Polarization of Cosmic-Ray Muons at Sea Level*†

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Cosmic-ray muons at sea level are known to be partially longitudinally polarized. The object of this experiment was to measure the positive muon polarization by stopping the muons in a copper absorber and observing the up:down asymmetry in the decay electrons. Measurements were also made with iron and sulfur as control absorbers to test the symmetry of the apparatus. Since this experiment is performed in zero magnetic field, it has been possible to verify that unmagnetized iron depolarizes positive muons by precessing their magnetic moments in the strong, randomly oriented magnetic fields of the domains. The up:down ratio of decay electrons was 1.14 ± 0.02 for copper, 0.98 ± 0.02 for iron, and 1.02 ± 0.03 for sulfur. On the basis of the two-component neutrino theory, the muon polarization is found to be $\geq(21\pm3)\%$, where the inequality is due to depolarization effects. On the basis of cosmic-ray data on the muon energy spectrum as a function of altitude, the polarization is expected to be about 30%. The data of the present experiment suggest that in the absence of external magnetic fields copper depolarizes positive muons, as there appears to be a decrease in the up:down ratio with time lived before decay.

I. INTRODUCTION

It has been determined experimentally that cosmic-ray muons are partially longitudinally polarized.[1,2] This partial polarization may be understood qualitatively in the following way. In the pion rest frame muons are emitted isotropically,[3] and these muons are 100% longitudinally polarized.[4] Positive and negative muons are polarized, respectively, antiparallel and parallel to their momenta.[5,6] In this experiment, we are only concerned with positive muons because negative muons get depolarized before they decay or interact.

The present experiment is designed so as to select only muons within a narrow range of velocities. To a sufficient approximation, they may be all considered to have the same velocity. A muon of given energy in the laboratory may have resulted from the decay of relativistic pions into angles with the pion momentum from 0° to 180° in the pion rest frame, since muons emitted backward in the pion rest frame will be transformed forward in the laboratory frame. Consider the extreme cases. A muon of given energy may be the product of a pion of energy E, decaying directly forward, or of a pion of energy $E'=1.75E$ decaying directly backward. Of course, all intermediate energies and directions of emission are also possible. A muon emitted backward (spin forward) will transform into the laboratory with spin forward, while a muon emitted forward will transform with spin still backward. Since the pion energy spectrum falls off rapidly with increasing energy, one expects to find more muons with spin directed backward (up), resulting from a more frequent forward decaying pion, than forward, from a less frequent higher energy pion decaying backward. It is apparent that the partial polarization of cosmic-ray muons depends upon the shape of the parent pion energy spectrum, and that by measuring the polarization at sea level, one has a method of obtaining this spectrum.

The present experiment is essentially a continuation of the preliminary investigation reported in reference 2, the main difference being that the apparatus has been enlarged to approximately six times the area of the original.

II. THEORY

The theory of the polarization of cosmic-ray muons has been developed by Hayakawa[7] and Primakoff.[2] The partial longitudinal polarization $(\boldsymbol{\xi}\cdot\hat{v}_1')$ of muons from pions decaying in flight is given by[2]:

$$(\boldsymbol{\xi}\cdot\hat{v}_1')=-|\boldsymbol{\xi}|\left(\frac{1}{vv'}\right)\left(1-\frac{\eta}{\epsilon\epsilon'}\right), \quad (1)$$

where the prime indicates the laboratory frame and the subscript one a unit vector. Here, $\boldsymbol{\xi}$ is the muon polarization vector in the muon rest frame, v and ϵ are, respectively, the muon velocity and total energy (in

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1 G. Clark and J. Hersil, Phys. Rev. **108**, 1538 (1957).

2 J. M. Fowler, H. Primakoff, and R. D. Sard, Proceedings of the International Conference on Mesons and Recently Discovered Particles, Padua-Venice, September, 1957 (unpublished), p. IV-107; J. M. Fowler, H. Primakoff, and R. D. Sard, Nuovo cimento **9**, 1027 (1958).

3 R. L. Garwin, G. Gidal, L. M. Lederman, and M. Weinrich, Phys. Rev. **108**, 1589 (1957).

4 G. R. Lynch, J. Orear, and S. Rosendorff, Phys. Rev. **118**, 284 (1960). This article contains a review of earlier muon asymmetry parameter measurements.

5 G. Culligan, S. G. F. Frank, J. R. Holt, J. C. Kluyver, and T. Massem, Nature **180**, 751 (1957).

6 P. C. Macq, K. M. Crowe, and R. P. Haddock, Phys. Rev. **112**, 2061 (1958).

7 S. Hayakawa, Phys. Rev. **108**, 1533 (1957).

units of its rest energy) in the pion rest frame, and η is the total pion energy (in units of its rest energy) in the laboratory frame.

