

Correction to the Debye-Hückel Theory*

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The correction to the equation of state of a classical electron gas is calculated by the method of Abe. The results are compared with those recently obtained by a different method by Bowers and Salpeter.

IN a recent paper, Bowers and Salpeter described a method for calculating the correlation energy of a classical one-component electron gas.¹ Their method employs the potential of mean force, and in first approximation leads to the expression

$$E_{\text{corr}}/kT = -\frac{1}{2}\lambda - \frac{1}{2}\lambda^2 \left[\frac{1}{2} \ln \lambda + \left(\gamma - \frac{2}{3} + \frac{1}{2} \ln 3 \right) \right], \quad (1)$$

where the dimensionless parameter λ (denoted by ϵ in reference 1) is equal to $(4\pi)^{\frac{1}{2}}(q^2\rho^{\frac{1}{3}}/kT)^{\frac{1}{2}}$.

The same problem has been considered by Meeron,² Friedman,³ and Abe,⁴ using methods involving the summation of certain infinite classes of diagrams. While the relation between these theories and that of Bowers and Salpeter is not yet clear, it is interesting to note that Abe obtained an expression equivalent to Eq. (1) by approximating an integral which he called S_2 .

We have calculated the contribution to the equation of state using Abe's original expression for S_2 , and we found that Eq. (1) is valid only for values of λ less than about 0.1; for $\lambda \geq 0.4$ it gives a correction of the wrong sign. On the other hand, our results are comparable to those which Bowers and Salpeter obtained from higher approximations which they denote by $A-T$ and $A'-T'$.

$$PV/RT = 1 + \frac{1}{3}(E_{\text{corr}}/kT) = 1 - \frac{1}{6}\lambda + S',$$

where

$$S' = \frac{\lambda}{16} + \frac{1}{4\lambda} \int_0^\infty \left[x^2 \exp\left(-\frac{\lambda e^{-x}}{x}\right) + \lambda x \exp\left(-x - \frac{\lambda e^{-x}}{x}\right) \right] dx. \quad (2)$$

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¹ D. L. Bowers and E. E. Salpeter, *Phys. Rev.* **119**, 1180 (1960).

² E. Meeron, *J. Chem. Phys.* **28**, 630 (1958), *Phys. Fluids* **1**, 139 (1958), and later papers.

³ H. L. Friedman, *Mol. Phys.* **2**, 23 (1959).

⁴ R. Abe, *Progr. Theoret. Phys. (Kyoto)* **22**, 213 (1959).

TABLE I. Values of S' calculated in four different ways.

λ	From Abe's S_2	From Eq. (1)	By Bowers and Salpeter	
			($A-T$)	($A'-T'$)
0.01	0.000 030 8	0.000 030 7		
0.02	0.000 103 1	0.000 099 7		
0.03	0.000 205 7	0.000 194 0		
0.04	0.000 332 8	0.000 306 6		
0.05	0.000 480 9	0.000 432 5		
0.06	0.000 647 5	0.000 568 1		
0.07	0.000 830 6	0.000 710 3		
0.08	0.001 028 6	0.000 856 5		
0.09	0.001 240 4	0.001 004 6		
0.10	0.001 464 9	0.001 152 4	0.001 5 ₈	0.001 6 ₀
0.20	0.004 247 6	0.002 299 1		
0.30	0.007 715 9	0.002 132 0		
0.33			0.009 0 ₆	0.009 8 ₉
0.40	0.011 639 2	-0.000 045 6		
0.50	0.015 893 7	-0.004 720 1		
0.60	0.020 402 5			
0.70	0.025 113 4			
0.80	0.029 989 2			
0.90	0.035 002 4			
1.00	0.040 131 8		0.034 6 ₇	0.036 9 ₇
2.00	0.095 302 8			
3.00	0.153 996 6			
4.00	0.214 238 3			
5.00	0.275 311 4			

The term $\frac{1}{6}\lambda$ is known as the Debye term; when this contribution is taken out of the correlation energy, the numbers given by Bowers and Salpeter retain only two significant figures, but they are still within about 10% of ours.

Values of S' are given in Table I for selected values of λ up to 5.0. Contributions from more complicated diagrams (S_3, S_4 , etc. in Abe's scheme) will presumably become more important when λ is greater than 1.0, but the relevant computations have not yet been performed.

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