

## Search for Neutrinoless Conversion of Muon into Electron\*

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A search has been made for the hypothetical reaction in which a muon near a nucleus is converted into an electron without production of neutrinos. Negative muons were stopped in a copper target. A magnetic spectrometer at right angles to the beam transmitted particles of the momentum expected for the electron (about 90 Mev/c on entry into the spectrometer). A long scintillation counter at the output of the spectrometer gave a pulse corresponding to the emerging particle's energy loss. Selection by both momentum and pulse-height eliminated particles heavier than the electron and greatly reduced the accidental background. In the main run, three events meeting the selection criteria were recorded, while the expected number of accidentals is  $0.23 \pm 0.04$ . Various alternative processes that would produce accepted events are considered and found to give expectation values even smaller than that for accidentals. Without further experimentation, we cannot decide whether the hypothetical reaction does or does not occur, but we can set an upper limit of  $(4_{-2}^{+3}) \times 10^{-6}$  on the ratio  $R$  of the reaction rate to that of normal absorption.

### I. INTRODUCTION

WE have carried out an experiment designed to detect the hypothetical reaction in which a negative muon near a nucleus is converted into an electron without production of neutrinos. Several theoretical estimates of the rate of this reaction have recently been made.<sup>1-5</sup> Weinberg and Feinberg<sup>1</sup> have emphasized that if the reaction results from structure in the  $\mu e \gamma$  interaction, different form factors are involved than those determining the  $\mu \rightarrow e + \gamma$  rate, so that the observed rarity of the latter reaction does not necessarily imply a low rate for the one under consideration. Using Primakoff's<sup>6</sup> fit to the observed negative-muon absorption rates in complex nuclei, and Hofstadter's<sup>7</sup> results on the nuclear form factor, we find that their expression (1) for the branching ratio  $R$  of the coherent reaction in which the nucleus recoils rigidly to the normal absorption reaction with emission of a neutrino,

$$R = \frac{\text{rate of } \mu^- + (Z, A) \rightarrow e^- + (Z, A)}{\text{rate of } \mu^- + (Z, A) \rightarrow (Z-1, A)^* + \nu}$$

has the value  $1.25 \xi_0^2 / \xi_\beta^2$  for copper. Here  $\xi_0^2$  and  $\xi_\beta^2$  are algebraic functions of the  $\mu e \gamma$  form factors and the  $\beta$ -decay coupling constants, respectively. The intermediate vector boson hypothesis predicts  $\xi_0^2 / \xi_\beta^2$  on the order of  $2.5 \times 10^{-6}$ . Less than one-seventh of the

reactions are expected to be incoherent, i.e., to excite the nucleus. In the coherent case, the electron is emitted with a unique energy of 103.6 Mev. In the incoherent reaction, the electrons are expected to have a spectrum some 20 Mev wide centered at 98.5 Mev.

The hypothesis of different muon and electron neutrinos absolutely forbids the neutrinoless conversion of a muon into an electron.<sup>8-11</sup>

The pioneer cosmic-ray experiment of Lagarrigue and Peyrou showed that  $R$  is at most a few percent.<sup>12</sup> More recently, Steinberger and Wolfe have searched for the reaction by counting particles with range greater than 24.5 g-cm<sup>-2</sup> polyethylene emerging from a 0.317-cm copper target within 0.2  $\mu$ sec of an incident negative meson.<sup>13</sup> In a run in which they expected 1.86 accidental counts and  $2.62 \times 10^3 R$  real counts,<sup>14</sup> no counts were recorded. Since the probability of obtaining no real events is  $\exp(-2.62 \times 10^3 R)$ , it is likely that  $R$  does not exceed  $10^{-3}$ .

In our experiment a copper target was placed at muon range in the negative-meson beam of the 184-in. synchrocyclotron. A 180-deg,  $n = \frac{1}{2}$  magnetic spectrometer was used to select particles with momentum in the neighborhood of 90 Mev/c.<sup>15</sup> A 45.7-cm long plastic scintillation counter receives particles emerging from the spectrometer; the pulse height is measured both with a gated twenty-channel pulse-height analyzer

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<sup>1</sup> S. Weinberg and G. Feinberg, Phys. Rev. Letters **3**, 111 and 244 (1959).

<sup>2</sup> S. P. Rosen, Nuovo cimento **16**, 7 (1960).

