tained using an independent technique with much narrower photon energy resolution.

J. J. Sakurai[2] has predicted that large polarizations near 90° c.m. can only be obtained in this energy region by interferences between first and second pion resonances of opposite parity, as suggested by Peierls.[10] G. Stoppini and C. Pelligrini,[12] and L. F. Landovitz and L. Marshall,[13] suggested other models which might also give rise to appreciable polarizations, even if the two resonances were both of even parity.

One of us (J. O. M.) has investigated the nature of most of these models. A qualitative examination of the multipole expansions for the cross sections and polarizations was supplemented by numerical calculations using simple resonance formulas for the resonant amplitudes and phases. It was concluded that only the model suggested by Peierls, in which the second state has odd parity and is photoproduced by electric dipole radiation, can consistently explain the angular distributions and polarizations observed in π^0 photoproduction. The distributions appear to contain material contributions from nonresonant states and from the third resonance. In some of the models, the sign of the polarization is inconsistent with the signs of the interference terms in the angular distribution. The small size of the $\cos\theta$ term in the angular distribution was found to be correctly predicted by the Peierls model, especially when nonresonant s waves are included. Typical results are shown in Fig. 2.

[12] G. Stoppini and C. Pellegrini, Proceedings of the Ninth Annual Conference on High Energy Physics, Kiev, 1959 (unpublished).
[13] L. F. Landovitz and L. Marshall, Phys. Rev. Letters **3**, 190 (1959).

PHYSICAL REVIEW VOLUME 122, NUMBER 4 MAY 15, 1961

Kinematical and Dynamical Resonances*

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(Received January 16, 1961)

A method is given to distinguish between the solutions of the dispersion relations corresponding to kinematical and dynamical resonances. It consists of studying the resonance energy as a function of the coupling constant. The method is illustrated for potential scattering, for charged scalar meson theory, and for resonances due to unstable particles.

I

A RESONANCE in the scattering of elementary particles is called *kinematic* if it is due to an intermediate unstable particle. In the description of such a resonance the unstable particle is put into the theory to begin with; in more conventional field theories a new field is introduced for the new particle. In contrast to this, a *dynamical resonance* arises solely from the nature of forces between the initial interacting particles and therefore must come out automatically from the dynamical equations without introducing a new particle.

In view of the discovery of several new resonances[1] in the strong interactions of mesons and hyperons, it is desirable to characterize these two types of resonances more fully and to distinguish them both theoretically and experimentally. An experimental characterization has been given by Chew[2] according to which the phase shift will change sign near the resonance if it is due to an unstable particle and for a dynamical resonance the phase shift, in general, will not change sign. Unfortunately, the dispersion theoretical treatment of the strong interactions does not distinguish between kinematical and dynamical resonances. This is due to a well-known ambiguity[3] of the solutions of the dispersion relations. The so-called extra solutions of the dispersion relations can be shown to have resonance character and to correspond to unstable intermediate states.[4] The question has been raised whether conventional field theories can produce any resonances or whether the observed resonances are due to the unstable particles[4] (composite, elementary, or excited states). If the second alternative is true, we may conjecture that the failure of the perturbation theory in strong interactions is not due to the largeness of the coupling constant but to the fact that hitherto such unstable intermediate states have not been considered.

In this note we give a method to characterize and distinguish the kinematical and dynamical resonances,

* Supported in part by the Air Force Office of Scientific Research.
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[1] M. Alston *et al.*, Phys. Rev. Letters **5**, 520 (1960); A. Abashian *et al.*, *ibid.* **5**, 258 (1960); M. Ferro-Lussi *et al.*, Bull. Am. Phys. Soc. **5**, 509 (1960).
[2] G. F. Chew, University of California Radiation Laboratory Report UCRL-9289 (unpublished), p. 56.
[3] L. Castillejo, R. H. Dalitz, and F. Dyson, Phys. Rev. **101**, 453 (1956).
[4] See for example, A. O. Barut and K. H. Ruei, Nuclear Phys. **21**, 300 (1960).

or the two types of solutions of dispersion relations, by studying the resonance energy as a function of the coupling constant and discuss their relations to bound states.

