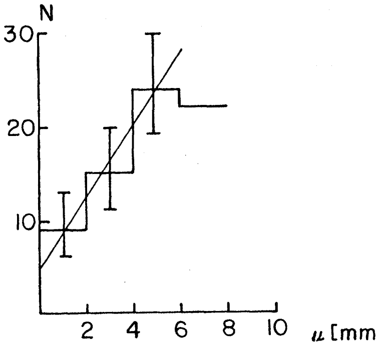

FIG. 7. Distribution of u for small values of u. Shown are combined results of this work and of reference 9.

Direct pair production affects the average distance $\langle u \rangle$ where the first secondary pair is found. If $\langle u \rangle_0$ is the average distance calculated without taking into account direct pair production, the actual average distance $\langle u \rangle$ is given in a good approximation ($\langle u \rangle \ll L$) by

$$\langle u \rangle = \langle u \rangle_0 (1 - 2\langle u \rangle_0 / \pi L). \tag{9}$$

Taking a value $L=16$ cm from theory, this leads to a 4.8% correction to the experimentally found distribution, if it is to be compared with $\langle u_0 \rangle$. This correction has already been included in our experimental results given in Table III.

7. CONVERSION LENGTH OF HIGH-ENERGY PHOTONS

The conversion length is the mean distance a photon travels before materializing. The conversion length λ of the primary high-energy photons starting the showers can be determined from the distribution of the origins of the primary electron-positron pairs inside the stack. In this investigation a total of 89 showers were used. For these showers the restriction $l_s > 7$ cm of Sec. 2 was dropped.

Let l be the distance the primary photon travels in the stack before converting. The integral distribution of l is plotted in Fig. 8, assuming that all the primary photons come from outside. There exists a tail in this distribution extending to very large distances l. This shows that a certain fraction of the photons was produced inside the stack in an interaction, the presence of which could not be detected at the origin of the first pair. Judging from those cases where a scan in the extrapolated line of flight of the photon was successful, we tend to believe that the events most likely to account for these photons are high-energy nuclear interactions with a small number of shower particles under rather wide angles. Before evaluating λ, a subtraction of this background must be made. It was assumed that the events responsible for these photons have the same distribution as the high-energy nuclear interactions in the stack. The distribution of l for nuclear interactions satisfying the acceptance criteria given in Sec. 2 is known experimentally.[37] It is roughly exponential with a slope of about 15 cm (determined mostly by the dimensions of the stack). The experimental distribution, Fig. 8, was therefore fitted by the sum of two exponentials, one of which, representing the background, has a slope of 15 cm. The slope of the other one was determined from the distribution itself at small distances l. The result of the best fit for the background curve is drawn in Fig. 8. The background represents about 18% of the events. After subtracting the background, the remaining points lie with good accuracy on a single exponential curve.

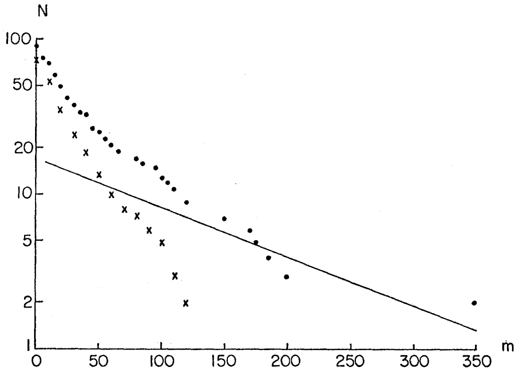

FIG. 8. Integral distribution of length l traveled by primary photon in the stack before converting. Solid curve: best fit for background. ×: integral distribution of l after subtraction of background.

The analysis rests furthermore on the assumption that each shower is started by exactly one photon coming from outside and not by several parallel photons of very small lateral separation ($\ll 1000\ \mu$). Each of the showers was carefully traced for at least several cm from

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[37] A. G. Barkow, B. Chamany, D. M. Haskin, P. L. Jain, E. Lohrmann, M. W. Teucher, and M. Schein (to be published).

the original pair origin and several plots of the lateral distribution were available. In no case was the presence of more than one photon detected. Such an event seems, furthermore, very unlikely since showers of parallel high energy photons show typical separations of the order of cm. Our stacks, which were exposed at high balloon altitudes, show a very small number of these multiple core events.