<sup>3</sup> J. Dreitlein, thesis, Washington University, Saint Louis, June, 1960 (unpublished).

<sup>4</sup> R. Gatto, Lectures on Some Questions of the  $V-A$  Theory, given at the International School of Physics of the Italian Physical Society, Varenna sul Lago di Como, Italy, June 29-July 11, 1959 (unpublished); N. Cabibbo and R. Gatto, Phys. Rev. **116**, 1334 (1959).

<sup>5</sup> B. Jouvet and J. C. Houard, Nuovo cimento **15**, 31 (1960).

<sup>6</sup> H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).

<sup>7</sup> B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev. **101**, 1131 (1956).

<sup>8</sup> E. M. Lipmanov, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1054 (1959) [translation: Soviet Phys.-JETP **37**(10), 750 (1960)].

<sup>9</sup> Y. Tanikawa and S. Watanabe, Phys. Rev. **113**, 1344 (1959).

<sup>10</sup> B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1751 (1959) [translation: Soviet Phys.-JETP **37**(10), 1236 (1960)].

<sup>11</sup> T. D. Lee and C. Yang, Phys. Rev. Letters **4**, 307 (1960).

<sup>12</sup> A. Lagarrigue and Ch. Peyrou, Compt. rend. **234**, 1873 (1952).

<sup>13</sup> J. Steinberger and H. B. Wolfe, Phys. Rev. **100**, 1490 (1955).

<sup>14</sup> The coefficient  $2.62 \times 10^3$  was calculated in reference 13 on the assumption that the reaction is incoherent. If it is coherent, the probability of the electron's emerging from the copper with enough energy to penetrate the range telescope is increased, and the coefficient of  $R$  needs to be somewhat larger. From the curve given in reference 13 for the detection probability as a function of energy and the Bethe-Heitler radiation cross section, we estimate that its value should be  $3.56 \times 10^3$  in this case.

<sup>15</sup> H. Kruger and K. M. Crowe, Phys. Rev. **113**, 341 (1959).

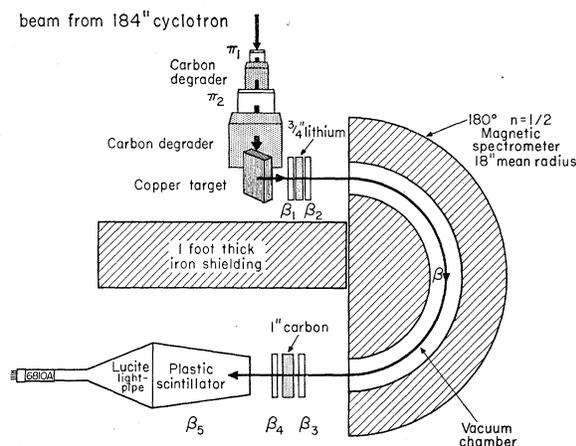


FIG. 1. Schematic drawing of the geometrical arrangement. The meson beam from the cyclotron is horizontal. Particles leaving the target horizontally at right angles to the beam enter the spectrometer. After being bent 180 deg they emerge horizontally into a long plastic scintillator used for energy discrimination.

and by photographing the amplified pulse on a cathode-ray oscilloscope. Selection of particles by both momentum and energy greatly reduces the spurious background and has permitted a reduction of the smallest detectable value of  $R$  by two powers of ten.

## II. APPARATUS

Figure 1 is a perspective sketch of the essential features of the arrangement. The 210-Mev/ $c$  negative external beam is filtered by 45.5 g-cm<sup>2</sup> graphite equivalent before striking the 5.61 g-cm<sup>2</sup> copper target placed at 45 deg to the beam direction. Scintillation counters  $\pi_1$  and  $\pi_2$  in the moderator respond to almost all of the beam particles, of which only some 10 to 20% are muons. The counter train  $\beta_1, \beta_2, \beta_3, \beta_4,$  and  $\beta_5$  selects particles that leave the target horizontally at 90 deg to the beam, traverse the spectrometer, and enter the long (47.9 g-cm<sup>-2</sup>) plastic scintillator. The 1.02 g-cm<sup>-2</sup> lithium and 4.00 g-cm<sup>-2</sup> graphite slabs between the  $\beta$  counters serve to reduce the accidental-coincidence rate. The average ionization energy loss of a 100-Mev electron between the middle of the target and the mouth of the spectrometer is 8.8 Mev. The resolution function of the spectrometer with this source and detector arrangement is an isosceles triangle of full width at half maximum,  $\Delta p/p_0$ , equal to 9.8%. During the main run, its center,  $p_0$ , was set at 90 Mev/ $c$ .

