large peaks of Na^{24} reported by Stelson and Preston[2] indicate: (a) appreciable interference of levels of opposite parity; (b) the presence of s-wave levels in the region around 400 kev; (c) a dominant s-wave dependence near 450 kev; and (d) a strong s-wave level around 540 kev. The present data together with the results obtained by the analyses are consistent with these indications. Each large level reported by Stelson and Preston[2] was found to consist of a number of levels of opposite parity. Two s-wave levels do occur near 400 kev and the dominant level near 450 kev appears to be an s-wave level. The widest s-wave level found in Na^{24} is located near 540 kev.

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Reduced Widths and Isotopic Spin Impurities of $\frac{1}{2}+$ States of N^{15}†

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The nuclear reactions $C^{14}(p,n)N^{14}$ and $C^{14}(p,\gamma_0)N^{15}$, for protons of less than 1.6 Mev involve the interference of two adjacent states having the same spin and parity ($\frac{1}{2}+$) but different isotopic spin ($T=\frac{1}{2}$ and $\frac{3}{2}$). By taking properly into account the effect of other, more distant $\frac{1}{2}+$ levels on the cross section near the interfering pair of levels, we are able to fit well the (p,n) cross section from the neutron threshold up to a proton energy of 1.6 Mev and to obtain reliable estimates for the reduced width amplitudes of the interfering pair, as well as for the physically significant phases of the amplitudes. Since the neutron decay of the $T=\frac{3}{2}$ state is "forbidden," the neutron reduced widths of the close-lying pair lead to a direct measure 4% of the isotopic spin impurity of the pair. The results found in the cross section analysis are compared to shell model calculations based on the N^{15} wave functions of Halbert and French, and reasonably satisfactory agreement is found.

I. INTRODUCTION

THE nuclear reactions $C^{14}(p,n)N^{14}$ and $C^{14}(p,\gamma_0)N^{15}$, for proton energies of less than 1.6 Mev, have several unusual features of importance for nuclear structure. The (p,n) reaction cross section has been measured at several laboratories[1–3] while the (p,γ_0) results are given by Bartholomew *et al.*[1] The $C^{14}+p$ cross sections are shown on Fig. 1 along with a diagram of the levels of N^{15} which lie at or near the energy reached by $C^{14}+p$. The information on the compound states of N^{15} has been found from a variety of reactions as summarized by Ajzenberg-Selove and Lauritsen.[4] Although we shall consider the effects due to other states in describing the $C^{14}(p,n)N^{14}$ reaction, our interest will focus on the two adjacent $\frac{1}{2}+$ states at excitation energies of 11.438 and 11.610 Mev in N^{15}. These states are known to have isotopic spin $T=\frac{1}{2}$ and $T=\frac{3}{2}$, respectively.[5] The latter level is therefore forbidden to decay to the ground state of N^{14} by neutron emission.

As we shall see below, the two adjacent $\frac{1}{2}+$ states

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[1] G. A. Bartholomew, F. Brown, H. E. Gove, A. E. Litherland, and E. B. Paul, Can. J. Phys. **33**, 441 (1955).

[2] R. M. Sanders, Phys. Rev. **104**, 1434 (1956).

[3] J. H. Gibbons and R. L. Macklin, Phys. Rev. **114**, 571 (1959). The authors are grateful to Dr. Gibbons and Dr. Macklin for sending their results before publication.

[4] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

[5] G. A. Bartholomew, A. E. Litherland, E. B. Paul, and H. E. Gove, Can. J. Phys. **34**, 147 (1956).

show a cross-section shape strongly affected by their interference with each other. From a many-level analysis of the cross-section shape it is possible to deduce the reduced width amplitudes of the interfering levels. Because in this case the interfering levels do not have the same isotopic spin, it will turn out that the reduced width amplitudes give a model-independent estimate of the magnitude and sign of the isotopic spin impurity of the levels.

The purpose of the present paper is to carry out the many-level analysis of the $C^{14}+p$ reactions for the energy range of interest and to compare the resulting reduced width amplitudes and isotopic spin admixtures with calculations based on shell-model wave functions.

A many-level fit of the $C^{14}(p,n)N^{14}$ cross section and the $C^{14}(p,\gamma_0)N^{15}$ cross section near the two interfering $\frac{1}{2}+$ states has already been made by Ferguson and Gove.[6] Although they obtained a good fit to the (p,γ_0) cross section, their fit to the (p,n) cross section was not very good (see below), nor did it yield a reliable value for the forbidden neutron width of the $T=\frac{3}{2}$ state. It is the ratio of this forbidden width to the allowed neutron width of the neighboring $T=\frac{1}{2}$ $(J,\pi=\frac{1}{2}+)$ state which will inform us directly concerning the isotopic spin impurity.

Ferguson and Gove obtained a good fit to the $C^{14}(p,\gamma_0)N^{15}$ reaction but not to the $C^{14}(p,n)N^{14}$ reaction because the bound $\frac{1}{2}+$ levels, which they did not consider, have an important effect on the latter but not on the former. The (p,γ_0) cross section is dominated by the $T=\frac{3}{2}$ resonance at $E_p(\text{lab})=1.5$ Mev which has both a very large proton width and a very large gamma-ray width. For the (p,n) cross section the product $\gamma_{\lambda p}\gamma_{\lambda n}$, of proton reduced width amplitude and neutron reduced width amplitude, is rather small for both the 1.3-Mev level and the 1.5-Mev level. In a very rough way this means that the resonances in the $C^{14}(p,n)N^{14}$ reaction at 1.3 Mev and 1.5 Mev are both "weak" resonances which interfere because they are so close together. However, a distant $\frac{1}{2}+$ state with a large value of $\gamma_{\lambda p}\gamma_{\lambda n}$ may have a considerable effect near the adjacent $\frac{1}{2}+$ states. From the known behavior[2,3] of the $C^{14}(p,n)N^{14}$ reaction near threshold it is possible to deduce the value of $\gamma_{\lambda p}\gamma_{\lambda n}$ for the contributing $\frac{1}{2}+$ state, and to determine roughly where the level lies. It will turn out that the effect of the "distant" level or levels is important for the $C^{14}(p,n)N^{14}$ reaction and that it arises principally from one (or perhaps two) of the several bound $\frac{1}{2}+$ levels of N^{15} lying below the two adjacent $\frac{1}{2}$ states which are seen in the $C^{14}(p,n)N^{14}$ reaction.

In Sec. II we discuss the methods for analyzing the cross section of interfering levels and derive an approximate formula which takes into account the interference of a distant third level with two close-lying levels of the same spin and parity. Section III uses the methods of Sec. II for the analysis of the $C^{14}(p,n)N^{14}$ and $C^{14}(p,\gamma_0)N^{15}$ reactions.

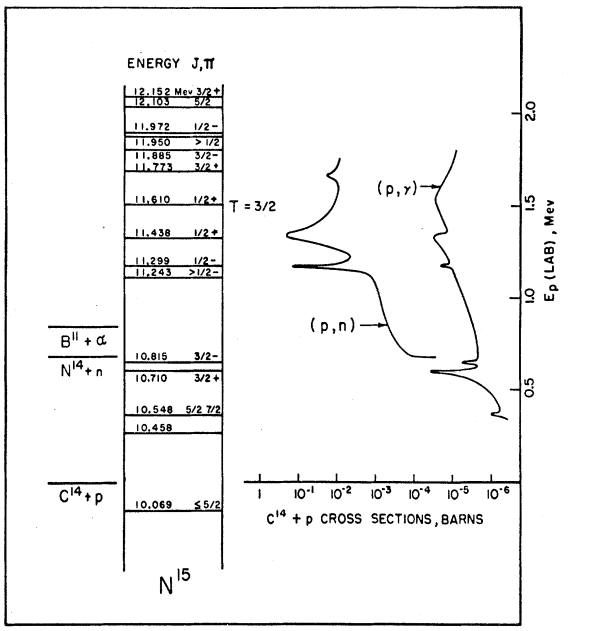

FIG. 1. The levels of N^{15} between 10 and 12 Mev and the $C^{14}+p$ cross sections.

The isotopic spin impurity of the close-lying pair of $\frac{1}{2}+$ states is directly related, in Sec. IV, to the ratio of the experimental reduced neutron width amplitudes of the pair. It is shown how this ratio can also be used to reduce the experimental reduced width amplitudes to "zero-order" amplitudes in which the isotopic spin impurity of the compound wave functions has been removed.

In Sec. V the experimental "zero-order" reduced-width amplitudes, including their relative phases, are compared with the calculations of Halbert and French[7] using the intermediate coupling shell model. The observed magnitude and sign of the isotopic spin impurity of the close-lying pair of levels are also compared with a prediction, based on a shell-model calculation of the Coulomb matrix element between the two states.

II. MULTILEVEL FORMULAS FOR THE $C^{14}+p$ REACTIONS

To describe the $C^{14}+p$ reactions below a proton energy of 1.6 Mev we need to consider the mutual interference of three or more $\frac{1}{2}+$ levels, only two of which lie in the energy interval for which the cross section is observed. The appropriate multilevel formulas will be derived from the general resonance theory of Wigner and Eisenbud.[8,9]

[6] A. J. Ferguson and H. E. Gove, Can. J. Phys. **37**, 660 (1959).

