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Momentum Spectrum of Muons*

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The momentum spectrum has been measured with a magnetic spectrometer for cosmic-ray particles incident vertically and at 68° zenith angle, in the range 2 to 70 Bev/c. The apparatus discriminates strongly against all particles but muons. The vertical muon spectrum is found to be in good agreement with the results reported by Pine *et al.* and by Ashton *et al.* No comparable data at 68° have been published. Assuming muon creation to occur entirely through pion decay, a single pion production spectrum accounts for the muon spectra at both zenith angles. The muon positive excess at energies of 8 to 50 Bev has also been obtained. The results ,while in general agreement with those of other workers, confirm the expected tendency for the positive excess to decrease with energy.

APPARATUS

THE measurements reported here were obtained with the Cornell magnet spectrometer, of which the details have been described by Pine *et al.*,[1] Roe,[2] and Roe and Ozaki.[3] The main differences between the experimental arrangements used by Pine *et al.* and by Roe and Ozaki are that the latter used three Conversi-type hodoscope chambers instead of cloud chambers for the fine momentum determination, and that they shortened the distances between the Geiger counter trays to accommodate a large multiplate cloud chamber. The arrangement used here is identical to the one described by Roe and Ozaki.[2,3] In fact, many of the present data were obtained concurrently with the data reported by Roe and Ozaki on direct pair production by muons.[3]

MUON SPECTRA

The momentum spectra are derived from both the data obtained with Geiger counters and those obtained with hodoscope chambers. The analysis of the Geiger counter data is based on 23 037 particles at 0° spectrometer orientation and 11 760 particles at 68° orientation. The high-momentum points in the spectrum are derived from measurements made with the hodoscope chambers, which were triggered only by the lowest-sagitta Geiger counter channel. A total of 1458 particles at 0° and 1020 particles at 68° has been analyzed with the hodoscope chambers. The procedure adopted for the reduction of data closely parallels the treatment of Pine, Davisson, and Greisen.[1]

The handling of the counter data may be summarized as follows. From the counting rates in various sagitta channels one obtains a trial sagitta distribution function $f(s)$, which represents a smooth fit to the data, corrected for the reduction of aperture at large sagittas, and subject to the condition $f(0)=0$. For each channel an acceptance function $g_i(s)$ is calculated from geometrical considerations. Knowledge of $f(s)$ and $g_i(s)$ permits the median sagitta s_{im} to be computed for each channel. Now one defines a factor k_i for each channel such that

$$k_i \int f(s) g_i(s) ds = f(s_{im}).$$

Multiplication of the observed rate in any channel by k_i yields the rate at the median sagitta of that channel. The differential sagitta spectrum so obtained is converted into a differential momentum spectrum by use of the relations

$$p_{im} = b/s_{im} \quad \text{and} \quad (dN/dp)_{p_{im}} = (s_{im}^2/b)(dN/ds)_{s_{im}},$$

with $b=9.8$ (Bev/c) counter diameters for the counter data and $b=120$ (Bev/c) mm for the hodoscope chambers.

The results are shown in Fig. 1. The vertical spectrum is normalized to the Rossi curve[4] at 5 Bev/c. The magnet aperture so determined fixes the normalization of the points at 68°.

The hodoscope data were grouped into sagitta channels of width 1.7 mm. The reduction procedure applied to the counter data is also used for the hodoscope data, except that the acceptance functions in this case are derived not from geometry alone, but primarily from an evaluation of the hodoscope measurement errors. The trial function $f(s)$ is obtained from the observed sagitta distribution, corrected for the variation with sagitta of the aperture of the counter channel triggering the hodoscope chambers, and subject to the condition $f(0)=0$.

A measure of error is obtained from the distribution of the coordinate $Y_j = C_j - Z_j$, where Z_j is the intersection of the particle trajectory with the plane of Geiger tray j, as predicted from the lateral coordinates observed in the hodoscope chambers, and C_j is the center position of the counter fired in that tray. In the

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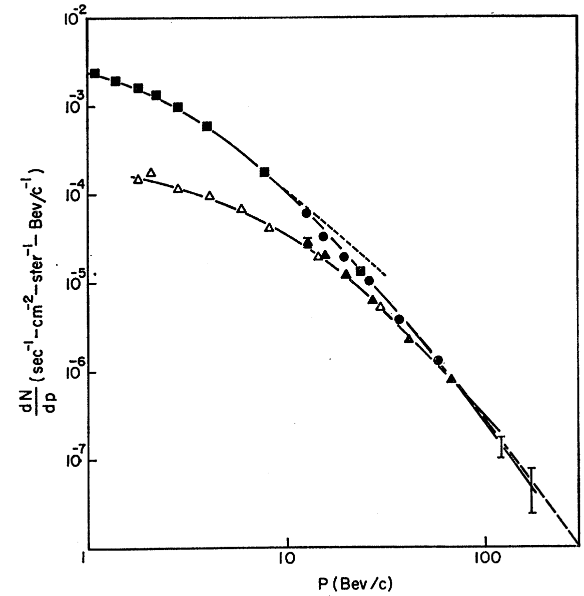

FIG. 1. The differential muon momentum spectrum at two zenith angles. Standard errors are shown only where they extend outside the points.