The circuitry is shown schematically in Fig. 2, with the amplifiers, discriminators, and scalers omitted (the outputs of all four coincidence circuits are recorded). The coincidence-anticoincidence circuits  $W_3$  and  $W_4$  are cascaded,  $W_4$  giving the output  $\beta_1\beta_2\beta_3\beta_4\beta_5\pi_1\pi_2$  corresponding to a particle not in prompt coincidence with a beam particle passing through the spectrometer and entering the long counter. The resolving times are approximately  $\pm 4 \times 10^{-9}$  sec. A pulse from  $\pi_1$  makes  $W_4$  inoperative for  $14 \times 10^{-9}$  sec;

one from  $\pi_2$  makes  $W_3$  inoperative for  $21 \times 10^{-9}$  sec. The output of  $W_4$  opens a gate about  $40 \times 10^{-9}$  sec long to the pulse-height analyzer (PHA) and triggers a  $20 \times 10^{-9}$ -sec/cm oscilloscope sweep. A positive pulse from the tenth dynode of the fourteen-stage photomultiplier tube looking at the long counter is fed through the gate circuit to the PHA and is also applied to the vertical-deflection system of the oscilloscope. The circuits recording the  $W_4$  and PHA outputs are gated by a  $1.1 \times 10^{-3}$ -sec pulse bracketing the cyclotron beam. This reduces the cosmic-ray contribution to the data by the factor  $(10^3/64 \times 1.1) = 14$ .

To calibrate the pulse-height-analysis system, we placed the long counter in the meson beam with 3 in. of graphite in front of it. Most of the particles entering it are then pions of about 81-Mev energy, which stop in the counter. They should give pulses corresponding to 81 Mev plus the energy loss by the charged particles in the star; the total light should correspond to about 110-Mev energy dissipation by a fast particle. The kinetic energy of the muons entering the counter is 101 Mev. They too stop in the counter. In most cases a decay electron is emitted so long after the muon stops that it does not contribute to the measured pulse. The electrons passing through the long counter lose 88 Mev by ionization; shower development is not negligible, as the length of the counter is 1.1 radiation lengths.

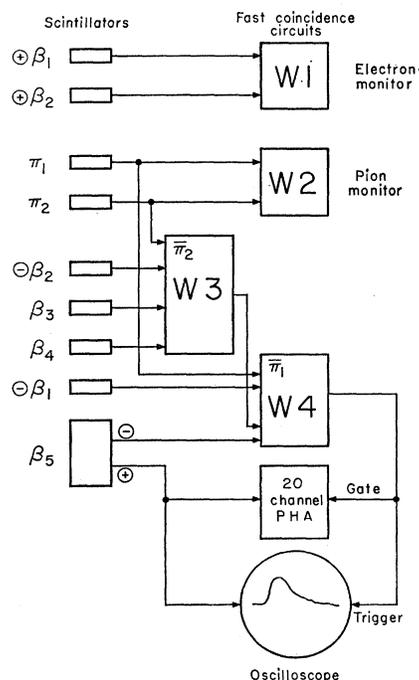


FIG. 2. Schematic diagram of the electronic circuits. Amplifiers, discriminators, scalars, and recorders are omitted. The cascaded fast coincidence-anticoincidence circuits  $W_3$  and  $W_4$  give an output corresponding to  $\beta_1\beta_2\beta_3\beta_4\beta_5\pi_1\pi_2$ . This output opens the gate to a 20-channel pulse-height analyzer and triggers the sweep of an oscilloscope on which the pulse is photographed, thus providing two independent measures of the pulse height in  $\beta_5$ .

The pulse-height distribution obtained is shown in Fig. 3(a). It peaks in channels 15, 16, and 17. Pulse-height distributions were also measured in the calibration runs (see below) with a carbon target, the spectrometer set at 43.3 Mev/c, and the long counter in its normal position. These correspond to the top of the  $\mu^- \rightarrow e^-$  decay spectrum in carbon. The distribution, summed over all the calibration runs during the main run, is shown in Fig. 3(b). The electron energy on entry into the long counter centers at about 35 Mev. The pulse-height distribution is seen to peak near channel 7. Comparing the distributions, we see that channel number of the peak corresponds roughly to the energy loss, and that the PHA system is adjusted properly for detecting electrons such as those expected to result from muon conversion. For 90 Mev/c momentum in the spectrometer, electrons reach the long counter with about 82-Mev kinetic energy, to give a pulse-height distribution peaking in channel 12. Muons would enter the long counter with only 17 Mev; pions would not even reach it.