For the actual evaluation of λ, Bartlett's method[38] was applied to the events after subtraction of the background. This method requires the knowledge of l_a, the total length available in the stack for the detection of the shower. This length is smaller than the potential length l_p, defined in Sec. 2, since a certain distance must be allowed for the shower development before it can be detected in the scan. Figure 9 shows the correlation between l and l_p. There are no events with $l_p - l < 6$ cm. This means that there is a minimum distance of about 6 cm from the origin of the first pair to the point where the cascade is discovered. The distribution of the distance between the primary pair origin and the scanning line is also in agreement with this. We have, therefore, used

$$l_a = l_p - 6 \text{ cm}.$$

The conversion length λ, as obtained from Bartlett's method, after a 3.6% correction for the air space between emulsions, is:

$$\lambda = 34 \pm 5 \text{ mm}.$$

It should be compared with the value $\lambda = 37$ from the electromagnetic theory. The average energy of the photons for which λ is stated is 320 Bev. Pinkau has applied the same method to a sample of 24 events of energy >10 Bev. His result of 43_{-18}^{+11} mm is also in agreement, both with theory and with this experiment.

8. ENERGY DISTRIBUTION OF THE PAIR ELECTRONS

The distribution function describing the way the energy is shared by the electron and positron of the primary high-energy pair can in principle be studied by relative multiple-scattering measurements. However, in most cases the development of the cascade prevents sufficiently accurate measurements. A more indirect approach to this problem consists of measuring the separation of the pair at a given distance from the origin which is equivalent to a scattering measurement with one cell. The distribution of the pair separation for a large number of primary pairs can then be compared with the distribution calculated from theory.

The separation of the two members of the original pair, projected into the plane of the emulsion, was measured at a distance of 1 cm from the origin of the pair. This projected separation will be denoted by y. Measurements were possible on 79 events. For 10 events no measurements could be made since the beginning of the cascade prevented positive identification of the original particles.

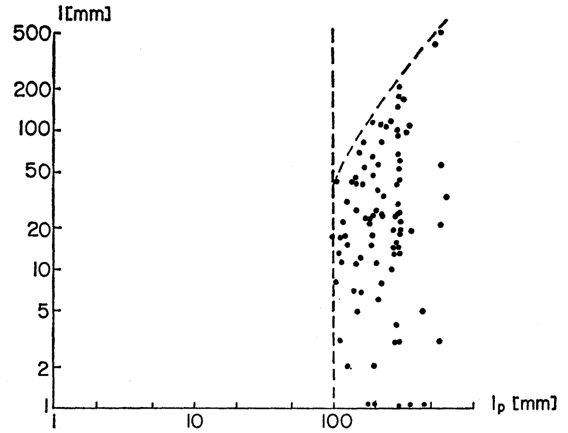

FIG. 9. Correlation between l and potential length l_p. Dashed curve: $l_p - l = 6$ cm.

At a distance of 1 cm from the pair origin, the original angle of divergence of the pair of order m/E can be neglected.[16–18] The distribution function y is only determined by multiple scattering. According to Molière[39–41] the distribution function for a particle of energy $E \gg mc^2$ which has traversed the path length Z, has the following form:

$$\Phi(\varphi)d\varphi = [(2/\pi^{\frac{1}{2}}) \exp(-\varphi^2) + B^{-1} f_1(\varphi) + \cdots] d\varphi, \quad (10)$$

where

$$\varphi = Ey'/kZ^{\frac{3}{2}} 2^{\frac{1}{2}}. \quad (11)$$

y' is the projected separation between the particle and the tangent to the trajectory of the particle at the origin ($Z=0$). k is a function which varies slowly with Z. Its value in nuclear emulsion and for $Z \approx 1$ cm is given in Sec. 2.

f_1 and B are functions which have been tabulated by Molière. The influence of f_1 was taken into account for the calculations, although it is very small. The influence of the higher order terms in Molière's expression, Eq. (10), can be completely neglected for the present purpose.

For calculating the theoretical distribution $T(y)$ of the projected separation y between the electron and the positron of the primary pair, the energy loss of the electrons by radiation must be taken into account. The problem was divided into two parts by considering the following two possibilities separately. First, the case where the electron and the positron lose $<30\%$ of their energy over the path of 1 cm. The energy loss was, in this case, treated by substituting for the initial energy of the particle an average effective energy which gives rise to the same scattering. This effective energy was

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[38] M. S. Bartlett, Phil. Mag. 44, 249 (1953).

[39] G. Molière, Z. Naturforsch. **3A**, 78 (1948).