[7] E. Halbert and J. B. French, Phys. Rev. **105**, 1563 (1957).

[8] E. P. Wigner, Phys. Rev. **70**, 15 (1946); *ibid.* **70**, 606 (1946); and E. P. Wigner and L. Eisenbud, *ibid.* **72**, 29 (1947).

[9] For a summary of the resonance theory the reader is referred to the review articles of A. M. Lane and R. G. Thomas, Revs.

In the Wigner-Eisenbud theory the general expression for a cross section, $\sigma_{cc'}$, proceeding from an incident channel c to an outgoing channel c', integrated over the possible angles of c' is

$$\sigma_{cc'}=\frac{\pi}{k_c^2}\sum_J g_J|\delta_{cc'}-U_{cc'}{}^J|^2, \quad (1)$$

where k_c is the relative momentum of the incident particle and the target nucleus, $U_{cc'}{}^J$ is the collision matrix component of a given total spin J (and parity) referring to the channels c and c', and the statistical spin factor g_J is given by

$$g_J=(2J+1)/(2I+1)(2i+1), \quad (2)$$

where I is the spin of the target nucleus and i the spin of the incident particle. In addition to being integrated over all angles, Eq. (1) has been averaged over the possible polarizations of the incident nucleon and the target nucleus.

The most useful form of the collision matrix components $U_{cc'}{}^J$ for the case of a small number of interfering compound states of the same spin and parity is:

$$U_{cc'}{}^J=e^{i(\varphi_c+\varphi_{c'})}[\delta_{cc'}+i\sum_{\lambda\lambda'}(\Gamma_{\lambda c})^{\frac{1}{2}}(\Gamma_{\lambda' c'})^{\frac{1}{2}}A_{\lambda\lambda'}], \quad (3)$$

in which the sums over the indexes λ and λ' include all the interfering levels.[10] In (3), $(\Gamma_{\lambda c})^{\frac{1}{2}}$ is the square root of the observed partial width $\Gamma_{\lambda c}$ taken with the sign appropriate to the corresponding reduced-width amplitude, that is,

$$(\Gamma_{\lambda c})^{\frac{1}{2}}\equiv(2P_c)^{\frac{1}{2}}\gamma_{\lambda c}, \quad (4)$$

where P_c is the penetration factor for the channel c. The φ_c of Eq. (3) are potential scattering phase shifts[10] which occur only in the elastic scattering cross section. The level matrix, $A_{\lambda\lambda'}$, of (3) is defined in terms of its inverse,

$$(A^{-1})_{\lambda\lambda'}=(E_\lambda-E)\delta_{\lambda\lambda'}+\Delta_{\lambda\lambda'}-\tfrac{1}{2}i\sum_c(\Gamma_{\lambda c})^{\frac{1}{2}}(\Gamma_{\lambda' c})^{\frac{1}{2}}, \quad (5)$$

where c runs over all channels and E_λ is the energy of the level λ. The matrix $\Delta_{\lambda\lambda'}$ leads to shifts of the resonance energies from the values E_λ and is defined by

$$\Delta_{\lambda\lambda'}=-\sum_c(b_c+S_c)\gamma_{\lambda c}\gamma_{\lambda' c}, \quad (6)$$

where the sum again runs over all channels and S_c is the shift function for the channel c and b_c the boundary condition number for the channel c.

In the $C^{14}(p,n)N^{14}$ reaction the formation of the $\frac{1}{2}+$ compound states involves only a single channel—s-wave protons incident on a spin-zero target nucleus—and hence g_J is equal to unity. The decay of these states by neutron emission to the 1^+ ground state of N^{14} may, on the other hand, involve either s-wave or d-wave neutrons. However, the d-wave neutron penetrability is at least 20 times smaller than the s wave for the energies we are considering and, correspondingly, the $N^{14}+n$ elastic-scattering data show that the d-wave neutron emission from these levels is very small.[11] We shall, therefore, assume that neutron emission from the $\frac{1}{2}+$ states is purely s wave. As will be shown later, a small contribution from d-wave neutrons does not affect the analysis below.

Just as the decay by neutron emission of the $\frac{1}{2}+$ states involves only one channel, so the gamma decay to the $\frac{1}{2}-$ ground state of N^{15} involves only $E1$ radiation. Hence the (p,n) and the (p,γ_0) cross section of C^{14}, for $\frac{1}{2}+$ compound states, may be written as

$$\begin{aligned}\sigma_{pn}&=(\pi/k_p^2)|\textstyle\sum_{\lambda,\lambda'}\Gamma_{\lambda p}{}^{\frac{1}{2}}\Gamma_{\lambda' n}{}^{\frac{1}{2}}A_{\lambda\lambda'}|^2,\\ \sigma_{p\gamma_0}&=(\pi/k_p^2)|\textstyle\sum_{\lambda,\lambda'}\Gamma_{\lambda p}{}^{\frac{1}{2}}\Gamma_{\lambda'\gamma_0}{}^{\frac{1}{2}}A_{\lambda\lambda'}|^2.\end{aligned} \quad (7)$$

We note that in the level matrix $A_{\lambda\lambda'}$, of (5) and (7) we need to retain only quantities pertaining to the incident proton channel and the outgoing neutron channel: The gamma-ray widths are much too small to have any effect on $A_{\lambda\lambda'}$.

Although result (7) gives accurate many-level formulas for the $C^{14}+p$ reactions proceeding through $\frac{1}{2}+$ states, to use it one must limit the sum over levels to some finite number or else approximate it in some other way. If we ignore, say, all but three of the $\frac{1}{2}+$ states then the correct three-level formulas are given by (7), where the level indexes refer to the three chosen levels. The formulas still require inversion of a three-by-three matrix with complex components, so that computations may be rather laborious. The results described in the next section were obtained with the accurate three-level formulas (7), using the Chalk River Datatron computer. However, to discuss the principal features of these results we shall derive simpler (but approximate) three-level formulas. For this purpose the following two approximations are made in (7):

(1) The level shift matrix (6) is ignored. As discussed in the next section, the boundary condition numbers b_c of the shift matrix are to be chosen so that each component of Δ vanishes at some energy near the narrow level [E_p(lab)$=1.31$ Mev]. If the shift function S_c+b_c of each channel c is expanded in a Taylor's series about that energy, the first term vanishes by definition: As is well known, the next term in the expansion can be completely taken into account as a factor multiplying each width, that is, as a "renormalization" of the widths which are phenomenological parameters. Therefore, as far as the shape of the cross section is concerned

Modern Phys. **30**, 257 (1958), and of E. Vogt, *Nuclear Reactions* (North-Holland Publishing Company, Amsterdam, The Netherlands, 1958).

[10] A straightforward derivation of (3) can be found in recent work by E. Vogt, Phys. Rev. **112**, 203 (1958), dealing with the application of the multilevel formulas to another problem.

[11] The s-wave character of the neutrons involved in the two interfering $\frac{1}{2}+$ levels is shown, first of all, by the strong interference dip in the $N^{14}+n$ total cross section just below the 11.438-Mev level. [See J. J. Hinchey, P. H. Stelson, and W. M. Preston, Phys. Rev. **86**, 483 (1952); and C. H. Johnson, B. Petree, and R. K. Adair, Phys. Rev. **84**, 775 (1951)]. Secondly, the angular distribution of elastically scattered neutrons from N^{14} at the 11.610-Mev level ($J\pi=\frac{1}{2}+$, $T=\frac{3}{2}$) is isotropic. [See J. L. Fowler and C. H. Johnson, Phys. Rev. **98**, 728 (1955).]

we are only ignoring quadratic and higher terms in the expansion of the shift function.

(2) We remember that two of the $\frac{1}{2}+$ states which we are considering lie in the energy interval of interest (Fig. 1) while the other state lies at some distance from this interval. The second approximation consists, then, in considering E_0-E (where $\lambda=0$ refers to the distant level) to be large compared to the level widths occurring in the matrix A^{-1}, (5). If E_1-E and E_2-E are also considered as large (as they would be near the neutron threshold) then the cross section is simply

$$\sigma_{cc'}\approx(\pi/k_c^2)\left|\sum_\lambda \Gamma_{\lambda c}^{\frac{1}{2}}\Gamma_{\lambda c'}^{\frac{1}{2}}/(E_\lambda-E)\right|^2, \tag{8}$$

where the sum refers to the three levels $\lambda=0$, 1, 2. Expression (8) is derived from Eqs. (7) and (5) by retaining in (7) only those terms of highest order in $(E_\lambda-E)$. On the other hand, if we are near the two adjacent $\frac{1}{2}+$ levels at $E_p(\text{lab})=1.31$ and 1.50 Mev ($\lambda=1$ and 2) so that E_1-E and E_2-E are of the same magnitude as the level widths but smaller than E_0-E, then, if we retain only terms of highest order in E_0-E, Eq. (7) reduces to a two-level formula involving the levels $\lambda=1$, 2. Combining (8) with this result yields the following approximate three-level formulas applicable to the $\frac{1}{2}+$ states over the whole energy interval shown in Fig. 1.