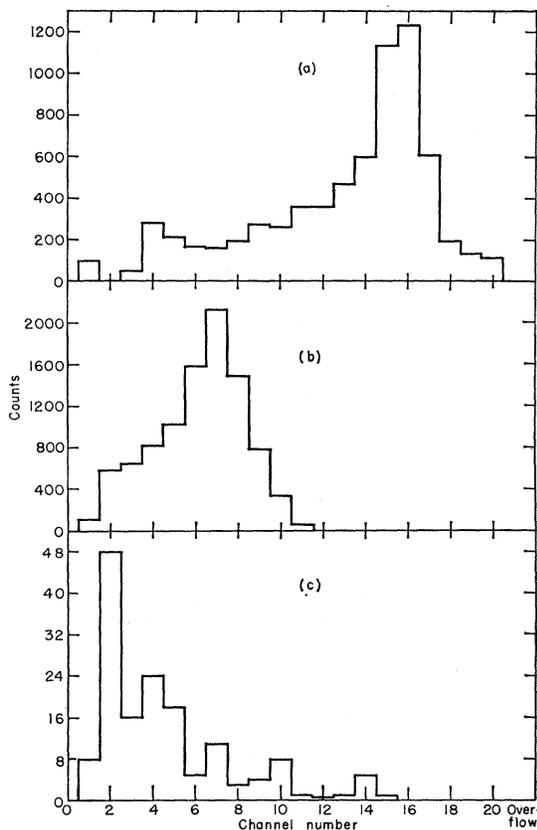


FIG. 3. Pulse-height distributions from the long counter. (a) Counter in the negative-meson beam, behind 13 g-cm<sup>-2</sup> graphite. The particles entering the counter are mostly pions of 80-Mev kinetic energy. (b) Counter in normal position, spectrometer set at 43.3 Mev/c, carbon target. The pulses are due to decay electrons from muons stopped in the target. (c) Counter in normal position, spectrometer set at 90 Mev/c, copper target. The PHA is gated by ( $\beta_3\beta_4\beta_5\pi_2\pi_1$ ) coincidences. In this case, the pulse-height distribution is that characteristic of accidental events.

The pulses photographed on the oscilloscope screen correlated in height with the number of the PHA channel in which the event was recorded; the larger the pulse, the higher the channel number.

### III. RESULTS

For an event to be counted in the data run, it was required that it be recorded in the gated scaler on the *W4* output and in one of channels 2 through 20 of the gated PHA, and that a pulse of appropriate height be photographed on the oscilloscope screen.

In the main run, which lasted 95.2 hr with the spectrometer set at 90 Mev/c, three events satisfying these criteria were found. One was in channel 6 and two were in channel 10.

In order to determine the number of muon conversions to be expected in this time, the run was interspersed with short runs in which the copper target was replaced by a graphite target of the same stopping power, and the spectrometer setting was reduced to 43.3 Mev/c. Denoting by  $\beta$  the number of recorded events, we have

$$\beta(43.3 \text{ Mev}/c, C) = S_C \times f_{\text{dec}} \times f_{>21} \times (\Omega/4\pi) \times \eta(43.3 \text{ Mev}/c, C) \times \epsilon. \quad (1)$$

Here  $S_C$  is the number of muons stopping in the carbon target;  $f_{\text{dec}}$ , the fraction of stopped muons that decay, is 0.92<sup>16</sup>;  $f_{>21}$ , the fraction of decays occurring after  $21 \times 10^{-9}$  sec, is 0.99<sup>17,18</sup>;  $\Omega$  is the effective solid angle of the spectrometer;  $\eta(43.3 \text{ Mev}/c, C)$  is the normalized response of the spectrometer, when set at 43.3 Mev/c, to the spectrum of decay electrons from the carbon target; and  $\epsilon$  is the efficiency of the counting system. During a normal run, on the other hand, we have

$$\beta(90 \text{ Mev}/c, \text{Cu}) = S_{\text{Cu}} \times f_{\text{int}} \times R \times f_{>21} \times (\Omega/4\pi) \times \eta(90 \text{ Mev}/c, \text{Cu}) \times \epsilon. \quad (2)$$

