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Relativistic Formula for the Spin Correlation Coefficient C_{KP}

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A formula due to Stapp for the spin correlation coefficient C_{KP} in nucleon-nucleon scattering is corrected. Formulas for the other triple-scattering and unpolarized-incident-beam correlation parameters are included. The formulas are applicable when the nonrelativistic scattering matrix formalism of Wolfenstein is used to analyze triple-scattering experiments at higher energies where relativistic effects are not negligible.
The definition of the correlation coefficients in the relativistic case is discussed.

A RELATIVISTIC theory of polarization phenomena applicable to nucleon-nucleon scattering has been given by Stapp.[1] He has shown that the nonrelativistic formalism due to Wolfenstein and Ashkin[2] may be used providing that, in addition to relativistic kinematics, certain "rotational corrections" are taken into account in deducing the formulas for observables. The corrections apply only to components of polarization in the scattering plane; as examples Stapp gave the formulas for $R(\theta)$ and the spin correlation parameter C_{KP}. However, the latter formula does not follow from his theory, and is not symmetrical under exchange of identical particles. The correct result is

$$I_0C_{KP}^{(\mathrm{rel})}=4\,\mathrm{Re}(ich^*)\cos(\alpha'-\alpha)-2\,\mathrm{Re}[(a-m)g^*]\sin(\alpha'+\alpha)+2\,\mathrm{Re}[(a+m)h^*]\sin(\alpha'-\alpha).\quad(1)$$

The scattering matrix has been expressed in Stapp's representation as

$$M=a+c(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)\cdot\mathbf{n}+m(\boldsymbol{\sigma}^1\cdot\mathbf{n}\boldsymbol{\sigma}^2\cdot\mathbf{n})+(g+h)(\boldsymbol{\sigma}^1\cdot\mathbf{P}\boldsymbol{\sigma}^2\cdot\mathbf{P})+(g-h)(\boldsymbol{\sigma}^1\cdot\mathbf{K}\boldsymbol{\sigma}^2\cdot\mathbf{K}).$$

I_0 is the differential cross section in scattering an unpolarized incident beam. Denoting the scattering and recoil angles, respectively, by θ, ϕ in the center-of-mass system and θ_L, ϕ_L in the laboratory, we define

$$\alpha=\theta/2-\theta_L,\quad \alpha'=\phi/2-\phi_L.$$

Of course, measuring ϕ in the sense opposite to θ, $\phi=\pi-\theta$.

Given the masses, energy, etc., for the collision, $\alpha=\alpha(\theta)$ is a function of θ only, so $\alpha'=\alpha(\pi-\theta)$. At 90° c.m., formula (1) reduces to that given by Stapp[1] (his is equivalent to replacing α' by α everywhere), so previous applications[3] of this result are not affected. However, since measurements of C_{KP} at smaller angles are planned for the near future, it seems worthwhile to point out this error at the present time. The calculations[4,5] which motivate this experiment will be slightly changed. Using the values of the scattering matrices[6] in Table V of reference 5, it is found that at 30° and 310 Mev, the relativistic corrections to C_{KP} are 0.1% for solution 1 and 6.6% for solution 2. The situation at 60° is similar. A rough calculation indicates that at 635 Mev the corrections will be typically of order 10%. Since the present formula differs from that of Stapp by terms of order $(\alpha'-\alpha)$, the additional relativistic corrections are in general an order of magnitude smaller than his.

It should be noted that there is some ambiguity in defining C_{KP} in the relativistic region. What is in fact measured[3] would more logically be denoted $C_{s_1s_2}$ where $\hat{s}_1$, $\hat{s}_2$ are unit normals in the scattering plane to the lab momenta $\mathbf{k}_1$, $\mathbf{k}_2$, of the outgoing particles. In the nonrelativistic limit $\hat{s}_1$, $\hat{s}_2$ coincide with $\mathbf{K}$, $\mathbf{P}$. This is the definition adopted here. There are two other possible definitions, but the parameters so defined have more complicated formulas and are not directly measurable.

Finally, the formulas for the other unpolarized spin correlation coefficients and triple scattering parameters are listed below. In each case the "experimental" definition has been adopted: $C_{PP}^{(\mathrm{rel})}=C_{k_1s_2}$, $C_{KK}^{(\mathrm{rel})}=C_{s_1\,-k_2}$.