To find the average polarization of the muon "beam," which is what we measure, it is necessary to average $(\xi\cdot\hat{v}_1')$ for a given v' over the pion energy spectrum of decaying pions. We assume the pion differential energy spectrum to be of the form:

$$N(\eta)d\eta=(\text{const.})N^{-\gamma}d\eta, \quad (2)$$

with $\gamma>0$. The polarization is now averaged over this energy spectrum:

$$\{(\xi\cdot\hat{v}_1')_{\text{av}}\}=-|\xi|\int_{\eta_{\min}}^{\eta_{\max}}\left(\frac{1}{vv'}\right)\left(1-\frac{\eta}{\epsilon\epsilon'}\right)N(\eta)\frac{d\eta}{\eta}\Big/\int_{\eta_{\min}}^{\eta_{\max}}N(\eta)\frac{d\eta}{\eta}. \quad (3)$$

Here, the limits of integration correspond to forward and backward decay:

$$\eta_{\min}=\epsilon'\epsilon(1+v'v), \quad (4)$$

and

$$\eta_{\max}=\epsilon'\epsilon(1-v'v). \quad (5)$$

The added factors of η^{-1} in Eq. (3) are necessary to account for the fact that the probability that a muon will have just the right energy to stop in our absorber decreases with increasing pion energy.[8]

By integrating Eq. (3) between these limits and denoting $\{(\xi\cdot\hat{v}_1')_{\text{av}}\}$ by δ, we obtain (for $\gamma>2$)

$$\delta=-|\xi|\left(\frac{1}{vv'}\right)\left\{1-\left(\frac{\gamma}{\gamma-1}\right)[1-(vv')^2]\times\frac{(1+vv')^{\gamma-1}-(1-vv')^{\gamma-1}}{(1+vv')^{\gamma}-(1-vv')^{\gamma}}\right\}. \quad (6)$$

Expanding Eq. (6) in powers of $(v'v)$ yields

$$\delta=-|\xi|\{\tfrac{1}{3}(\gamma+1)(vv')+\cdots\}, \quad (7)$$

as an approximate relation between δ and γ.

It should be noted that Eq. (6) does not contain ϵ' explicitly and depends on the muon laboratory energy only through v', which is essentially equal to 1.0 in the relativistic region. Hence, the expression Eq. (6) for the polarization in the laboratory frame applies to any spectrum of muon energies provided only that the energies are large compared to 105.7 Mev.

The partial longitudinal polarization of the muon "beam" was found by stopping the muons in a suitable absorber (in this case, copper) and observing the asymmetry of the muon decay electrons. Such a "beam" of partially polarized muons is expected to give an angular distribution of electrons of the form:

$$1+a\delta\cos\psi, \quad (8)$$

where a is an asymmetry coefficient[9] depending upon the decay electron energy and the experimental configuration, and ψ is the angle between the muon momentum upon entering the absorber and the decay electron momentum. The up:down asymmetry (R) of decay electrons has the form:

$$R=\frac{1+|\delta|\bar{a}}{1-|\delta|\bar{a}}, \quad (9)$$

where $\bar{a}$ is an effective asymmetry coefficient.

III. EXPERIMENT

Figure 1 is a schematic illustration of the apparatus. A filter consisting of 345.9 g cm^{-2} of lead (approximately 200 g cm^{-2} air equivalent) is provided and requires that the muon have a minimum momentum of 540 Mev/c to enter the absorber. The three absorbers used are copper (33.9 g cm^{-2}), iron (30.0 g cm^{-2}), and sulfur (12.7 g cm^{-2}). Iron and copper stop particles with momenta up to 593 Mev/c and sulfur up to 572

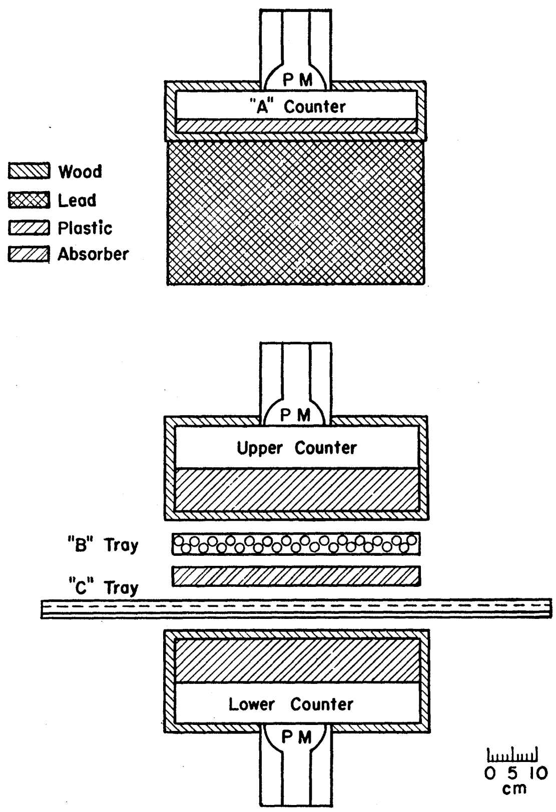

FIG. 1. Schematic diagram of the apparatus showing arrangement of counters, filter, and absorber.

[8] The factor noted here was omitted in references 2 and 7 but has been correctly included in the calculation by I. I. Gol'dman, Soviet Phys.—JETP 34(7), 702 (1958); J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1017 (1958).

[9] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957).