Here  $S_{\text{Cu}}$  is the number of muons stopping in the copper target;  $f_{\text{int}}$ , the fraction of stopped muons that interact, is 0.93<sup>19</sup>;  $f_{>21}$ , the fraction of interactions taking place after  $21 \times 10^{-9}$  sec, is 0.88<sup>17,19</sup>; and  $\eta(90 \text{ Mev}/c, \text{Cu})$  is the normalized response of the spectrometer when set at 90 Mev/c to the spectrum of conversion electrons produced in the copper. One can now determine  $\Omega \times \epsilon$  from the carbon data, obtaining

$$\beta(90 \text{ Mev}/c, \text{Cu}) = R \times \beta(43.3 \text{ Mev}/c, C) \frac{S_{\text{Cu}}}{S_C} \times 0.90 \times \frac{\eta(90 \text{ Mev}/c, \text{Cu})}{\eta(43.3 \text{ Mev}/c, C)}. \quad (3)$$

The spectrometer response functions,  $\eta$ , were computed numerically. The Landau distribution of ionization

<sup>16</sup> F. R. Stannard, Phys. Rev. Letters 4, 523 (1960).

<sup>17</sup> J. C. Sens, Phys. Rev. 113, 679 (1959).

<sup>18</sup> R. A. Reiter, T. A. Romanowski, R. B. Sutton, and B. G. Chidley, Phys. Rev. Letters 5, 22 (1960).

<sup>19</sup> F. E. Holmstrom and J. W. Keuffel, University of Utah, Salt Lake City, (private communication).

losses, Heitler's radiation straggling distribution, and the calculated triangular resolution function of the instrument were assumed.<sup>20</sup> For the coherent conversion process (with a unique initial electron energy of 103.6 Mev),  $\eta(90 \text{ Mev}/c, \text{Cu})$  is 0.23. For the decay electrons from carbon, a calculation based on the positive muon spectral shape gives  $\eta(43.3 \text{ Mev}/c, \text{C})=0.067$ . We use 3.5 for the ratio of the response functions. For the ratio of the numbers of stoppings,  $S_{\text{Cu}}/S_{\text{C}}$ , we take the ratio of the target thicknesses in muon stopping power, as read from range-momentum curves. In the first part of the run, the carbon calibration target ( $3.76 \text{ g-cm}^{-2}$ ) was somewhat thin, and the ratio is 1.12. In the latter part of the run, another carbon target was used, for which the ratio is 1.00. The  $\beta(43.3 \text{ Mev}/c, \text{C})$  rate in the first part was  $31.1 \pm 0.4 \text{ min}^{-1}$ ; in the second part,  $39.3 \pm 0.6 \text{ min}^{-1}$ . The corresponding running times at 90 Mev/ $c$  with the copper target were  $3.36 \times 10^8 \text{ min}$  and  $2.25 \times 10^8 \text{ min}$ . With these values Eq. (3) becomes

$$\beta(90 \text{ Mev}/c, \text{Cu}) = 6.5 \times 10^5 R. \quad (4)$$

This is the expected number of recorded events in the 95.2-hr run, to be compared with the three that were found.

Measurements of the various counting rates show that the accidental coincidences are due overwhelmingly to overlapping of real  $(\beta_1\beta_2)$  events with real  $(\beta_3\beta_4\beta_5)$  events. The pulse-height distribution obtained when the PHA is gated by  $(\beta_3\beta_4\beta_5\pi_2\pi_1)$  is shown in Fig. 3(c). It is a falling one, with  $41/143 = (29 \pm 4)\%$  of the pulses occurring in channels 6 through 20 and  $18/143 = (13 \pm 3)\%$  of the pulses in channels 10 through 20. The effective-duty-cycle-resolving-time factor was determined periodically by comparing the  $(\beta_2\beta_3\beta_4\pi_2)$  rate with the product of the  $(\beta_2\pi_2)$  and  $(\beta_3\beta_4\pi_2)$  rates; the quotient was found to be essentially constant at  $37 \times 10^{-10} \text{ min}$ . The  $(\beta_2\beta_3\beta_4\pi_2)$  rate monitored during the run was found to correlate closely with the product  $(\beta_1\beta_2) \times (\beta_3\beta_4\beta_5)$ , so that its average value over the whole run could be used to determine the effective average accidental rate. The result is  $(1.4 \pm 0.15) \times 10^{-4} \text{ min}^{-1}$ , or a total of  $0.80 \pm 0.09$  events, in channels 2 through 20. From the pulse-height distribution of Fig. 3(c), it follows that  $0.23 \pm 0.4$  accidental events are expected in channels 6 through 20, and  $0.10 \pm 0.03$  in channels 10 through 20.