$$\sigma_{pn}\approx(\pi/k_p^2)\left|\frac{\Gamma_{0p}^{\frac{1}{2}}\Gamma_{0n}^{\frac{1}{2}}}{E_0-E}+\frac{\Gamma_{1p}^{\frac{1}{2}}\Gamma_{1n}^{\frac{1}{2}}(E_2-E)+\Gamma_{2p}^{\frac{1}{2}}\Gamma_{2n}^{\frac{1}{2}}(E_1-E)}{(E_1-E-\frac{1}{2}i\Gamma_1)(E_2-E-\frac{1}{2}i\Gamma_2)+\frac{1}{4}\Gamma_{12}^2}\right|^2, \tag{9}$$

$$\sigma_{p\gamma_0}\approx(\pi/k_p^2)\left|\frac{\Gamma_{0p}^{\frac{1}{2}}\Gamma_{0\gamma_0}}{E_0-E}+\frac{\Gamma_{1p}^{\frac{1}{2}}\Gamma_{1\gamma_0}^{\frac{1}{2}}(E_2-E)+\Gamma_{2p}^{\frac{1}{2}}\Gamma_{2\gamma_0}^{\frac{1}{2}}(E_1-E)+iR}{(E_1-E-\frac{1}{2}i\Gamma_1)(E_2-E-\frac{1}{2}i\Gamma_2)+\frac{1}{4}\Gamma_{12}^2}\right|^2, \tag{10}$$

where

$$R\equiv\tfrac{1}{2}[(\Gamma_{1n}^{\frac{1}{2}}\Gamma_{2\gamma_0}^{\frac{1}{2}}-\Gamma_{2n}^{\frac{1}{2}}\Gamma_{1\gamma_0}^{\frac{1}{2}})(\Gamma_{1n}^{\frac{1}{2}}\Gamma_{2p}^{\frac{1}{2}}-\Gamma_{2n}^{\frac{1}{2}}\Gamma_{1p}^{\frac{1}{2}})]. \tag{11}$$

In (9) and (10) the "mixed width," Γ_{12}, is defined by

$$\Gamma_{12}\equiv\sum_c \Gamma_{1c}^{\frac{1}{2}}\Gamma_{2c}^{\frac{1}{2}}, \tag{12}$$

where c runs over all channels (two in our case). In expressions (9) and (10) the first term in the absolute square yields the direct contribution of the distant level to the cross section, the second yields the contribution of the levels 1 and 2, including their mutual interference,[12] and the cross term gives the interference of the distant level with the adjacent pair.

If the interference of more than one distant $\frac{1}{2}+$ level is to be taken into account then the approximate cross-section expressions are the same as expressions (9) and (10), except that we replace the amplitude $(\Gamma_{0p}\Gamma_{0n})^{\frac{1}{2}}/(E_0-E)$ of the distant level by $\sum_\lambda(\Gamma_{\lambda p}\Gamma_{\lambda n})^{\frac{1}{2}}/(E_\lambda-E)$, where the sum applies to all the distant levels. Although the analysis given below is made for three interfering levels, this replacement will be useful in comparing the observed amplitude of the distant level with that predicted by the shell model.

The interference between resonances determines the relative signs of some of the reduced width amplitudes, as is evident from Eqs. (7) and (8). We inquire first what information about the relative signs is physically significant and then show that this is given by the analysis. We are dealing with three compound levels each of which is connected with the same pair of final states. The proton and neutron amplitudes $(\gamma_{\lambda p},\gamma_{\lambda n})$ form a 3×2 dimensional array and there are $(2)^6=64$ possible arrangements of signs. However, a change of all the signs in one row or in one column would correspond simply to a phase change of a final state or a compound state, which is of no physical significance. Of these five operations only four are independent (since a sign change in all the rows is equivalent to a sign change in all the columns) and thus there are only $64/2^4=4$ physically significant sign combinations. In other words, the phases of the compound states and of the final states may be used to fix the sign of four of our six reduced width amplitudes. In the remainder of this paper we shall define γ_{0n}, γ_{1n}, γ_{2n}, and γ_{2p} to be positive (i.e., $\Gamma_{0n}^{\frac{1}{2}}$, $\Gamma_{1n}^{\frac{1}{2}}$, $\Gamma_{2n}^{\frac{1}{2}}$, and $\Gamma_{2p}^{\frac{1}{2}}$ are positive). Then the four physically significant sign arrangements correspond to the four possible sign combinations of γ_{0p}, γ_{1p}.

It is readily established that the cross-section analysis determines all the physically significant phases. From the general many-level formula, [(3) and (1)], it is easily verified that each cross section remains unchanged if the phase of a final state or a compound state is changed (one must note that the level matrix component, $A_{\lambda\lambda'}$, changes sign along with either the state λ or the state λ'). If we then adopt the phase convention given above (γ_{0n}, γ_{1n}, γ_{2n}, and γ_{2p} positive) we see that the cross-section analysis determines the sign of γ_{0p}, γ_{1p}. Thus, in Eq. (8), the sign of $\Gamma_{1p}^{\frac{1}{2}}$ is determined by the value of the second term under the absolute value sign; the interference of the first term with this second one determines the sign of $\Gamma_{0p}^{\frac{1}{2}}$. In a rough way we can say that (within our phase convention) γ_{1p} is positive if the interference of the level $\lambda=1$ with the level $\lambda=2$ is destructive at energies in between E_1 and E_2 (and constructive elsewhere); similarly γ_{0p} is positive if the

[12] The mutual interference of two levels of different total isotopic spin is to be contrasted with the failure, as in Eq. (1), of two levels of different total angular momentum to interfere with each other. To explain this difference we note that our reaction channels are polarized in isotopic spin space. On the other hand, the interference between different values of J has disappeared from Eq. (1) only because (1) has been averaged over all possible polarizations of the spins of the incoming and outgoing particles. If it were possible to construct beams without polarization in isotopic spin space the interference between states of different T would disappear.

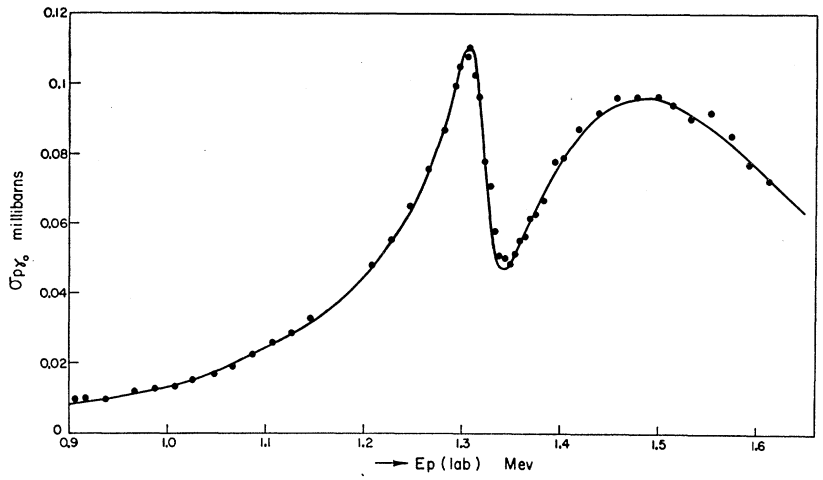

FIG. 2. The $C^{14}(p,\gamma_0)N^{15}$ cross section as a function of the proton energy in the laboratory system. The points are the data of Bartholomew *et al.*[1] and the solid line is the computed contribution from the two $\frac{1}{2}+$ states responsible for the two peaks.

interference of the level $\lambda=0$ with the level $\lambda=2$ is destructive in between E_0 and E_2. The interference of the level $\lambda=0$ with the level $\lambda=1$ cannot be fixed separately: it is destructive in between E_0 and E_1 if the signs of γ_{0p} and γ_{1p} are the same and constructive if not. This picturesque relation between the physically significant phases and cross-section behavior is accurate only for energies which are many level widths away from the resonance energies of each of the interfering levels so that approximate formulas, such as (8), are valid.

III. ANALYSIS OF THE CROSS SECTIONS

The $C^{14}(p,\gamma_0)N$ data reaction which we shall use are those of Bartholomew, Brown, Gove, Litherland, and Paul,[1] which was also used by Ferguson and Gove[6] in their analysis. Figure 2 shows the data along with the theoretical fit discussed below.

There are three independent sets of experimental data[1–3] for the $C^{14}(p,n)N^{14}$ reaction. The three sets do not agree well with each other: Apart from small energy shifts ($\sim$20 kev) between the sets of data the shape of the cross section near the 1.31 Mev resonance is different for the three sets. For our analysis we have accepted the Oak Ridge data of Gibbons and Macklin[3] (shown in Fig. 3), which is the most recent. It appears everywhere to coincide with either the Chalk River data[1] or the Wisconsin data of Sanders.[2] The qualitative features of the analysis below are not affected by the choice of data although the values of the reduced widths obtained are. Only part of the difference (see Table I) between our results and those of Ferguson and Gove (who used the Chalk River data) are due to the choice of data.