Mev/c. Copper[10] was used to determine the up:down asymmetry. Iron (soft unmagnetized iron[11]) was used as a control to find out if there are inherent asymmetries in the geometry or electronics. Iron is expected to completely destroy the polarization by making the muon spin precess rapidly in the large magnetic fields of the domains.[12] Sulfur,[13] which is known to depolarize[14] almost completely, was also used as a control.

The detectors labeled A, upper (U) and lower (L) in Fig. 1 are scintillation counters consisting of plastic scintillators,[15] each of which is viewed by a single 5-in. photomultiplier tube (DuMont type 6364). Counter A consists of four $1\times10\times10$ in. pieces arranged together to form a single $1\times20\times20$ in. scintillator. The U and L counters are identical and the scintillators are $3\frac{5}{8}\times20\times20$ in., each consisting of four pieces of scintillator $3\frac{5}{8}\times10\times10$ in.

The B and C Geiger trays consist of twenty-six 1×20 in. and thirty-one 1×36 in. Geiger tubes, respectively, staggered to minimize the possibility of particles passing between the counters undetected. The walls of the tubes are 0.015-in. brass.

The surfaces of the scintillators used in A, U, and L were machined smooth, but not polished, to reduce light loss by internal reflection. To enhance the reflectivity, the inside surfaces of the counter boxes were painted with several coats of a flat white paint.[16]

It is important for the counters to have a pulse-height response which is as uniform as possible, i.e., particles of the same energy should give the same pulse height regardless of where they strike the scintillator. Counters of this geometry, in general, give larger pulses for particles incident near the center than near the edges. In this case, for minimum-ionizing particles, the difference in pulse height is about 15% between the edges and the center. This is because most of the light coming from particles near the edges of the counter is scattered at least once, losing intensity with each scattering, before striking the photocathode, while somewhat more of the light particles near the center is incident directly on the photocathode. In order to minimize this effect, circular disks of aluminum foil were attached to the surface of the scintillators, between the scintillator and phototube, to reduce the amount of direct light from particles near the center. To determine the optimum disk diameter, pulse-height spectra were taken for minimum-ionizing particles incident near the edge and near the center of the counter, for disks of various sizes. A diameter of 9 in. gave practically the same spectrum for the two positions, indicating that uniformity was essentially complete. The resolution for minimum-ionizing particles incident uniformly over the counter was improved from 23% (full width of the differential pulse-height peak at half-maximum) to 16% with less than 10% decrease in gain, by using the disks.

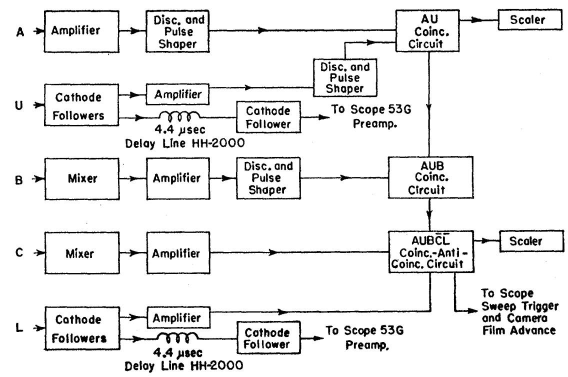

FIG. 2. Block diagram of electronic apparatus.

Figure 2 shows a block diagram of the electronics. We require for a muon entering the counter telescope a triple coincidence between A, U, and B, and C and L in anticoincidence. Muons stopping in the lower half of a B-tray Geiger tube or the upper half of a C-tray Geiger tube will also satisfy these conditions, but this is of no consequence since the tubes have brass walls.

In Table I the counting rates, measured and computed, are listed for the counters and various combinations in which they were used. The observed rates are from a typical run using iron as an absorber, and the calculated rates are based on sea level data.[17] The A counter was mounted on top of the lead filter with only about 4.0 g cm^{-2} of light material above it. However, the counter was biased so that very little of the soft

TABLE I. Typical counting rates.

Counter designation	Observed rate (sec^{-1})	Calculated rate (sec^{-1})
A	4.05×10^{1}	3.45×10^{1}
U	3.69×10^{1}	3.84×10^{1}
B	1.30×10^{2}	3.84×10^{1}
C	4.49×10^{2}	9.26×10^{1}
L	3.76×10^{1}	3.84×10^{1}
AU	7.07	8.75
AUB	4.96	6.16
$AUB\bar{C}\bar{L}$	1.90×10^{-1}	1.29×10^{-1}
Muon decay rate in U or L	1.47×10^{-3}	1.84×10^{-3}

[10] The copper absorber consisted of six $\frac{1}{4}\times20\times20$ in. plates of 99.998% pure copper.

[11] The iron absorber consisted of six $\frac{1}{4}\times20\times20$ in. plates of "Armco Magnetic Ingot Iron," available from the American Rolling Mill Company, Middletown, Ohio.

[12] H. Primakoff (private communication, 1957).

[13] The sulfur absorber consisted of three $2\frac{1}{2}\times6\frac{5}{8}\times20$ in. blocks, molded from sublimed, N.F. grade, powdered flowers of sulfur, available from Mallinckrodt Chemical Works, St. Louis, Missouri. The absorber had an average density of 1.96 g cm^{-2} and was in the rhombic crystalline form.