II

The scattering amplitude due to an unstable state is given in second-order perturbation theory by[4]

$$J_{+}=\frac{\langle q'|H_1|\alpha\rangle\langle\alpha|H_1|q\rangle}{E-E_\alpha-(a+ib)}, \tag{1}$$

where q and q' are the initial and final states, α the unstable intermediate state, E_α the energy of the intermediate state, and

$$a+ib=\int\frac{dq''\langle\alpha|H_1|q''\rangle\langle q''|H_1|\alpha\rangle}{E-E''+i\epsilon}. \tag{2}$$

We write the amplitude (1) in the following form:

$$J_{+}=a_1\lambda/(1-X),\quad \lambda=g^2, \tag{3}$$

with

$$X=(E_\alpha+a+ib)/E;\quad \lambda a_1=\langle q'|H_1|\alpha\rangle\langle\alpha|H_1|q\rangle/E.$$

Equation (1) is also valid if the total energy of the system E is partly discontinuous (i.e., bound states). The poles of J_+ will correspond to the bound states which will lie in the unphysical region for scattering, $E<E_0$, where E_0 is the energy corresponding to a bound state of zero binding energy. $E>E_0$ is the physical region for scattering. Splitting (2) into a principal part and a part containing $i\pi\int\delta(E-E'')$, we get

$$a=P\int dq''|\langle\alpha|H_1|q''\rangle|^2/(E-E'');$$
$$b=\pi\int dq''\,\delta(E-E'')|\langle\alpha|H_1|q''\rangle|^2. \tag{4}$$

In the unphysical region $a<0$, $b=0$. Hence the poles of J_+ in this region are given by $E=E_\alpha+a<E_\alpha$. In the physical region $b>0$, a takes its minimum negative value at $E=E_0$. At $E=0$, $a<0$. We expect that the numerator of the integrand of a has a maximum in the neighborhood of $E=E_\alpha$. a becomes positive at higher energies and approaches zero as $E\to\infty$. From (3) the resonance energy is given by $E_r=E_\alpha+a$. Putting $a=\lambda f(E_r)$, we get

$$\lambda=(E_r-E_\alpha)/f(E_r). \tag{5}$$

We first plot a as a function of E_r, i.e., $f(E_r)$; then using (5) we see that and the form of λ as a function of the resonance energy is given schematically as in Fig. 1. If the α state has a definite angular momentum so that one may consider a partial wave scattering amplitude, and if we make the reasonable assumption

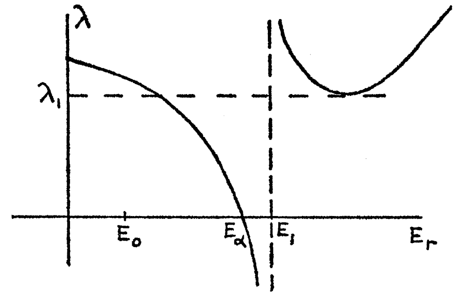


FIG. 1. Resonance energy E_r as a function of the coupling constant λ for kinematical resonances.

that there is only one resonance of that given angular momentum, then the first branch of the curve between $E_r=0$ and $E_r=E_\alpha$ is relevant. The resonance energy increases with *decreasing* coupling constant and, as one may expect, the resonance can occur for arbitrarily small coupling constants.[5]

III

We compare now this behavior with that of the dynamical resonance. Consider first the dynamical resonances in potential scattering. The scattering amplitude $f(E,\tau)$ where τ is the momentum transfer, or simply the partial wave amplitude $f_l(E)$ can be put in the form of Eq. (3):

$$f_l(E)=a_{1l}\lambda/(1-X_l)=\sum_{n=1}^{\infty}a_{nl}\lambda^n, \tag{6}$$

where we have put $H_1=\lambda V$ and

$$X_l=X_l^{(1)}+iX_l^{(2)}.$$

By the unitarity condition $X_l^{(2)}=[k/(2l+1)]a_{1l}\lambda$. $X_l^{(2)}=0$ in the unphysical region, $E<0$, by the reality condition in this region, where the bound states are given by

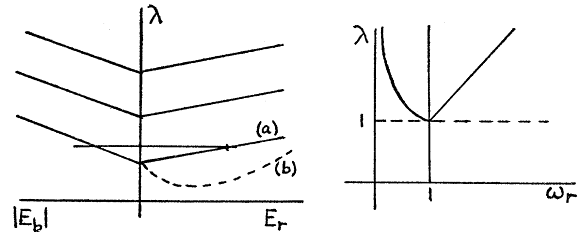
$$X_l(E,\lambda)=1; \tag{7}$$

the corresponding energies will be denoted by $E_b\leqslant 0$. In the physical region the resonances—if they exist—are given by

$$X_l^{(1)}(E_r,\lambda)=1,\quad E_r\geqslant 0. \tag{8}$$

The form of the curves (7) and (8) is given schematically in Fig. 2(a). Bound-state energies (E_b) will increase monotonically with increasing λ. Since $X_l^{(1)}$ is a continuous function of E, the two curves coincide at $E=0$. By Eq. (6) there is no resonance as $\lambda\to 0$, assuming that the series (6) converges; at the other end, for any finite λ and for a large class of potentials (regular at origin and sufficiently rapidly going to zero at infinity) the phase shift is different from $\pi/2$ as $E\to\infty$ for both relativistic and nonrelativistic potential scattering.[6] Thus the only way of having a resonance

[5] For such a resonance in weak interactions see S. L. Glashow, Phys. Rev. 118, 316 (1960).
[6] A. O. Barut and K. H. Ruei, J. Math. Phys. 2, 181 (1961).