Rough values of some of the partial widths of the $\frac{1}{2}+$ levels can be obtained directly from the observed peak cross sections and the full widths at half maximum of

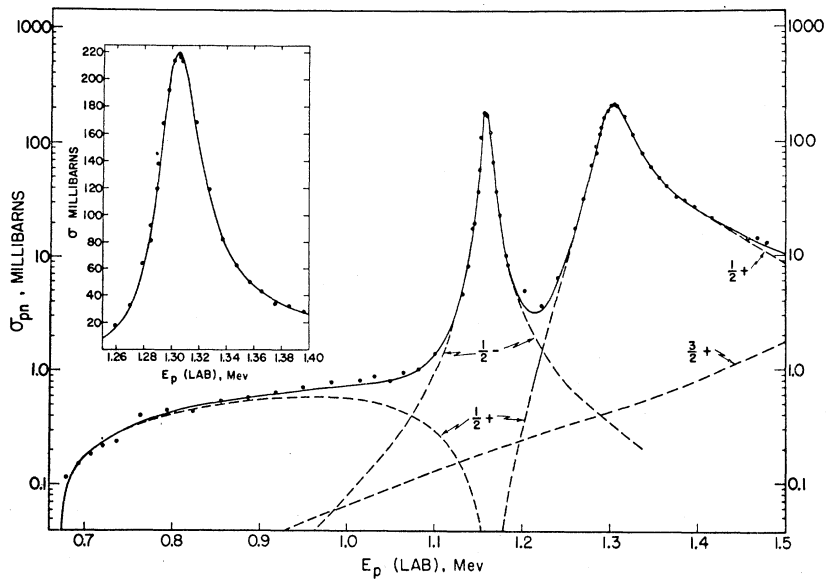

FIG. 3. The $C^{14}(p,n)N^{14}$ reaction as a function of the proton energy in the laboratory system. The points are the data of Gibbons and Macklin.[3] The solid line is the computed cross section. Various contributions to the computed cross sections are also given (broken curves) and labeled by the spin and parity of the states giving rise to them. The inset gives part of the data and the computed curve in greater detail.

the resonances. The parameters determined roughly in this way are:

(1) $\Gamma_{1n}\approx\Gamma_1$ from the full width ($\sim$40 kev) at half maximum of the resonance at E_p(lab)$=1.31$ Mev (see either Fig. 2 or Fig. 3). The fact that the neutron width makes up by far the largest part of the total width is known, for example, from the $N^{14}+n$ elastic scattering of this resonance.

(2) Γ_{1p} from the peak cross section of the (p,n) data and from Γ_{1n}. The peak cross section is known[2] to within an absolute value of 30%.

(3) $\Gamma_{2p}\sim\Gamma_2$ from the full width at half maximum ($\sim$400 kev) of the $T=\frac{3}{2}$ level as observed in the (p,γ_0) reaction (Fig. 2). It is clear from the figure that Γ_2 is very large, but it is not easily estimated with accuracy.

(4) $\Gamma_{1\gamma_0}$ from the value of $\Gamma_{1p}\Gamma_{1\gamma_0}/\Gamma_1^2$ given by the peak cross section of the (p,γ_0) reaction at E_p(lab)$=1.31$ Mev and from Γ_1 and Γ_{1p} above.

(5) $\Gamma_{2\gamma_0}$ from the value of $\Gamma_{2\gamma_0}\Gamma_{2p}/\Gamma_2^2\approx\Gamma_{2\gamma_0}/\Gamma_2$ given by the peak cross section of the (p,γ_0) reaction at E_p(lab)$=1.50$ Mev, and from Γ_2 above.

The level width Γ_{2n}, which is closely related to the isotopic spin impurity, can only be determined by an accurate fit to the (p,n) cross section.

Before analyzing the contribution of the $\frac{1}{2}+$ states to the (p,n) cross section we must account for the contribution to the cross section of levels of different spin and parity. From various data[4] estimates of (or upper limits for) the widths for particle emission have been made for each of the levels shown in Fig. 1. Apart from the $\frac{1}{2}+$ states, the only levels which may make an appreciable contribution to the cross section are the $\frac{1}{2}-$ level at 11.299 Mev and the $\frac{3}{2}+$ levels at 10.710, 10.806, 11.773, and 12.152 Mev: no other contribution is large enough to appear on Fig. 3 (on which the ordinate goes down to 4×10^{-5} b). The level at 11.243 Mev of unknown spin is very sharp ($\Gamma_n<3$ kev, $\Gamma_p<0.01$ kev) and could make a contribution very near its resonance energy. However, no such sharp small resonance is seen at E_p(lab)$=1.10$ Mev so that Γ_p is probably considerably smaller than 0.01 kev.

The contribution of the $\frac{1}{2}-$ level at E_p(lab)$=1.159$ Mev has been calculated, as shown in Fig. 3. The fit shown there takes into account the large effect of target thickness on this narrow level. The level widths resulting from the fit are $\Gamma_p=7.7$ kev, $\Gamma_n=0.77$ kev (in the center-of-mass system) corresponding to reduced widths of $\gamma_p^2=34.3$ and $\gamma_n^2=1.8$ kev.

The (p,n) cross section from the neutron threshold up to E_p(lab)$=1.0$ Mev (Fig. 3) must be accounted for by the $\frac{3}{2}+$ or $\frac{1}{2}+$ states, which can emit s-wave neutrons. Since the $\frac{3}{2}+$ states are formed by d-wave protons—unlike the $\frac{1}{2}+$ states, which are formed by s waves—it seems likely that the latter would contribute much more to the cross section near threshold than the former. This can be shown to be so. There are two $\frac{3}{2}+$ states which lie above the neutron threshold of Fig. 1, one at 11.773 Mev and the other at 12.152 Mev. The contribution of these two levels to the (p,n) cross section is shown in Fig. 3 by the broken curve labeled $\frac{3}{2}+$. Their mutual interference is assumed to be constructive in Fig. 3, so that the estimate shown there may be much too large. The $\frac{3}{2}+$ states which lie above the one at 12.152 Mev all lie above 13 Mev and are not therefore expected to make any appreciable contribution to the (p,n) cross section. The $\frac{3}{2}+$ states which lie at 10.710 Mev and 10.806 Mev (just below the neutron threshold) cannot make a major contribution to the (p,n) cross section near threshold. The 10.806-Mev level is only 36 kev below the threshold and appears[13] to contribute significantly to the low-energy neutron cross sections of N^{14}. However, because of the very small proton width of this level[13] it cannot contribute appreciably to the (p,n) cross section except within a few kev of the threshold. On the other hand, the 10.710-Mev level has a large proton width.[13] Because of its proximity to the threshold the contribution from this level begins to fall off shortly above the threshold, unlike the data, even if the interference of the

TABLE I. Parameters of the $\frac{1}{2}+$ states of N^{15}. All results are given in the center-of-mass system.

A. The two levels at 11.43 and 11.61 Mev

	Present analysis	Ferguson and Gove	Present analysis (isotopic spin impurity removed)	Shell-model calculations
E_1, Mev	11.452	11.444	11.452	12.73
E_2, Mev	11.647	11.622	11.647	12.34
$\Gamma_{1\gamma_0}$, ev	4.0	4.2		
$\Gamma_{2\gamma_0}$, ev	25.0	19.2		
Γ_{1p}, kev	6.9	6.8		
Γ_{2p}, kev	475.0	400.9		
Γ_{1n}, kev	31.0	34.6		
Γ_{2n}, kev	1.75	4.0		
γ_{1p}, $(\text{Mev})^{\frac{1}{2}}$	−0.091	−0.0888	0.056	0.108
γ_{2p}, $(\text{Mev})^{\frac{1}{2}}$	0.667	0.6084	0.670	0.562
γ_{1n}, $(\text{Mev})^{\frac{1}{2}}$	0.141	0.1482	0.144	0.122
γ_{2n}, $(\text{Mev})^{\frac{1}{2}}$	0.0314	0.0472	0	0

B. Other $\frac{1}{2}+$ levels

	Shell-model calculations				
Measured energy,[a] Mev	E_λ, Mev	$\gamma_{\lambda p}$, $(\text{Mev})^{\frac{1}{2}}$	$\gamma_{\lambda n}$, $\text{Mev}^{\frac{1}{2}}$	$\gamma_{\lambda p}\gamma_{\lambda n}$, Mev	Exptl $\gamma_{\lambda p}\gamma_{\lambda n}$, Mev
5.31	6.51	−0.571	−0.157	0.075	b
8.32 or 9.06	9.75	−0.188	0.864	−0.163	−0.131 or −0.094
	14.40	−0.082	0.176	−0.015	
	16.32	0.471	0.128	0.061	

[a] The levels observed could be either $\frac{1}{2}$ or $3/2+$.

[b] As discussed in the text, the analysis of the data determines $\Sigma_\lambda \gamma_{\lambda p}\gamma_{\lambda n}/(E_\lambda-E)$, where the sum should run over all the levels and E is an energy near the neutron threshold ($E\sim11$ Mev). The experimental value of the sum is $+0.049$. The calculated contributions from the four levels are, respectively: -0.013 ($E_\lambda=5.31$ Mev), $+0.061$ ($E_\lambda=8.32$ Mev), -0.004 ($E_\lambda=14.4$ Mev), $+0.011$ ($E_\lambda=16.32$ Mev), where experimental energies are used for the first two levels. The calculated sum is thus $+0.055$; if we use $E_\lambda=9.06$ instead of $E_\lambda=8.32$ for the nearest bound level, the calculated sum is $+0.078$.