[14] R. A. Swanson, Phys. Rev. **112**, 580 (1958).

[15] Material manufactured by Allied Research Associates, Inc., Boston, Massachusetts.

[16] Plasticoat available from Coating Laboratories, Inc., Tulsa, Oklahoma.

[17] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).

component was counted. All of the scintillation counters were made insensitive to local radioactivity by discriminating against small pulses. U and L were under the filter and their rates were consequently somewhat lower than A. The U rate was smaller than the L rate because U was better shielded by the filter. The B and C rates were higher than expected due to local radioactivity. The AU, AUB, and $AUB\bar{C}\bar{L}$ rates have been computed following the method given in reference 18, and the expected muon decay rate into U and L was calculated from the results of the machine calculation of the asymmetry coefficient (Sec. IV).

Data were taken by photographing the face of a Tektronix 531 cathode-ray oscilloscope. The outputs from U and L are delayed approximately 4.4 μsec and fed into the balanced inputs of a type 53G Tektronix preamplifier; one input deflecting the electron beam up and the other down. $AUB\bar{C}\bar{L}$ pulses from the coincidence-anticoincidence circuit triggered the 1-μsec/cm horizontal sweep (≈ 10 μsec long). Since the U and L pulses are delayed, they reach the scope preamplifier after the sweep has started and the pulse due to the the muon traversing U appears on the sweep first followed by a decay electron pulse, if there is one, in the same direction as U for an upward decay and in the opposite direction for a downward decay.

The film was analyzed by measuring the differential pulse height and time distributions of the delayed pulses. Only pulses occurring 0.625 ± 0.015 μsec after the start of the muon pulse and with amplitude corresponding to an energy greater than 6 Mev were analyzed. The raw data in 0.625-μsec channels are listed in Table II.

The U and L counters were connected rigidly together and mounted on tracks so that they could be rolled away from the absorber and their relative positions interchanged by rotating them through 180° on pivots. This arrangement makes it possible to average out inherent asymmetry due to differences in the two counters and their associated electronics, i.e., differences in gain, resolution, etc.

Data were taken in runs approximately 24 hr long, the counters being interchanged three times during the run giving two separate, and equal, periods with each counter in the two positions. Before and after every run the gains of the two counters, and their associated electronics (up to the scope preamplifiers) were checked by taking an integral spectrum of minimum ionizing cosmic rays in each counter. The absorbers were interchanged between runs as a further precaution against systematic errors, copper being alternated with either iron or sulfur.

TABLE II. Differential time distribution of electron pulses.

Channel number	Iron		Sulfur		Copper	
	Lower	Upper	Lower	Upper	Lower	Upper
0	1394	1380	813	752	1476	1801
1	1005	993	505	547	1167	1227
2	729	704	346	425	816	1035
3	579	593	279	310	647	747
4	446	412	198	201	490	554
5	345	326	164	146	424	446
6	274	266	137	132	286	300
7	216	206	104	85	267	255
8	150	129	55	69	167	166
Totals	5138	5009	2601	2667	5740	6531

[18] A. M. Conforto and R. D. Sard, Phys. Rev. **86**, 465 (1952).

TABLE III. Background.

	Number of sweeps	Background		Running time (hr)
		Lower	Upper	
Iron	1.05×10^6	148	153	1007.0
Sulfur	0.330×10^6	47	48	817.6
Copper	1.09×10^6	154	160	1394.8

Background was measured by increasing the sweep speed to 10 μsec/cm and running the experiment as before. Only U and L pulses more than 23.4 μsec after the start of the sweep were analyzed to insure that only background pulses were counted. This gave an effective sweep length of 74.8 μsec compared to 5.625 μsec for the data sweeps. In a run of 147.3 hr giving 1.40×10^5 slow sweeps, there were 262 L pulses and 272 U pulses. By using this background rate, we calculate the numbers listed in Table III.

IV. EVALUATION OF THE ASYMMETRY COEFFICIENT

We wish to compute the flux of decay electrons expected in U and L from muons decaying within the absorber. The problem involved here is to find, for muons of given polarization, the number of decay electrons per second one would expect in U and L for all possible muon angles of incidence, absorber depths, decay electron energies, and directions of emission. The number of muons decaying into U and L may be written:

$$I=\int\cdots\int\{[dN(U,\psi,\delta)][P(\theta_\mu)dA\,d\Omega_\mu dz]\times[R(r,U)]\}dr. \quad (10)$$

Here, the first term to the right of the integral signs is the decay electron energy-angle distribution from muons at rest with polarization δ, the second term gives the number of muons per second entering the apparatus and coming to rest in an element of thickness dz of the absorber, and the last term gives the differential range distribution for an electron of energy U.