Key: ●, present experiment, 0°, hodoscope data; ■, present experiment, 0°, Geiger counters; ▲, present experiment, 68°, hodoscope data; △, present experiment, 68°, Geiger counters; I, and — — —, Ashton *et al.*[6]; ————, Pine *et al.*[1] and Owen and Wilson[5]; - - -, Rossi.[4]

absence of measurement error (counter inefficiency near the walls being negligible), the Y values would be uniformly distributed from -0.5 to $+0.5$ counter diameter. The departure from such a rectangular Y distribution provides a measure of the random error of measurement.

Y values were calculated at all three Geiger-tray planes for each of the 2478 trajectories considered in the analysis of the hodoscope data. Those tracks leading to Y values of more than two counter diameters were rejected as gross errors, due to mistaken identity of tracks or misreading of coordinates in one of the chambers. The over-all precision, derived from the Y distributions, corresponds to a standard error of 1.7 mm of sagitta. Knowledge of this error of measurement makes possible the calculation of the acceptance functions of the hodoscope sagitta channels.

The conversion of the sagitta distribution into points on the differential momentum spectrum proceeds as with the Geiger counter data. The normalization is obtained from the lowest-sagitta point of the counter data, since this counter channel triggered the hodoscope chambers. The results are shown in Fig. 1.

The full lines in Fig. 1 represent a smooth fit to our data. For the 0° spectrum the full line also represents the best fit to the data of Pine *et al.*[1] and of Owen and Wilson,[5] whose measurements extend up to 175 and 20 Bev/c, respectively. Also indicated in the figure are the early data summarized by Rossi[4] and the recent measurements of Ashton *et al.*[6] Our results are in agreement with the latter measurements over the entire energy region covered by the present experiment. At higher energies the curve published by Ashton *et al.* lies slightly above that given by Pine *et al.*, but we have reproduced the experimental points of Ashton *et al.* in the momentum region between 100 and 300 Bev/c to show that the statistical errors make the discrepancy insignificant. Below 100 Bev/c, where the errors are much smaller, the agreement of the three independent experiments is highly gratifying, and increases confidence also in the points obtained at 68°.

ORIGIN OF THE MUONS

Barrett *et al.*[7] have derived an expression for the differential muon spectrum in terms of a pion production spectrum, under the assumptions that the primary particles are incident isotropically on the top of the atmosphere, that the pions retain the directions of their producers, and that muons of energy E arise from decay of pions of energy E/r. Assuming further that the absorption mean free paths of pions and of their

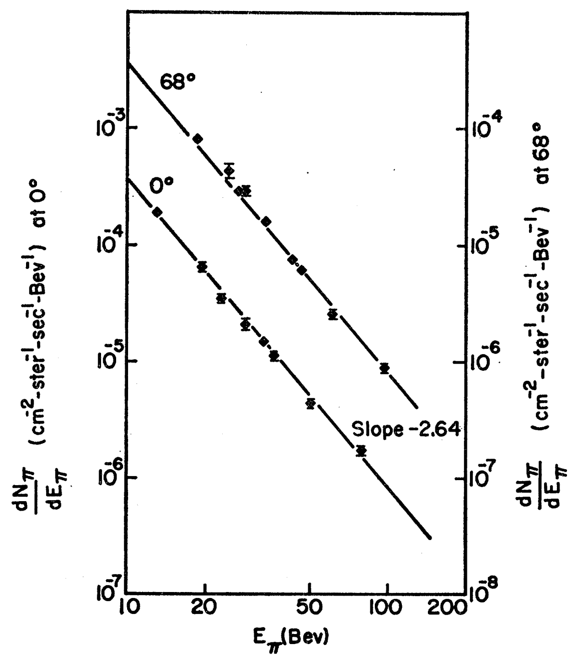

FIG. 2. Inferred differential production spectrum of pions. The 68° results have been shifted upwards by a decade to avoid overlapping of the two sets of points. Standard errors are shown where they extend outside the points. The full line represents the best fit to the data of Pine *et al.*[1]

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[6] F. Ashton, G. Brooke, M. Gardener, P. J. Hayman, D. G. Jones, S. Kisdnasamy, J. L. Lloyd, F. E. Taylor, R. H. West, and A. W. Wolfendale, Nature **185**, 364 (1960).