[13] D. F. Hebbard and D. N. F. Dunbar, Phys. Rev. **115**, 624 (1959).

level with the higher lying $\frac{3}{2}+$ states is taken into account in an optimal manner. We therefore neglect the effect of the 10.710-Mev level in Fig. 3 and conclude that its neutron-reduced width (not known) must be reasonably small. No other lower lying $\frac{3}{2}+$ state should make an appreciable contribution to the (p,n) cross section in Fig. 3. The level at 10.458 Mev could be $\frac{3}{2}+$ but it is known[13] to have a very small proton width. A $\frac{3}{2}+$ level lying below 10 Mev could not have an important effect because of its distance.

In the (p,n) cross section of Fig. 3 the difference between the data and the sum of the $\frac{1}{2}-$ and $\frac{3}{2}+$ contributions must be accounted for by the $\frac{1}{2}+$ levels. In addition to the two $\frac{1}{2}+$ levels at 11.438 and 11.610 Mev there are only two or three other $\frac{1}{2}+$ levels in N^{15} below 14 Mev, and all of them lie below 9.062 Mev. We shall neglect the $\frac{1}{2}+$ states above 14 Mev in the analysis —the shell-model calculations given below assign small particle widths to these distant high-lying levels.

Because the bound $\frac{1}{2}+$ levels are so distant the analysis does not distinguish between them. According to the approximate formulas (10) and (8) above, their effects may be lumped together in one term, $4P_pP_n[\sum_\lambda \gamma_{\lambda p}\gamma_{\lambda n}/(E_\lambda-E)]$, of the cross-section formula, the square bracket being essentially a constant, the one free parameter concerning these levels. For sake of definiteness we shall consider only one bound level in the analysis so that we can employ the three-level formula above. The final result we get for $\gamma_{0p}\gamma_{0n}/(E_0-E)$ will, however, be compared to the shell-model calculation of $\sum_\lambda \gamma_{\lambda p}\gamma_{\lambda n}/(E_\lambda-E)$, where the sum runs over all the bound $\frac{1}{2}+$ levels.

In addition to determining accurate values of the parameters γ_{1n}, γ_{1p}, and γ_{2p} whose rough values are known from the peak heights and widths of the (p,n) and (p,γ_0) cross sections, the accurate analysis of the contribution from $\frac{1}{2}+$ states to the (p,n) cross section on Fig. 3 should give us values of γ_{2n}, of $\gamma_{0p}\gamma_{0n}$, and of the signs of γ_{0p}, γ_{1p} (with the sign convention given above).

The level $\lambda=1$, for which both γ_{1n} and γ_{1p} are roughly known, cannot account by itself for the "shoulder" in the (p,n) cross section on its high energy side—that is near $E_p(\text{lab})=1.5$ Mev in Fig. 3. The bound level $\lambda=0$ and the $T=\frac{3}{2}$ level $\lambda=2$ must together account for this shoulder and they must together interfere destructively with the level $\lambda=1$ near $E_p(\text{lab})=1.1$ Mev as well as account for the cross section near threshold. To determine γ_{2n} we must find out what part, if any, of the "shoulder" at 1.50 Mev arises from it. The distant bound $\frac{1}{2}+$ levels could, in principle, give rise to the "shoulder" through their interference with the level $\lambda=1$. However, it turns out that the contribution to the "shoulder" from this latter source cannot be large. If it were, then the (p,n) cross section near threshold would be much larger than is observed. Consequently the level $\lambda=2$ must account for most of the "shoulder." Together with the rough values of Γ_{1p}, Γ_{1n}, and Γ_{2p} derived above this effect determines a rough value of Γ_{2n}, and fixes γ_{1p} to be negative.

Having rough values of Γ_{1p}, Γ_{1n}, Γ_{2p}, and Γ_{2n}, the (p,n) cross section can now be used to determine many of the resonance parameters accurately. We begin by noting that from the rough values of the above parameters and the negative sign of γ_{1p} we conclude that the levels $\lambda=1$ and $\lambda=2$ interfere destructively below $E=E_1$, so that their combined contribution near the neutron threshold is no more than about 1% of the observed (p,n) cross section there. (Without interference, the contribution from the two levels would be larger than the observed (p,n) cross section near threshold). Consequently the value of $\gamma_{0n}\gamma_{0p}$ is determined very accurately by the (p,n) cross section below $E_p(\text{lab})=1.0$ Mev. Furthermore, γ_{0p} must be negative—otherwise one would not obtain the required destructive interference of the three $\frac{1}{2}+$ levels in the vicinity of $E_p(\text{lab})=1.1$ Mev. Having determined $\gamma_{0n}\gamma_{0p}$, accurate values of Γ_{1p}, Γ_{1n}, Γ_{2p}, and Γ_{2n} are found by fitting the shape of the (p,n) cross section accurately between $E_p(\text{lab})=1.2$ and 1.5 Mev, as shown in Fig. 3.

From the fact that $\Gamma_{1p}\ll\Gamma_{1n}$ and $\Gamma_{2n}\ll\Gamma_{2p}$ we observe that Γ_{1p} and Γ_{2n} may be varied considerably without effecting a large change in Γ_1, Γ_2, or Γ_{12}. The approximate three-level formula, (9), for the (p,n) cross section then shows that a simultaneous increase (or decrease) of $(\Gamma_{0n}\Gamma_{0p})^{\frac{1}{2}}$, Γ_{1p}, and Γ_{2n} merely increases (or decreases) the absolute value of the (p,n) cross section. In Fig. 3 the peak cross section at $E_p(\text{lab})=1.31$ Mev has been chosen to be 220 mb compared to the absolute value measured by Sanders,[2] of (315 ± 100) mb. Our analysis showed that the calculated (p,n) cross section could be increased by as much, perhaps, as a 100 mb, before the increased values of Γ_{1p} and Γ_{2n} would destroy the good fit to the shape of the cross section which is shown in Fig. 3. An accurate measurement of the absolute value, of the (p,n) cross section would allow $(\Gamma_{0p}\Gamma_{0n})^{\frac{1}{2}}$, Γ_{1p}, and Γ_{2n} to be determined accurately.

Analysis of the $C^{14}(p,\gamma_0)N^{15}$ cross section provides a little more information on the level parameters than is given earlier. Because of the large value of Γ_{2p} and $\Gamma_{2\gamma_0}$ the distant $\frac{1}{2}+$ levels are much less important in the (p,γ_0) cross section than in the (p,n). Extensive investigation showed that the level $\lambda=0$, even for moderately large values of $\Gamma_{0\gamma_0}$, has no important effect on the (p,γ_0) cross sections of Fig. 2; the minor effects of this level could easily be simulated by small changes in the parameters of the other two $\frac{1}{2}+$ levels. It is evident from (10) that the small but finite (Γ_{2n}) has only a small effect on the (p,γ_0) cross section. Similarly Γ_{1p} enters into the (p,γ_0) cross section mainly in the product $\Gamma_{1p}\Gamma_{1\gamma_0}$, and any change in Γ_{1p} can be compensated for by changing $\Gamma_{1\gamma_0}$. Thus the (p,γ_0) cross section does not determine the three parameters which "scale" the (p,n)

cross section. Of course, Γ_{1n} is accurately determined by the shape of both the (p,γ_0) and the (p,n) cross sections: Γ_{2p} is only very roughly determined by the width of the very broad level $\lambda=2$, on Fig. 2. A more accurate value of Γ_{2p} results from the value of the (p,γ_0) cross section at the interference minimum near $E_p(\text{lab})=1.35$ Mev in Fig. 2. The term R [expression (11)] is only slowly energy dependent and fixes the value of this minimum. With the (p,n) cross section scaled as in Fig. 3 [to 220 mb at $E_p(\text{lab})=1.31$ Mev, the same value calculated by Ferguson and Gove[6]], our value of Γ_{2n} is a factor of two smaller than that obtained by Ferguson and Gove. This change in Γ_{2n} has sufficient effect on R so that Γ_{2p} must be increased to a value 20% larger than that of Ferguson and Gove. The result for Γ_{2p} is not strongly effected by the "scaling" of the (p,n) cross section.[14]

The value of the level parameters found from the cross section fits of Figs. 2 and 3 are given in Table I, where they are compared to the shell model calculations discussed below.

The only level widths which are determined accurately (to within a few percent) are Γ_{1n}, Γ_{2p}, and $\Gamma_{2\gamma_0}$. The relative values of $(\Gamma_{0p}\Gamma_{0n})^{\frac{1}{2}}$, Γ_{2n}, Γ_{1p}, and $\Gamma_{1\gamma_0}^{-1}$ are accurately determined but their absolute value is known, roughly, only to within the 30% error associated with the absolute value of the (p,n) cross section.