The electron energy-angle distribution in which we are interested is that which gives the fraction of muons, with polarization δ, decaying into electrons, with energy between U and $U+dU$ at an angle ψ with the muon momentum into solid angle $d\Omega_e$. Such a distribution is given in the two-component neutrino theory[19] by

$$dN(U,\psi,\delta)=\{2U^2[(3-2U)-\delta\cos\psi(1-2U)]\times d\Omega_e\}(1/4\pi), \quad (11)$$

where U is in units of 53 Mev and (see Fig. 3) $d\Omega_e=\sin\theta_e d\theta_e d\phi_e$. It is necessary to find the connection between θ_μ, θ_e, ϕ_μ, ϕ_e, and ψ. This is

$$-\cos\psi=\sin\theta_e\cos\phi_e\sin\theta_\mu\cos\phi_\mu+\sin\theta_e\sin\phi_e\times\sin\theta_\mu\sin\phi_\mu+\cos\theta_e\cos\theta_\mu. \quad (12)$$

On assuming the current of incident muons varies as the square of the cosine of the zenith angle,[17] the number of positive muons per second stopping in an element of thickness dz of absorber is $P(\theta_\mu)dAd\Omega_\mu dz=i_v\cos^2\theta dAd\Omega_\mu dz$. Here, dA is the projection of the element of area $dxdy$ normal to the direction of muon incidence ($dA=dxdy\cos\theta_\mu$).

Upon emission an electron must traverse a thickness of absorber, determined by the depth at which the meson stopped and the direction of emission, before entering the counter. We represent the differential range distribution by $R(r,U)$, i.e., $R(r,U)$ gives the probability that an electron will stop in each element dr of path length. $R(r,U)$ is a very complicated function because of electron range straggling and must be evaluated numerically. Fortunately, electron range-straggling tables[20] are available and these were used in evaluating Eq. (10). On collecting results, we have

$$I=(1/4\pi)\int\cdots\int\{2U^2[(3\times 2U)-\delta(\sin\theta_e\cos\phi_e\times\sin\theta_\mu\cos\phi_\mu+\sin\theta_e\sin\phi_e\sin\theta_\mu\sin\phi_\mu+\cos\theta_e\cos\theta_\mu)]\times(dU\sin\theta_e d\theta_e d\phi_e)\}\{i_v\cos^3\theta_\mu\sin\theta_\mu d\theta_\mu d\phi_\mu\times dxdydz\}\{R(r,U)\}dr, \quad (13)$$

which we write for convenience

$$I_\pm=I_1\pm|\delta|I_2, \quad (14)$$

where I_1 and I_2 are independent of δ. The plus and minus signs refer to the flux of decay electrons in U and L, respectively.

Equation (14) yields the up:down asymmetry of decay electrons as

$$R=\frac{I_+}{I_-}=\frac{1+|\delta|(I_2/I_1)}{1-|\delta|(I_2/I_1)}. \quad (15)$$

On comparing Eqs. (9) and (15), we obtain: $\bar{a}=I_2/I_1$.

The necessity of using range-straggling tables and the finite geometry of our apparatus make direct evaluations of Eq. (13) impossible, and the integration must be carried out, for the most part, numerically. This has been done using the Washington University IBM-650 data processing machine. This evaluation gives $\bar{a}=0.31\pm0.01$. Errors enter into I_2 and I_1 in the same way and cancel in first order.

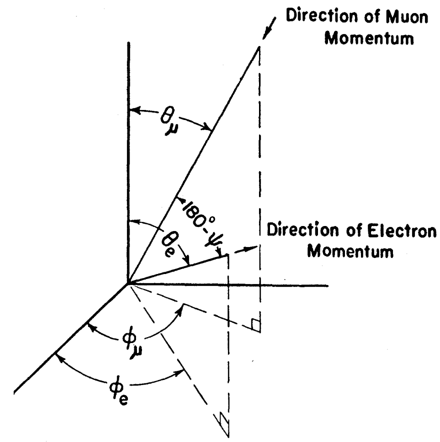

FIG. 3. Schematic diagram illustrating the decay of a muon.

V. REDUCING OF DATA

The differential time distributions (Table II) are not in general expected to be simple exponentials because of the presence of negative muon decays and background pulses. Most of the negative muons stopping in the absorbers annihilate with nuclei. Those decaying, however, give an isotropic[21] background that must be taken into account.

On dividing the time scale into equal channels of width Δ, starting t_0 after time zero, and labeling the channels by integers ($i=0, 1, 2, \cdots$) we have for the average number of decay electrons in the ith channel:

$$\bar{Y}_i=A\alpha_i+C, \quad (16)$$

with

$$A=N_{\mu^+}G(+)\exp[-\lambda_0 t_0](1-\exp[-\lambda_0\Delta]), \quad (17)$$

$$\alpha_i=\exp[-\lambda_0 i\Delta]\left\{1+\frac{N_{\mu^-}}{N_{\mu^+}}\exp[-(\lambda_--\lambda_0)t_0]\frac{\lambda_d}{\lambda_-}\times\frac{1-\exp[-\lambda_-\Delta]G(-)}{1-\exp[-\lambda_0\Delta]G(+)}\exp[-(\lambda_--\lambda_0)i\Delta]\right\}, \quad (18)$$

and C is the background per channel. Here, $G(\pm)$ is the average detection efficiency for electrons from positive and negative decays, respectively, $N_\mu^\pm$ is the number of positive and negative muons stopped by the absorber, and the ratio $N_\mu^-N_\mu^+=45/55$[22]; λ_0 is the

[19] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[20] J. E. Leiss, S. Penner, and C. S. Robinson, "Tables of Electron Range Straggling in Carbon," University of Illinois, Urbana, Illinois (unpublished).