[40] G. Molière, Z. Naturforsch. **10A**, 177 (1955).

[41] H. A. Bethe, Phys. Rev. **89**, 1256 (1953).

calculated by suitably averaging over the path and the energy distribution. One has next to fold in the energy partition function of the positron and electron. The result is given in reference 18. This group contributes about 50% of the cases.

The second group consists of those events where the electron or the positron or both lose $>30\%$ of their energy over the path of 1 cm. The probability of a given total energy loss by an electron traversing a given amount of material has been calculated by Bethe and Heitler.[19] This calculation uses an approximation which is not admissible for the case of very large energy losses ($\gtrsim 60\%$ of the original energy). For the distance of 1 cm (0.35 radiation units) it is sufficient to consider a given large ($>60\%$) energy loss as being essentially due to the emission of only one photon of high energy. For energy losses $<60\%$ the distribution given by Bethe and Heitler was used. For the calculation of the separation it can be assumed that the energy loss occurs in one step which is equally likely to lie anywhere within the 1 cm track interval under consideration. One has then to integrate over all points along the path and the energy spectrum of the electrons. The problem of calculating the scattering of a particle, the energy of which changes as a function of the path length, has been solved by Molière[39] and Eyges.[42] For the calculations of this work the expressions of reference 18 were used. The distribution of y' can then be calculated. Finally, one has again to fold in the energy distribution between the electron and the positron for the case that only one of them lost $>30\%$ of its energy and for the case that both lost $>30\%$ of their energy. Finally, one has to combine the distributions of both cases stated at the beginning.

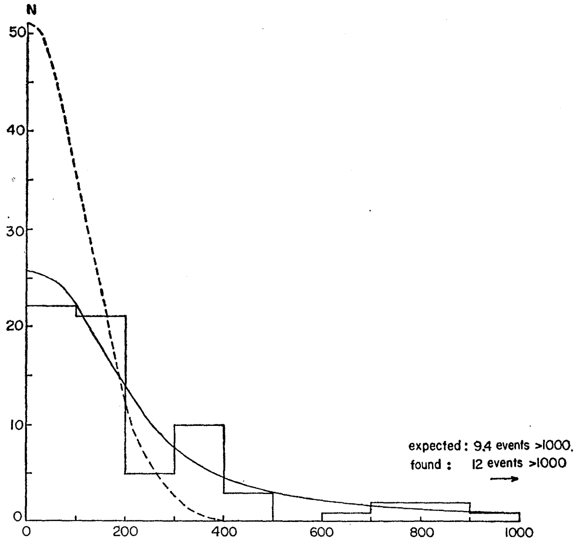

FIG. 10. Distribution of the projected separation y between electron and positron of the original pair 1 cm from the origin. Abscissa: E_0y in Bev μ. E_0: primary energy of pair. Solid curve: theoretical distribution $T(E_0y)$. Dashed curve: distribution assuming equipartition of energy between electron and positron and neglecting energy loss by radiation.

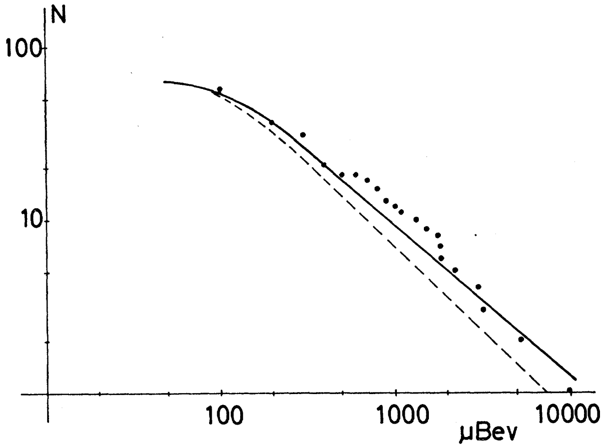

FIG. 11. Integral distribution of E_0y. Solid curve: theoretical distribution. Dashed curve: theoretical distribution neglecting radiation loss.