The effect of the level shift on the cross section depends slightly on the value of the boundary condition number b_p accompanying the proton shift function (6). [The s-wave neutron shift vanishes identically.] The expression b_p is chosen so that each component of the shift matrix vanishes at the narrow level, $\lambda=1$. If we alter b_p slightly, e.g., so that Δ vanishes at $E_p(\text{lab})=1.40$ Mev instead of at $E_p(\text{lab})=1.305$ Mev, the peak cross section at the latter energy is reduced by 5% with, *mutatis mutandis*, no other change in the cross section. The effect is due to the nonzero value of Δ_{12} at the narrow peak. To minimize the possible effect of ignored $\frac{1}{2}+$ levels on the cross section we should choose Δ to vanish identically at $E_p(\text{lab})=1.305$ Mev. In the expansion of the shift function about this energy the linear term which "renormalizes" the widths is large. Part of the difference between our widths and those of Ferguson and Gove[6] is due to this source.

IV. MAGNITUDE OF THE ISOTOPIC SPIN IMPURITY

Having deduced with some care the experimental value of γ_{2n}, we shall use it now to determine the isotopic spin impurity of the $T=\frac{3}{2}$ state ($\lambda=2$). We do not need to consider isotopic spin impurities in the $A=14$ target or residual nuclei because it is well known[15] that the admixtures in the low-lying states are entirely negligible.

The nonzero value of the forbidden reduced-width amplitude γ_{2n} cannot be attributed to an admixture of isotopic spins arising in the exit channel, $N^{14}+n$. Since this channel involves neutrons the only electromagnetic interaction in the channel space is the small one involving the anomalous magnetic moment of the neutron. Even for a charged particle, the Coulomb admixture of isotopic spins in the channel space will be small compared to that arising within the compound nucleus; the Coulomb energy of a particle integrated from the nuclear radius to infinity is only a few percent[15] of the total Coulomb energy inside the nucleus. The nonzero value of γ_{2n} must be attributed to the admixture of a $T=\frac{1}{2}$ state (or states) into the $T=\frac{3}{2}$ level.

If in the charge-independent limit a state of N^{15} is $X_{\lambda T}$, the actual wave function will be

$$[1+\sum_{\lambda' T'}(\alpha_{\lambda T;\lambda' T'})^2]^{-\frac{1}{2}}(X_{\lambda T}+\sum_{\lambda' T'}\alpha_{\lambda T;\lambda' T'}X_{\lambda' T'}), \quad (13)$$

the $\alpha_{\lambda T;\lambda' T'}$ denoting the amplitudes of the impurity states. In the present case only T values $\frac{1}{2}$, $\frac{3}{2}$ need be considered since the lowest $T=\frac{5}{2}$ state would be many Mev higher. Besides this, when we are dealing with an isolated close-lying pair of levels of the same (J,π) but different T it follows that the admixing interaction simply admixes into one state a multiple of the other—the sums in expression (13) are limited to the pair of levels, in our case, the levels 1 and 2 of the $C^{14}(p,n)N^{14}$ reaction. This argument could fail if the matrix element linking the $\lambda=2$, $\lambda=1$ wave functions were very small compared with that to distant $\frac{1}{2}+$ states. This possibility we consider later on the basis of a calculation of the matrix element. The assumption that $\alpha_{1,\frac{1}{2};2,\frac{3}{2}}(\equiv-\alpha_{2,\frac{3}{2};1,\frac{1}{2}})$ is the only nonzero admixture coefficient in expression (13) establishes a connection between γ_{2n}, γ_{1n} and the impurity amplitude. We have simply

$$\gamma_{2n}/\gamma_{1n}=\frac{\text{amplitude of } T=\frac{1}{2} \text{ basic state in the level } \lambda=2}{\text{amplitude of } T=\frac{1}{2} \text{ basic state in the level } \lambda=1}=\alpha_{21}, \quad (14)$$

where we have dropped the isotopic spin indexes from $\alpha_{2,\frac{3}{2};1,\frac{1}{2}}$ for convenience.

Result (14) determines α_{21} from the observed values of γ_{2n} and γ_{1n} and is independent of any nuclear model. Using the reduced width amplitudes of Table I, we find[16] $\alpha_{21}=0.22$. This magnitude of α_{21} implies that the expectation value of the isotopic spin operator taken in the level $\lambda=2$ has only a 5.0% probability of assuming the value $\frac{1}{2}$. No direct derivation of this sort for the isotopic

[14] The level $\lambda=0$ does not affect the value of R. In the accurate three-level formula the R of expressions (10) and (11), which may be labeled R_{12} because it refers to the levels $\lambda=1$ and 2, is replaced by $R\rightarrow(E_0-E)R_{12}+(E_2-E)R_{10}+(E_1-E)R_{20}$. Because of the large value of E_0-E and R_{12} (from its dominant term $\Gamma_{1n}\Gamma_{2p}{}^{\frac{1}{2}}\Gamma_{2\gamma_0}{}^{\frac{1}{2}}$) the terms containing R_{10} and R_{20} are unimportant. This has been verified by computations with the three-level formula.

[15] W. M. MacDonald, Phys. Rev. **101**, 271 (1956).

[16] Again, the magnitude of $\alpha_{21}{}^2$ has the 30% uncertainty of the absolute value of the cross section.

spin impurity has been made previously. Other derivations of the impurity come mainly from the γ transitions of self-conjugate nuclei[17] and have always suffered from the shortcoming that an average value of the unretarded matrix element must be used in making the determination.

We stress that γ_{2n}/γ_{1n} is the only quantity in our analysis which is directly related to an isotopic spin impurity. Thus γ_{1n} and γ_{1p}, for example, are not related through a Coulomb matrix element because the C^{14} and N^{14} ground states are not members of the same isotopic spin triplet.

The admixture of the states has resulted in a finite value of the otherwise-forbidden neutron width for the $T=\frac{3}{2}$ state. In other words, it produces reduced width amplitudes which are different from the "zero-order" ones that would pertain if there were no breakdown of charge independence. The zero-order ones are, however, significant for us since in the next section we compare the results with those of a charge-independent shell model, and it is therefore worthwhile to determine them. This is quite simple to do since we are only considering the mutual admixing of the close-lying pair. It follows then by the properties of a 2− dimensional orthogonal matrix that $\alpha_{21}=-\alpha_{12}$ and thus we determine the zero-order amplitudes of the level $\lambda=1$ as well as those of $\lambda=2$. The significant thing here is that the small admixture of the $T=\frac{3}{2}$ state into its $T=\frac{1}{2}$ neighbor causes a large change in the proton amplitude γ_{1p}. As shown in Table I, we find a zero-order value of γ_{1p} which is different in sign from the experimental value of γ_{1p}.

V. COMPARISON OF THE RESULTS WITH SHELL-MODEL CALCULATIONS

In this section we compare the reduced-width amplitudes and the isotopic spin impurity amplitude determined by the reaction analysis to the amplitudes calculated by a shell model. The calculation makes use of the wave functions calculated several years ago by Halbert and French[7] and used in their study of the positive parity levels of N^{15}. In this calculation the positive parity levels were regarded as arising from the three "first excited" configurations, $(1s)^4(1p)^{10}2s$, $(1s)^4(1p)^{10}2d$, and $(1s)^3(1p)^{12}$; the Hamiltonian included a Rosenfeld-Yukawa two-particle interaction and a single-particle spin-orbit force; the wave functions were derived via a full-scale so-called "intermediate-coupling" calculation. The results of this calculation showed rough general agreement with experiment for the energy levels and somewhat better agreement for the available reduced widths.

It happens that the earlier calculation produced an isolated pair of $\frac{1}{2}+$ levels with $T=\frac{1}{2}$, $\frac{3}{2}$ which at that time were associated with the 11.43- and 11.61-Mev levels which concern us here. We shall make the same association but we stress at the outset that we have no very strong reason to believe that the association is a good one as far as the $T=\frac{1}{2}$ level is concerned. For one thing, there is experimental evidence[4,7] that one or two low-spin $T=\frac{1}{2}$ levels which are not predicted by the model lie in the domain 8–11 Mev; for another thing, while the 11.61-Mev state is both empirically and theoretically a simple one (being described quite well as an s particle coupled to the C^{14} ground state), the 11.43-Mev state is apparently complex and one must wonder then whether the model will describe it well even if we do assume that the theoretical $T=\frac{1}{2}$ level should be associated with the 11.43-Mev level.