[21] V. L. Telegdi, Washington University Colloquium, May, 1959 (unpublished). A. E. Ignatenko, L. B. Egorov, B. Khalupa, and D. Chultem, Soviet Phys.—JETP **35**(8), 792 (1959); J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1131 (1958).

[22] G. Puppi, *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1956), Vol. 3, p. 341.

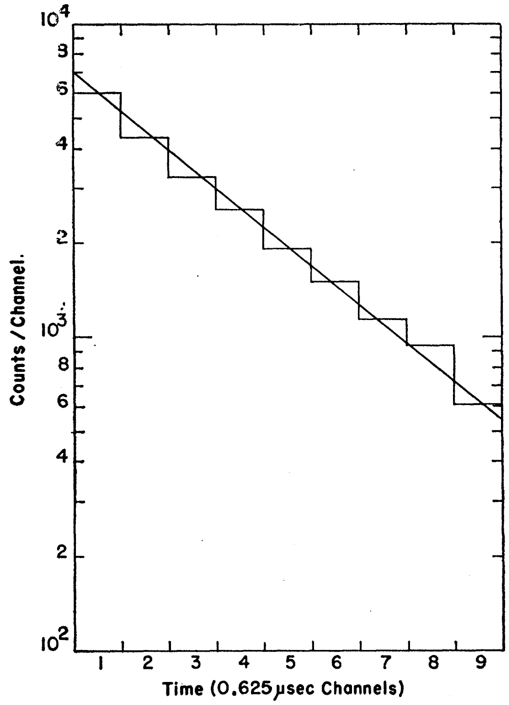

FIG. 4. Time distribution of positrons from iron and copper absorbers combined. Shown with a 2.20-μsec mean life curve.

decay constant for positive muons (2.20 ± 0.015 μsec)$^{-1}$,[23] λ_- is the disappearance rate of negative muons and is the sum of the decay rate from Bohr orbits (λ_d) and the nuclear interaction rate (λ_i). It should be noted that λ_0 and λ_d are not the same. In general, λ_d varies from material to material approaching λ_0 for small Z. Values of the disappearance and interaction rates have been measured at the University of Chicago synchrocyclotron.[24] Since the first channel starts 0.625 μsec after time zero, i.e., $t_0=0.625$ μsec, the last term in the curly brackets [Eq. (18)] is negligible for copper and iron. In sulfur, however, about 14% of the observed decays are due to negative muons and this term cannot be neglected. The function

TABLE IV. Summary of results.

		$A\pm\sigma_A$	$C\pm\delta C$	$R=A$ (upper)/ A (lower) Exper.	Theor.
Iron	Upper Lower	1300±19 1330±19	17.0±1.1 16.4±1.0	0.98±0.02	1.00
Sulfur	Upper Lower	637±12 625±12	5.3±0.5 5.2±0.5	1.02±0.03	1.01
Copper	Upper Lower	1700±22 1490±20	17.8±1.1 17.1±1.1	1.14±0.02	...

[23] J. Fischer, B. Leontic, A. Lundby, R. Meunier, and J. P. Stroot, Phys. Rev. Letters **3**, 349 (1959).
[24] J. C. Sens, Phys. Rev. **113**, 679 (1959).

α_i for sulfur involves the ratio $G(-)/G(+)$, but this is 1.00 since sulfur depolarizes the muons completely.

We need the up:down ratio (R) of decays. This is equal to the fraction of positive muons counted by U, $G(+,\text{upper})$ divided by the fraction counted by L, $G(+,\text{lower})$, Eq. (17) yields for this ratio:

$$\frac{G(+,\text{upper})}{G(+,\text{lower})}=\frac{A(\text{upper})}{A(\text{lower})}. \tag{19}$$

Thus we need to extract A(upper) and A(lower) from the data. The data was analyzed following the least-squares procedure of reference 2. In the present case, only a one parameter fit is required since the background was measured experimentally. On using this procedure, we obtain

$$A=\frac{\sum_i \alpha_i - C\sum_i \alpha_i/Y_i}{\sum_i \alpha_i^2/Y_i}, \tag{20}$$

and

$$\sigma_A=[\sum_i \alpha_i^2/Y_i]^{-\frac{1}{2}}, \tag{21}$$

for the statistical error in A. Here, Y_i is the number of pulses in the ith channel. The data in Table II were analyzed using this method and the results are summarized in Table IV. The total number of positive decay electron pulses recorded is $(1-\exp[-\lambda_0\Delta])^{-1}A$, and the time distribution of the iron and copper data combined is shown plotted in Fig. 4, where it is compared with a 2.20 μsec mean life curve. Using the maximum-likelihood procedure of Peierls,[25] these data yield 2.20 ± 0.04 μsec for the most probable muon mean life.