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producers are equal, these authors obtain

$$M(E)=\frac{F(E/r)}{r}\frac{E_0}{E_0+E\cos\theta},$$

where $M(E)$ is the muon differential spectrum, $F(E)$ is the pion differential production spectrum, $r=m_\mu/m_\pi=0.76$, and $E_0=m_\pi c^2 rH/c\tau_0=90$ Bev, H and τ_0 being the equivalent thickness of the atmosphere and the mean lifetime of pions at rest. This expression permits a simple conversion of the muon spectrum into a pion production spectrum if corrections for muon decay and ionization losses are taken into account. We have assumed for all muons a constant ionization loss of 2.2 Mev-cm^2/g, and losses by decay corresponding to transit times of 53 μsec for vertically incident muons and 200 μsec for muons incident at 68°.

The results of this analysis are plotted in Fig. 2. The full lines represent the best fit given by Pine *et al.*[1] to their data at 0°, and correspond to $F(E_\pi)=0.156(E_\pi)^{-2.64}$ (sec cm^2 sr Bev)$^{-1}$. It is seen that our points derived from the 0° data are in excellent agreement with this expression. The same spectrum is an excellent fit to the high-energy points obtained at 68°. At 100-Bev pion energy, the term $E_0/(E_0+E\cos\theta)$ describing the competition between decay and absorption of the pions is forty percent larger at 68° than at 0°; therefore the agreement of the inferred pion spectra at these two angles confirms the pion origin of most of the muons. If the muons were produced mostly by $K-\mu$ decay instead of $\pi-\mu$ decay, the constant E_0 in the above expression would be about 500 Bev instead of 90 Bev and the derived production spectra would not be in agreement at the two zenith angles. The assumption of direct muon production by cosmic-ray primaries would increase the discrepancy.

The low-energy points on the pion production spectrum at 68° lie slightly above those deduced at 0°. However, the agreement of these points is not much influenced by the assumed mechanism of muon origin. For these points, the corrections for muon decay and energy loss in crossing the atmosphere at 68° are quite large, and our assumption of a fixed height of production is inadequate. The effect of integrating over the atmosphere, taking into account muon production at greater depths, would be to move these points down a little. Hence this apparent discrepancy is not significant.

POSITIVE EXCESS

The positive excess η, defined as the difference of positive and negative muon intensities divided by their sum, has been determined from both the counter data and the hodoscope chamber data. In the highest-momentum channels the acceptance functions allow some mixing of negative particles in nominally positive channels, and *vice versa*. The knowledge of the sagitta spectrum and the acceptance functions permitted correction for this effect to be made. To obtain better statistical accuracy the data obtained at the two spectrometer orientations have been combined, and some of the sagitta channels have also been grouped together. The values of η are plotted in Fig. 3 at the median energy of each group, along with the results of other experiments.[1,5,8–10] The present data, while in general agreement with the other measurements, show a tendency for η to decrease with energy, as is expected if the average multiplicity of meson production by primary protons increases with energy. For this effect, it does not matter whether the multiplicity is a property of single collisions or the result of a cascade process.

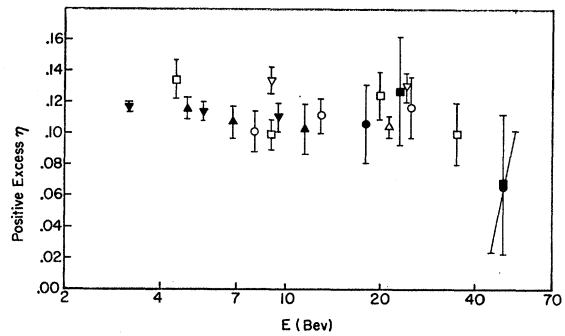

FIG. 3. Positive excess of muons vs muon energy.
Key: ●, present experiment, hodoscope data; ○, present experiment, Geiger counter data; ■, Pine *et al.*,[1] cloud chamber data; □, Pine *et al.*, Geiger counter data; ▲, Owen and Wilson[5]; ▼, Beretta *et al.*[8]; △, Filosofo *et al.*[9]; ▽, Brode and Weber.[10]

ACKNOWLEDGMENTS

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