The wave functions given by the earlier calculation appear in a complete LS representation; TSL are specified for the $A=15$ state $(s^4p^{10}l)$ and simultaneously for the s^4p^{10} subset of particles[18] (we label the latter values $T_\beta S_\beta L_\beta$ and, of course, $l=2s$ or $1d$). We write then

$$\Psi_J{}^T=\sum_{S,L,l}\sum_{T_\beta,S_\beta,L_\beta}K^{(T)}(S,L,T_\beta,S_\beta,L_\beta,l)\times\{\Phi_{T_\beta S_\beta L_\beta}(s^4p^{10})\times l\}_{TSL},\quad(15)$$

where the K's are the expansion coefficients which are supplied to us, Φ is an $A=14$ function, $\times$ denotes vector coupling to a resultant TSL and $\{\ \}$ denotes antisymmetrization. The $A=14$ states we write as

$$\Phi_{J_0}{}^{T_0}=\sum_{S_0,L_0}M_{S_0,L_0}{}^{T_0}\Phi_{T_0S_0L_0}(s^4p^{10}).\quad(16)$$

It is now a simple matter to show[19] that the reduced width amplitude is

$$\gamma(l,Z)/\gamma^{sp}=C(T_0,\tfrac{1}{2},T;Tz_0,Tz-Tz_0)\times\sum_{S,L,T_\beta}\sum_{S_\beta,L_\beta}K^T(S,L,T_\beta,S_\beta,L_\beta,l)\times M_{S_\beta,L_\beta}{}^{T_0}(-1)^{L_\beta+l-L}\times U(lL_\beta JS;LZ)U(L_\beta S_\beta Z\tfrac{1}{2};J_0S),\quad(17)$$

where γ^{sp} is the single-particle amplitude, $\sim(\hbar^2/m_ca_c^2)^{\frac{1}{2}}$, which may perhaps be regarded as a free parameter; U is a normalized Racah coefficient, C a Clebsch-Gordan coefficient, and l the transferred angular momentum. l and Z are both unique in the present case and may be ignored in writing γ. The phase convention adopted here is that appropriate to the earlier calculations; we may need to modify it later. According to Eq. (16), the $T=\frac{3}{2}$

[17] See, for example, the review by D. H. Wilkinson, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958).

[18] The s^3p^{12} amplitude is negligible for states below 20 Mev.

[19] See, for example, the review by M. H. MacFarlane and J. B. French, Revs. Modern Phys. **32**, 567 (1960), noting that the isotopic spin-coupling factor is not included in their definition of reduced width.

proton reduced-width amplitude is simply

$$\gamma_p{}^{T=\frac{3}{2}}/\gamma^{\mathrm{sp}}=(\tfrac{1}{3})^{\frac{1}{2}}K(^{42}S;{}^{31}S;s)M(^{31}S) -\tfrac{1}{3}\{K(^{44}P;{}^{33}P;s) -2^{\frac{1}{2}}K(^{42}P;{}^{33}P;s)\}M(^{33}P) \quad (18)$$

(where we have rearranged the labels in the coefficients) and the other cases are just as simple.

To produce numerical values for the γ's we must, of course, also have $A=14$ functions. It will turn out that the uncertainties in the $A=14$ functions will render impossible a really meaningful comparison between experiment and theory. We can reasonably investigate the magnitudes of the widths and the Coulomb admixing amplitude but the phases will be uncertain. Any reasonable calculation will produce a C^{14} ground-state wave function with about equal S and P amplitudes and with a positive relative phase (with the phase convention used in the $A=15$ calculations which is, in fact, the standard one used in all theoretical discussions of $A=14$). The trouble is that a direct analysis of stripping experiments[19] gives instead a function which is very largely S or very largely P (with once again a positive relative phase). For the N^{14} function it seems pretty clear that the major amplitude is D but, in particular, the sign of the small S component is uncertain. The usual shell model calculation gives S, P, D amplitudes with signs $(-, +, +)$, respectively; the C^{14} β-decay argument is not compelling,[20] and we are left with an uncertainty about the N^{14} S-state amplitude. In the following material we produce numerical values using, for definiteness, $A=14$ functions produced by Sherr[21] (with a positive S N^{14} amplitude and roughly equal S and P C^{14} amplitudes), and we test in each case whether the $A=14$ uncertainty can seriously affect the results, particularly the signs of the amplitudes. For good measure we calculate amplitudes also for two $T=\frac{1}{2}$ states which, according to the shell-model calculations lie above the interacting pair. Our conclusions, such as they are, would be unchanged even if we included all the higher theoretical $\frac{1}{2}+$ states.

The results of the shell-model calculations are given in Table I. The value of γ^{sp} used in evaluating the shell-model reduced widths was simply $(\hbar^2/2m_ca_c^2)=0.98$ $(\mathrm{Mev})^{\frac{1}{2}}$. It is suggested by the table that the $\frac{1}{2}+$ levels lying above the isolated interacting pair make a negligible contribution to the cross section in the energy region of interest to us [since the corresponding values of $\Gamma_{\lambda p}{}^{\frac{1}{2}}\Gamma_{\lambda n}{}^{\frac{1}{2}}/(E_\lambda-E)$ are very small]. The two low-lying levels, on the other hand, are predicted to give a quite sizable contribution, most of the effect coming from the second $\frac{1}{2}+$ state which Halbert and French associated with the 8.32-Mev level (but which, in fact, could correspond instead to the 9.06-Mev state). The calculated magnitude of $\sum_\lambda \Gamma_{\lambda p}{}^{\frac{1}{2}}\Gamma_{\lambda n}{}^{\frac{1}{2}}/(E_\lambda-E)$ compares satisfactorily with those determined by the analysis.

Next, about the phases. With $\gamma_{2n}=0$ there are only two physically distinct phase combinations for the amplitudes. The other degrees of freedom (remember that we must have four) are given by the phase of the admixing amplitude α_{21}. As they stand, the calculated phases in Table I agree with the experimental ones; if they had not we could have attempted by wave-function phase changes to make them agree. However, the admixing amplitude is calculated to be negative instead of positive as experimentally observed.

This disagreement is not significant (nor, indeed, would be an agreement). It turns out that the calculated phase of γ_{1n} is "unstable" with regard to the $A=14$ functions and would change its sign if we used the intermediate-coupling function calculated without attention to the β-decay requirement. And if we use a C^{14} function which is predominantly S state (as suggested by the stripping experiments), the calculated γ_{1p} changes its sign. We can then combine these with a phase for the 11.43-Mev theoretical state and produce agreement for all the phases.

The admixing amplitude is related to the Coulomb matrix element between the close-lying $\frac{1}{2}+$ states by

$$\alpha_{21}=\frac{\langle\Psi_{\frac{1}{2}}{}^{(0)}H_{\mathrm{Coul}}\Psi_{\frac{1}{2}}{}^{(0)}\rangle}{(E^{(0)})_{\frac{1}{2}}-(E^{(0)})_{\frac{1}{2}}}. \quad (19)$$

The evaluation of the Coulomb matrix element is a rather formidable job and not very interesting. It is best accomplished by making liberal use of the properties of tensor operators and tensor products including their behavior under the particle $\rightleftarrows$ hole transformation. We shall just make a few comments of slightly more general interest and then give the result.

The matrix element to be evaluated is that of a two-body operator between two antisymmetric wave functions, each composed of three groups of equivalent particles. This matrix element can be shown to equal a much simpler one, namely, one in which we ignore the antisymmetrization between groups and deal instead with separately antisymmetrized groups of particles, each group having definite particle numbers. If H_{ij} were the original two-body operator we should now instead have to use H_{ij}' where $H_{ij}'=H_{ij}$ if i, j are in the same group, $=H_{ij}(1-P_{ij})$ if they are not (P_{ij} permutes the particles i and j). The matrix element then breaks up into parts, some of which represent the internal interactions in a group, the others the interaction between groups.

The Coulomb operator is[15]

$$\frac{e^2}{4}\sum_{i<j}[1+2\{t_z(i)+t_z(j)\}+4t_z(i)t_z(j)]\frac{1}{r_{ij}}, \quad (20)$$

where r_{ij} is the distance between particles i and j and

[20] E. Baranger and S. Meshkov, Phys. Rev. Letters **1**, 30 (1958).
[21] R. Sherr, J. B. Gerhart, H. Horie, and W. F. Hornyak, Phys. Rev. **100**, 945 (1955).

$t_Z(i)$ is the z component of the isotopic spin operator of the nucleon i. The first term of (20) is an isotopic spin scalar which can be ignored, the second is an isotopic spin vector, the third contains both a scalar $[\frac{1}{3}\mathbf{t}(i)\cdot\mathbf{t}(j)]$ and a second rank tensor $[(\frac{2}{3})^{\frac{1}{2}}[\mathbf{t}(i)\times\mathbf{t}(2)]^2_0]$.

It is tempting to make a long-range approximation to the matrix element of (20) whereby, for each pair of particles with orbital angular momenta l, l', we replace $(r_{ij})^{-1}$ by its mean value, averaging also with respect to L with, of course, a $(2L+1)$ weighting. With this approximation we can replace $(r_{ij})^{-1}$, in the matrix element of (20), by $F^0(l,l';l,l')$, the usual $K=0$ Slater integral. The operator now is immediately expressible in terms of the isotopic spin operators of the various groups (the exchange interaction between the groups of course vanishing) and the whole evaluation of the matrix element is trivial. This would work for a matrix element diagonal in isotopic spin space, but it fails in an off-diagonal matrix element, such as that of Eq. (19), because it will turn out that the various F^0 values which will enter are quite closely the same (in the present case they vary less than $\pm 10\%$): On the other hand the matrix element must vanish when all F^0's become equal, as pointed out by Neudachin,[22] because it can then be written in terms of multiples of T_Z and $(T_Z)^2$.