VI. RESULTS

On taking into account possible depolarization effects, Eq. (9) becomes:

$$R=\frac{1+\bar{a}|\delta|K_{\text{atm}}K_{\text{mod}}K_{\text{abs}}}{1-\bar{a}|\delta|K_{\text{atm}}K_{\text{mod}}K_{\text{abs}}}. \tag{22}$$

Here, K_{atm}, K_{mod}, and K_{abs} are correction factors for depolarization in the atmosphere, moderator, and absorber, respectively. Hayakawa[7] has made theoretical estimates of the depolarization due to Coulomb scattering in the slowing down and stopping of muons. For an experiment very similar to this one, he estimates that muons are depolarized by about 5% in traversing the atmosphere and lead moderator, i.e., $K_{\text{atm}}K_{\text{mod}}\approx0.95$. Depolarization due to Coulomb scattering in the absorber is also estimated and found to be negligible. However, it is possible that muons are depolarized to some degree by the absorber after they are brought to rest. Depolarization of this kind could be caused by weak magnetic fields, of the order of a few gauss,

[25] R. Peierls, Proc. Roy. Soc. (London) **A149**, 467 (1935).

within the absorber. In Fig. 5, the time distributions (Table II) have been divided into three 1.875-μsec channels and the experimental up:down ratio per channel with the estimated background subtracted off is plotted against time. (The errors shown are standard statistical deviations.) The apparent decrease in the up:down ratio for copper, which is much smaller for sulfur or iron, would seem to indicate that there may very well be an internal depolarizing mechanism in copper. Cyclotron experiments[26,27] using copper as a muon absorber have not indicated any effects of this kind noted here. However, these experiments all use magnetic fields of the order of 50 to 100 gauss to turn the muon magnetic moments, and such fields could overcome small depolarization effects.

It should be noted that the present experiment is the only zero-field experiment reported to date, using copper, in which the time distributions of the decay electrons have been measured.

From the value of R for copper (Table IV) and the calculated asymmetry coefficient $\bar{a}$ we have from Eq. (22):

$$|\delta| = 0.21\pm0.03/(K_{\text{atm}}K_{\text{mod}}K_{\text{abs}}), \tag{23}$$

or

$$|\delta| > 0.21\pm0.03. \tag{24}$$

It is apparent from Eq. (23) that $|\delta|$ depends strongly on the magnitude of the depolarization effects. Unfortunately, our measurement of the depolarization in copper lacks sufficient accuracy to give a meaningful correction factor. On assuming that $|\delta|$ decreases exponentially with time, one obtains from Fig. 5 the value $K_{\text{abs}}=0.7_{-0.2}^{+0.3}$ within the limits of error. A factor of 0.70 would raise $|\delta|$ to 0.30. However, in view of the uncertainties involved, we are compelled to state $|\delta|$ as the inequality Eq. (24) until an accurate measurement of K_{abs} has been made in some future experiment.

Since the muons are produced at various altitudes,[28] the polarization measured is an average over a spectrum of muon energies between 2.64 and 0.403 Bev; corresponding to muon ranges of 1.2×10^3 g cm^{-2} and 2.0×10^2 g cm^{-2}, respectively. By using Eqs. (4) and (5), we find that these muons are produced by pions with energies between 4.62 and 0.407 Bev.

Strictly speaking, Eq. (2) is only valid over a limited range of energies.[22] However, Eq. (2) is a good approximation to the pion spectrum between 1 and 5 Bev with $\gamma\approx2.4$. Over 90% of the observed muons are from pions in this region; relatively few muons are from pions below 0.6 Bev, where the slope of the spectrum decreases rapidly with increasing energy.

For such high energies v' is a very slowly varying function of energy, and it is a good approximation to assume $v'=1.0$. In this energy region, $vv'\approx0.27$ and $|\delta|>0.21\pm0.03$. Equation (6) gives $\gamma>1.3\pm0.3$, and with $K_{\text{atm}}K_{\text{mod}}=0.95$, the inequality becomes $\gamma>1.4\pm0.3$. This result is consistent with the value (1.00 ± 0.6) obtained for γ by Clark and Hersil[1] who found $|\delta|=0.19\pm0.06$. Alikhanian and associates[29] have reported $|\delta|=0.21\pm0.08$ for muons of momentum 0.35 Bev/c on entry into the apparatus.

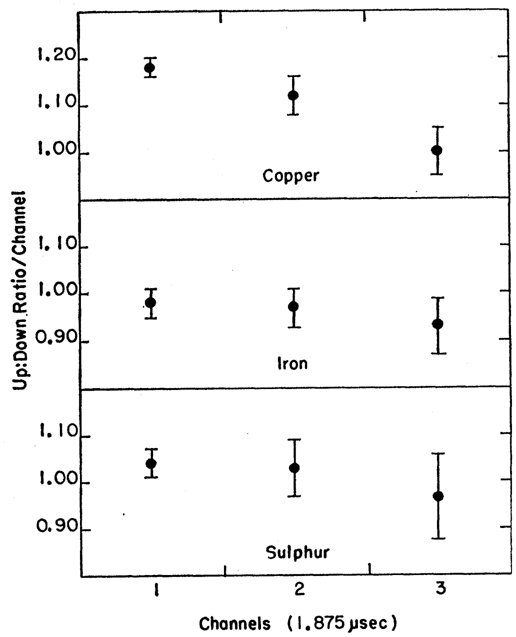

FIG. 5. Experimental up:down ratio per channel plotted against time, showing possible depolarization in copper.