The resulting distribution $T(y)$ is shown in Figs. 10 and 11. It depends only on the product E_0y, where E_0 is the primary energy of the shower. For comparison also the distribution of y expected for an equipartition of energy between the electron and the positron is given. The distribution for larger values of E_0y is very different from the Gaussian curve representing equipartition of energy. The behavior of $T(E_0y)$ for large values of E_0y is of the form:

$$T(E_0y) \rightarrow [A+B\ln(E_0y)](E_0y)^{-2}, \tag{12}$$

which can be approximated by $(E_0y)^{-1.87}$ for $Z=10$ mm and for (E_0y) between 500 and 50 000 Bev μ. The comparison with the experimental data is also shown in Figs. 10 and 11. There is good agreement between theoretical distribution and experiment. From this we conclude that there is no appreciable discrepancy between the theoretical pair energy distribution function and experiment at photon energies of about 300 Bev. At this energy both the BH and the LPH theory still give the same result. Included in Fig. 11 there is also the theoretical distribution function without considering large energy losses by bremsstrahlung for comparison. This effect is quite important for the behavior of the distribution function for large values of E_0y. The experimental points fit much better the distribution which includes the influence of energy losses $>30\%$, indicating that the theoretical cross sections for radiation loss $>30\%$ of the electron energy must be correct within a factor of about 2. This supplements the evidence on the bremsstrahlung spectrum obtained in Sec. 5.

9. SPECIAL EVENTS

A list of 5 peculiar showers found in the four stacks of nuclear emulsion is given in Table IV. They are all produced by singly charged particles entering the stack

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[42] L. Eyges, Phys. Rev. **74**, 1534 (1948).

TABLE IV. List of "μ-meson" events.[a]

No., Stack	Energy of shower (Bev)	y (μ)	p_t (Mev/c)	l (mm)	Zenith angle
1, T	230	5.2	90	57	117°
2, T	140	4.0	38	53	100°
3, T	280	5.7	570	70	22°
4, G	110	7.7	63	170	173°
5, M	180	0	···	62	28°

[a] y: lateral separation between incident singly charged track and the first pair. l: length the singly charged track traveled in the stack before the first pair occurred.

from the outside. Each travels a large distance l in the stack before the first electron-positron pair occurs. This distance is much larger than could be expected from Eq. (3), making a reasonable allowance for fluctuations. In all cases the first pair is of very high energy (>20 Bev) and actually appears to start the shower. The angle between the singly charged incoming particle and the pair can be measured for all events but No. 5. Assuming that the shower is produced only by the first pair, one can calculate the transverse momentum p_t of the photon producing the first pair. Its magnitude, as listed in Table IV, rules out the production of the first photon by an electromagnetic process experienced by an electron. It would be consistent with the value expected for the bremsstrahlung of a μ meson, and also with production by a nuclear interaction, although in this case a somewhat higher value of p_t would be expected. The line of flight of the photon can be extrapolated back to the intersection with the singly charged particle. No visible interaction was found at this point or at any point along the track of the singly charged particle. The possibility that the events Nos. 3 and 5 are nuclear events of type $0+1p$ cannot be excluded, although this seems rather unlikely from the prong number statistics of high-energy nuclear events found in the stacks.[36] Events No. 1, 2, and 4 have a zenith angle of $>90°$ in the stack. Thus it seems very unlikely that they were produced during the flight in the stratosphere. Among more than 200 nuclear and normal electronic events found in our stacks not a single case was observed with a zenith angle $>90°$. The most likely explanation for events No. 1, 2, and 4 seems therefore production by a high-energy bremsstrahlung photon from a μ meson while the stacks were stored on the ground. A precise quantitative estimate of the expected number of such events is difficult due to a large uncertainty in scanning efficiency, storage conditions, and angular distribution of the μ-meson flux. It appears that the observed number of events is in agreement within a factor of about 3 with a rough estimate of the expected number of events. These phenomena point out the interesting possibility to investigate processes of high-energy transfer by μ mesons in nuclear emulsions exposed on the ground.

A possible case of a double pair production ($\gamma \rightarrow e^+ + e^+ + e^- + e^-$) was observed (among a total of about 100 primary pairs investigated). The energies of the four particles are (in Bev): 170, 10, 0.02, 0.007. There is the possibility that this event is due to the chance coincidence of the primary pair and a secondary pair produced by chance very close to the original pair. From the distribution of secondary pair origins shown in Fig. 7 and the experimental uncertainty of 25 μ in the position of the origin of the four electrons, the probability of a chance coincidence is $<6\times10^{-4}$ for a single event, and $<6\%$ for finding such an event among the ≈ 100 showers investigated. This probability is sufficiently small to make the explanation by a chance coincidence unlikely, although it cannot be ruled out.

ACKNOWLEDGMENTS

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