We record now the various contributions to the matrix element. In the numerical evaluation we take $e^2\nu^{\frac{1}{2}}=0.97$ Mev, where $\exp(-\frac{1}{2}\nu r^2)$ is the exponential factor in the single-particle radial wave functions. The $(p^{10}l)$ contribution is uncertain by 3 or 4 kev because we have not evaluated it precisely. The contributions are:

$$\begin{aligned}
(p^{10})\ &\text{vector}=-114\ \text{kev},\\
(p^{10})\ &\text{tensor}=+12\ \text{kev},\\
(s^4,l)\ &\text{vector}=+46\ \text{kev},\\
(s^4,p^{10})\ &\text{vector}=-48\ \text{kev},\\
(p^{10},l)\ &\text{vector}=+78\ \text{kev},\\
(p^{10},l)\ &\text{tensor}=-8\ \text{kev},\\
&\text{Total}\approx -34\ \text{kev}.
\end{aligned}$$

Note that the tensor contributions are very much smaller than the vector contributions. This will almost always turn out to be the case when we are dealing with many particles, as may be seen easily by considering the long-range limit. If we have a group of m particles, then $\sum_{i<j}\{t_Z(i)+t_Z(j)\}\rightarrow(m-1)T_Z$; the explicit m dependence does not occur with the tensor interaction. The smallness of the tensor contribution can be directly seen from the binding energies of $T=1$ isobaric triplet states. For $T_Z=1, 0, -1$ the Coulomb energies are respectively $S+V+t$, $S-2t$, $S-V+t$ where S, V, t are the scalar, vector, and tensor terms. For the nine or so cases where the energies are known, t is just a few percent of v (and thus of course the plots of Coulomb energies vs T_Z are almost straight lines).

A general comment about the magnitude of the Coulomb matrix elements is in order. For a pair of particles we have $\langle e^2/r_{12}\rangle\approx(3/2)e^2(\nu/2\pi)^{\frac{1}{2}}\sim 600$ kev, and this, at first sight should be the order of magnitude of a Coulomb matrix element. Because of the long-range cancellation discussed above it is effectively reduced by almost an order of magnitude, say to 100 kev. Besides this there is an additional wave-function overlap factor which, so to speak, looks after the space and spin behavior of the functions. For the overlap of the first five theoretical $\frac{1}{2}+$, $T=\frac{1}{2}$ states with the lowest $T=\frac{3}{2}$ state this factor is found to be respectively, 0.7, 0.2, -0.1, 0.1, -0.6. We can therefore say, rather safely, that 100 kev is about the largest off-diagonal matrix element that we can expect, that an order of magnitude smaller than that is quite probable and that the matrix element can have either sign. This conclusion strongly supports our belief that the isotopic spin admixture in one member of the isolated doublet comes, essentially, completely from the other member. Hence our determination of the experimental admixture, in Sec. IV, is quite accurate.

For a pair of levels 195 kev apart, a 5% admixture would require a matrix element of about 40 kev so that our calculated value of 34 kev is quite reasonable. Moreover, the argument earlier that 100 kev is a reasonable upper limit for a Coulomb matrix element strongly supports our view that the isotopic spin admixing comes, essentially completely, from the mutual interaction of the two close-lying levels; for a 100-kev matrix element cannot appreciably admix in the more distant levels. We thus believe that, to within the present experimental uncertainty in the cross section magnitudes, we have given a trustworthy model-independent evaluation of the isotopic spin admixture.

Unfortunately there are few other possible reactions involving the two close-lying levels, whose cross sections are affected by the isotopic spin impurity. One possibility is the $C^{14}(p,\alpha)B^{11}$ cross section (or its inverse) whose threshold occurs at $E_p(\text{lab})=0.835$ Mev. Since $\gamma_{2\alpha}$ must also arise from the $T=\frac{1}{2}$ impurity of the $T=\frac{3}{2}$ level we have $\gamma_{2\alpha}/\gamma_{1\alpha}=\gamma_{2n}/\gamma_{1n}$. Consequently, to the extent that we can neglect all $\frac{1}{2}+$ levels except the admixed pair, we have the following relation between the (p,α) and the (p,n) cross sections of C^{14}, at each energy E.

$$\sigma_{p,\alpha}(E)=\frac{P_\alpha(E)}{P_n(E)}\frac{\gamma_{1\alpha}{}^2}{\gamma_{1n}{}^2}\sigma_{p,n}(E), \tag{21}$$

where $P_\alpha(E)$ is the p-wave penetrability for alpha particles and $P_n(E)$ the s-wave penetrability for neutrons. Because of the small value of $P_\alpha(E)$ in the neighborhood of the two levels the (p,α) cross section is only of the order of microbarns. Apart from the penetrability ratio the two cross sections should have exactly the same shape.

[22] V. G. Neudachin, Zhur. Ekspl. i Teoret. Fiz. **31**, 892 (1956) [translation: Soviet Phys.—JETP **4**, 756 (1957).

The neutron reduced width γ_{2n}^2 of the $T=\frac{3}{2}$ level is so small that this level has a very small effect on the elastic scattering of neutrons by N^{14}. The elastic scattering of protons by C^{14}, which has never been measured, would be useful in verifying the parameters of the above analysis. More importantly, an accurate absolute cross-section measurement of either the (p,p) or (p,n) cross sections of C^{14} would remove the substantial uncertainties in the experimental values of γ_{1p}, γ_{2n} and hence of the isotopic spin impurity.

VI. ISOTOPIC SPIN IMPURITIES FROM PARTICLE-WIDTH RATIOS IN OTHER REACTIONS

For other reactions the extraction of isotopic spin impurities from particle-width ratios may not always be as simple as in our case, where the admixed levels lie very close together. An interesting example has been studied recently by Hebbard.[23] In this example two fairly close-lying 1− states of O^{16}—at 12.43 Mev ($T=0$) and 13.09 Mev ($T=1$)—can be studied by the reactions $N^{15}(p,\gamma_0)O^{16}$, $N^{15}(p,\alpha_0)C^{12}$, and $N^{15}(p,\alpha_1)C^{12*}$. It had been shown earlier by Bloom *et al.*[24] that the admixture of the 13.09-Mev state with the low-lying 1−, $T=0$ state at 9.58 Mev was very small and hence could not account for the forbidden alpha-particle widths of the 13.09-Mev level, in spite of the very large alpha-particle reduced width of the 9.58-Mev level. It may therefore be reasonable to suggest, as Hebbard[23] does, that the isotopic spin impurities of the two 1− levels observed by Hebbard arise from their mutual admixture. This suggestion is borne out by the results of the cross-section analysis[23] (which included the interference of the two levels).

Referring to the higher lying level as $\lambda=2$ and the other one as $\lambda=1$, the ratios $\gamma_{2\alpha_0}^2/\gamma_{1\alpha_0}^2$, $\gamma_{2\alpha_1}^2/\gamma_{1\alpha_1}^2$, and $\Gamma_{1\gamma_0}/\Gamma_{2\gamma_0}$ are almost the same (∼0.15). However, Hebbard's analysis[23] found the interference of the levels, between their resonance energies, to be constructive for the (p,α_0) reaction, but assumed that the interference is destructive for (p,α_1) and destructive for (p,γ_0). If the assignment of the isotopic spin impurity to the mutual admixture of the two levels is correct, then wherever (p,α_0) exhibits constructive interference (p,α_1) must also; and since in the (p,α) cases it is the $T=0$ admixture in the $T=1$ state which allows an otherwise forbidden reaction and in the (p,γ_0) reaction the forbidden reaction proceeds by the $T=1$ admixture in the $T=0$ state (and since $\alpha_{12}=-\alpha_{21}$), we see that the (p,γ_0) reaction should show destructive interference. Just as in (21) above, one should find here:

$$\sigma_{p\alpha_1}(E)=\frac{P_\alpha(E-4.43\text{ Mev})}{P_\alpha(E)}\frac{\gamma_{1\alpha_1}^2}{\gamma_{1\alpha_0}^2}\sigma_{p\alpha_0}(E). \quad (22)$$

A careful analysis of these reactions might very well show the similarity in shape of the (p,α_1) and (p,α_0) reactions and would constitute an excellent verification of the above interpretation of the particle widths.

In Li^6 the second $T=1$ state (2+) at 5.35 Mev lies close to the broad ($\Gamma\sim0.6$ Mev) 2+, $T=0$ state at 4.5 Mev. The $He^4(d,He^4)D$ or $He^4(d,He^5)H^1$ reactions have never been studied at the appropriate energy so that the widths of the $T=1$ state are not known. The reaction energies (∼6 Mev for alpha particles in the lab) are now easily accessible to tandem accelerators, allowing these reactions to be studied in detail. In this case, however, things are greatly complicated by the fact that, if we regard the levels as belonging essentially completely to the s^4p^2 configuration, the first-order Coulomb matrix element vanishes between them; for it is easy to see that the Coulomb interaction, being central, is diagonal in isotopic spin in this configuration. The Coulomb admixing would then be a second-order effect, though not necessarily of second order in the Coulomb interaction, since the nuclear interaction would admix states of higher configurations. One then expects the admixing to be very small, but until the experiment is done it is not profitable to speculate further about it.

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[24] S. D. Bloom, B. J. Toppel, and D. H. Wilkinson, Phil. Mag. **2**, 57 (1957).