VII. CONCLUSIONS

The results of this experiment indicate that as expected the positive muon "beam," stopping in a thin absorber at sea level, is partially longitudinally polarized. On the basis of the two-component theory, the magnitude of this partial polarization is found to be $>(21\pm3)\%$. By assuming a power law for the parent pion differential energy spectrum, the slope of this curve is found to be $>1.4\pm0.3$ which is consistent with other results in this energy region.

There is evidence that, in the absence of external magnetic fields, copper depolarizes stopped muons at a rate corresponding to a relaxation time of several μsec. The experimental result is consistent with the value $\gamma\approx2.4$ obtained from other experiments of a different type[22] only if there is indeed a relaxation of the polarization in copper of an amount suggested by the data in Fig. 5. In effect, the polarization at sea level with $\gamma=2.4$ is 0.30, while with $K_{\text{abs}}=0.7$ the

[26] J. M. Cassels, T. W. O'Keefe, M. Rigby, A. M. Wetherell, and J. R. Wormald, Proc. Phys. Soc. (London) **A70**, 543 (1957).
[27] M. Weinrich, Columbia University Nevis Cyclotron Laboratory Report No. 56, 1958 (unpublished).
[28] M. Sands, Phys. Rev. **77**, 180 (1950).
[29] A. I. Alikhanian, *Proceedings of the Moscow Cosmic-Ray Conference* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. I, p. 317.

experimental result becomes 0.30. At present, however, the magnitude of this effect is not known accurately enough to correct $|\delta|$ and γ.

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Angular Distribution of Shower Particles from 1000-Bev Nucleon Alpha Particles on Emulsion Nuclei

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Twenty-eight interactions of α particles were located in a 22-liter stack of nuclear emulsion by tracing back showers of minimum ionization particles to their origins. The angular distributions of 17 α particles with a dip angle $\leqslant 20°$ are presented. The inelasticity for these 17 interactions shows large fluctuations for individual events and its mean value is 30%. The angular distributions of these α particles have been transferred into a system in which they are roughly symmetric. The degree of anisotropy of the angular distributions is in disagreement with a hydrodynamical model of nucleon-nucleus collisions. The detailed analysis of the angular distribution of composite stars for events with a high degree of anisotropy of secondaries in the center-of-mass system shows that the shape of the angular distribution is in agreement with the predictions of the "two-fireball" model of multiple meson production, both for nucleon-nucleon and nucleon-nucleus collisions.

1. INTRODUCTION

At present there are not enough experimental data available to show which one, among the several existing theories of Fermi,[1] Heisenberg,[2] Landau,[3] and the "two-center model,"[4–6] can best explain very high energy nucleon-nucleon interactions. The main difficulty is that the energy available in accelerators is not yet high enough ($\sim$30 Bev) to make any systematic studies. So, for the study of high-energy nuclear interactions, cosmic rays are the only source of high-energy particles. Since the flux of the high-energy particles is much less than of the low-energy particles in cosmic radiation, our present experimental knowledge is based on a small number of events.

In order to observe high-energy interactions, nuclear emulsion is generally used. It has the advantage of being light in weight and can thus be easily flown to very high altitudes. The only disadvantage in using nuclear emulsion is that it consists mainly of heavy elements, and consequently very few ($<5\%$) of the collisions of the primary particles with the emulsion nuclei are with free protons. Thus in nuclear interactions in emulsion, we are concerned primarily with nucleon-nucleus collisions, in which several target nucleons may take part. The generally accepted practice for selecting "jets" which are due to nucleon-nucleon interaction is to select those events which have no heavy prongs at all or only those events which have not more than three or four heavy prongs. This is believed to be the best approximation to a collision between a nucleon and a free proton or between a nucleon and only one bound nucleon at the periphery of a heavy nucleus without any visible excitation of the rest of the nucleus. We may point out here that these criteria used for the nucleon-nucleon interaction are not quite safe, because the emission of neutrons alone as a result of some excitation of a nucleus cannot be excluded entirely. As yet there exists no procedure by which it could be ascertained without any doubt whether the collision is a nucleon-nucleon interaction or not.

About one-third of the primary jets produced by cosmic ray particles are initiated by α particles rather than protons. In such collisions it is reasonable to regard the incident α particle as four separate nucleons, each of which, when it interacts, emits mesons from its individual center-of-mass system. On the average, these separate center-of-mass systems will have the same velocity in the laboratory system, and the assumption is made of symmetrical emission of mesons from a com-

[1] E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950); Phys. Rev. **81**, 683 (1951).
[2] W. Heisenberg, Z. Physik **126**, 569, (1949); **133**, 65, (1952).
[3] S. S. Belenki and L. D. Landau, Nuovo cimento Suppl. **3**, 15 (1956).
[4] P. Ciok, T. Coghen *et al.*, Nuovo cimento **8**, 166 (1958).
[5] G. Cocconi, Phys. Rev. **111**, 1699 (1958).
[6] K. Niu, Nuovo cimento **10**, 994, (1958).