(a) (b)

FIG. 2. Resonance and bound state energy as a function of the coupling constant for dynamical resonances. (a) Potential scattering; three bound states have been assumed. (b) Charged scalar meson theory.

for $E\rightarrow\infty$ is $\lambda\rightarrow\infty$. This gives us the general form of the resonance energy as a function of λ. If there is a single resonance for fixed angular momentum and fixed λ, then only curve (a) will appear.

If we increase λ, for a given potential, there may occur several bound states. One would expect then, for a fixed λ, n resonances if there are n bound states. For fixed λ the phase shift at $E=\infty$ is zero.[6] According to a theorem of Levinson,[7] the phase difference $\delta(0)-\delta(\infty)$ is equal to $n\pi$ if there are n bound states, none of them with binding energy zero; and it is equal to $(n-\frac{1}{2})\pi$ if one of the n bound states has zero binding energy. Therefore the phase difference between two adjacent resonances must be π and at $E=0$ the phase shift is discontinuous at the resonance points, as can be seen from Fig. 2.

Exactly the same behavior is found in charged scalar meson theory.[3] The amplitude for the Serber-Lee point-source model of this theory in one-meson approximation can be put in the form

$$e^{i\delta}\sin\delta/k=(\lambda/\omega)/(1-X);\; X=(\lambda/\omega)[1-(1-\omega^2)^{\frac{1}{2}}]. \quad (9)$$

The bound-state curves are given by $\lambda=\omega/[1-(1-\omega^2)^{\frac{1}{2}}]$ and in the physical region $(\omega>1)$ the resonance curves are given by $\lambda=\omega$ [Fig. 2(b)].

Thus *the dynamical resonances occur in connection with the bound states* and the resonance energy increases with *increasing* coupling constant. The resonance in the very low-energy neutron-proton scattering is precisely of this type.

IV

The method of analytic continuation of the coupling constant (i.e., coupling constant as a function of the resonance energy) discussed above has been actually constructed to deal with the solutions of dispersion relations. Given a solution of the dispersion relation, it is not clear whether we are dealing with the true solution or an extra solution. It is important to distinguish between these two types of solutions because one can either eliminate the extra solutions by suitable conditions, or, as the case may be, select the relevant extra solution if the resonance is due to an unstable particle. An important case in this connection is the 33-resonance in pion-nucleon scattering. A study of the analytic continuation of the coupling constant in the dispersion-type equation of Chew and Low[8] for the static p-wave meson-nucleon scattering reported separately.[9]

[7] N. Levinson, Kgl. Danske Videnskab. Selskab., Mat.-fys. Medd. **25**, No. 9 (1949).

[8] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[9] A. O. Barut and K. H. Ruei (to be published).

PHYSICAL REVIEW VOLUME 122, NUMBER 4 MAY 15, 1961

Internal State of a Gravitating Gas*

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(Received January 15, 1958; revised manuscript received December 28, 1959)

The significance of a theory of gravitational equilibrium of concentrated masses is discussed in connection with possible general relativistic effects in white dwarf stars. The covariant form of phase space and Liouville's theorem is developed, using the canonical equations for a particle under gravitational and electromagnetic forces. The dynamical isotropy of the ideal fluid is formulated, and the associated equations of state and allowed streaming patterns are found. A covariant kinetic theory yields general relativistic forms for the Maxwell and Fermi distributions in the case of thermal equilibrium, and limits their streaming to rigid motion. Rotating fluids are studied in comoving coordinates, and the problem of determining their gravitational equilibrium is reduced, in most cases of physical interest, to a simple standard form with constant density and vorticity.

I. INTRODUCTION

THIS paper is one of a series on the theory of great concentrations of gaseous matter capable also of rapid circulation. When a test particle falling freely from infinity towards the center of a resting mass reaches a speed comparable to that of light, the concentration of matter can be called great. When the

* This work was supported in part by the Office of Naval Research. Part of the work is based on an otherwise unpublished Ph.D. thesis presented by one of the authors (G.E.T.) to the University of Minnesota in 1951. A preliminary account of Secs. II and III was reported to the American Physical Society [Phys. Rev. **86**, 621(A) (1952)].