$$I_0C_{PP}^{(\mathrm{rel})}=2\,\mathrm{Re}[(a+m)h^*]\cos(\alpha'-\alpha)+2\,\mathrm{Re}[(a-m)g^*]\cos(\alpha'+\alpha)-4\,\mathrm{Re}(ich^*)\sin(\alpha'-\alpha).$$

$$I_0C_{KK}^{(\mathrm{rel})}=2\,\mathrm{Re}[(a-m)g^*]\cos(\alpha'+\alpha)-2\,\mathrm{Re}[(a+m)h^*]\cos(\alpha'-\alpha)+4\,\mathrm{Re}(ich^*)\sin(\alpha'-\alpha).$$

$$I_0R=(|a|^2-|m|^2)\cos(\theta-\theta_L)+2\,\mathrm{Re}[ic(a^*-m^*)]\sin(\theta-\theta_L)-4\,\mathrm{Re}gh^*\cos\theta_L,$$

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[3] E.g., A. Ashmore, A. N. Diddens, and G. B. Huxtable, Proc. Phys. Soc. (London) **73**, 957 (1959).
[4] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).
[5] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).
[6] It may be useful to point out the following misprints: In reference 5, the scattering matrices appropriate to phase shift solutions 1 and 2 have been tabulated incorrectly in Table V. M^* has been tabulated in place of M; this will be corrected if the columns headed B, $\bar{C}'$, G, H, and N are relabelled $-B$, $-\bar{C}'$, $-G$, $-H$, and $-N$, respectively. Calculated values of C_{KP} will then agree with Fig. 6 of this reference. In reference 4, Eq. (2.19),

$$I_0A = -(|a|^2-|m|^2)\sin(\theta-\theta_L) +2\,\mathrm{Re}[ic(a^*-m^*)]\cos(\theta-\theta_L) +4\,\mathrm{Re}gh^*\sin\theta_L,$$

$$I_0R' = (|a|^2-|m|^2)\sin(\theta-\theta_L) -2\,\mathrm{Re}[ic(a^*-m^*)]\cos(\theta-\theta_L) +4\,\mathrm{Re}gh^*\sin\theta_L,$$

$$I_0A' = (|a|^2-|m|^2)\cos(\theta-\theta_L) +2\,\mathrm{Re}[ic(a^*-m^*)]\sin(\theta-\theta_L) +4\,\mathrm{Re}gh^*\cos\theta_L.$$

As in the nonrelativistic case there is a relation among the triple scattering parameters, which becomes

$$(A+R')/(A'-R)=\tan\theta_L.$$

The equality between $C_{PP}^{(\mathrm{rel})}$ at θ and $C_{KK}^{(\mathrm{rel})}$ at $(\pi-\theta)$ also holds for identical particles.

the denominator of the expression for α^{JP} should be $2E(2J+1)$. α^{JP} is independent of L.

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Determination of Pion-Pion Scattering Amplitudes Satisfying Dispersion Relations and Unitarity

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A method is developed for determining the partial-wave scattering amplitude in terms of the unitarity condition and the known branch cuts and poles of the inverse amplitude. The method is applied to the problem of pion-pion scattering and an implicit solution to the pion-pion partial-wave amplitude is derived for any angular momentum state and for both elastic and inelastic scattering. With the aid of this solution the low-energy resonance behavior of the pion-pion scattering system is studied by neglecting all inelastic processes and concentrating on S and P waves. It is found that a P-wave resonance with a position and width required by nucleon electromagnetic structure can be determined in terms of two parameters. An iteration procedure is described that is applicable when the P wave dominates the equations and this procedure determines the contribution of the unphysical cut. The first iteration of the unphysical cut is numerically integrated on the IBM 709, and the results show that the shift of the resonance position due to the unphysical branch cut can be neglected.

1. INTRODUCTION

It has been conjectured by Mandelstam[1,2] that two-particle scattering amplitudes can be expressed in terms of a double spectral representation. The scattering amplitudes can be analytically continued into the complex plane as a function of the energy and momentum transfer variables and this leads to dispersion relations for the partial-wave amplitudes which satisfy the unitarity condition in a particularly simple form. It would seem that in principle this representation provides a complete dynamical description of scattering systems.

It has become evident that a more reliable description of pion-pion interaction is required if we are to understand the phenomena of strong interactions and the electromagnetic structure of the nucleon.[3,4] Chew and Mandelstam have used the double representation to formulate an approximation method for low-energy elementary particle scattering. By using the unitarity condition and the "effective-range" approximation, Chew and Mandelstam obtain a system of coupled nonlinear integral equations from the partial-wave dispersion relations for pion-pion scattering.[5–7] In the special case of dominant S-wave scattering and also in the case of dominant P-wave scattering, it has been shown that classes of solutions exist for the nonlinear integral equations. For P-wave dominant solutions a cutoff is required due to the singular nature of the Chew-Mandelstam equations, and the unphysical cuts are replaced by a corresponding series of poles.[8]

In the following a general method is developed which determines the partial-wave amplitude for a scattering problem in terms of the known branch cuts and the unitarity condition.[9] The method is applied to the

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