Evidence has recently been obtained<sup>19,21</sup> indicating that the decay rate of negative muons stopped in iron is some 10 to 20% greater than that for copper. The experimental effect is an increase in the yield of delayed electrons of energy greater than some 10 to 20 Mev. To test the very remote possibility that the effect is due to a high rate of neutrinoless conversion in iron, we took data for a short time with the copper target replaced by an iron one of the same thickness. In 46.0

minutes of running time with the spectrometer set at 90 Mev/ $c$ , no events were observed. The expected number, in terms of  $R_{\text{Fe}}$ , is  $5.5 \times 10^3 R_{\text{Fe}}$ . To account for a 10% increase in the yield of fast electrons,  $R_{\text{Fe}}$  would need to have the value  $10^{-2}$ , which gives an expected number of recorded events of 55. Evidently, neutrinoless conversion is ruled out as an explanation of the increased electron yield from iron. The conversion rate in iron does not exceed that in copper by several powers of ten.

#### IV. DISCUSSION

It is evident from Eq. (4) that  $R$  does not exceed  $10^{-5}$  in order of magnitude—otherwise many more events would have been recorded.

Since three events were recorded in channels 6 through 20, as compared with  $0.23 \pm 0.04$  expected accidentals, it is very unlikely that all three of them are accidental.

To draw a more definite conclusion from the experiment, we must assess the likelihood of the various alternative physical mechanisms that would produce real coincidences.

A natural limit to the sensitivity of the experiment is provided by the high-energy tail of the  $\mu^- \rightarrow e^- + \nu + \bar{\nu}$  spectrum, which results from the orbital motion of the muon at the moment of decay. Measurements of the rates with the copper target in position and the spectrometer set at 43.3 Mev/ $c$  were interspersed in the run; the result was  $\beta(43.3 \text{ Mev}/c, \text{Cu}) = 76 \pm 5 \text{ hr}^{-1}$ . The observed  $\beta(90 \text{ Mev}/c, \text{Cu}) = 0.032 \pm 0.018 \text{ hr}^{-1}$  is down by only a factor of  $(2.4 \pm 1.4) \times 10^3$ , while the tail of the muon decay spectrum makes a contribution that is down by a factor of at least  $3 \times 10^6$ .

We have considered the possibility of scattering of a 100-Mev electron into the spectrometer. Most of the scattering is elastic, with a cross section of about  $4.5 \times 10^{-31} \text{ cm}^2$  per Cu atom<sup>7</sup>; the probability of scattering into the  $1.8 \times 10^{-2} \text{ sr}$  solid angle of the spectrometer is then  $6.1 \times 10^{-10}$  per electron. The proportion of electrons in the beam has been measured on other occasions, and the number of electrons reaching the copper target is certainly less than the number of muons stopping there. The latter figure is estimated from the carbon target, 43.3 Mev/ $c$ , data to be  $1.3 \times 10^9$  in the 95.2-hr running period. Less than 10% of the electrons are in a 10-Mev band at 100 Mev. An upper limit on the expected number of scattered electrons is therefore 0.08. This does not take into account the reduction in detected electron scattering due to the anticoincident  $\pi$  counters. It is very unlikely that electron scattering accounts for any of the three recorded events.

Another possibility is that the events might be due to a contamination by negative pions. This was suggested by the observation that with  $10.6 \text{ g-cm}^{-2}$  C removed from the moderator so as to place the target at pion range, a considerably larger rate— $2.6 \pm 0.9 \text{ hr}^{-1}$

<sup>20</sup> Hans Kruger, thesis, University of California Radiation Laboratory Report UCRL-9322, December, 1960 (unpublished).

<sup>21</sup> D. Yovanovitch, Phys. Rev. **117**, 1580 (1960).

in channels 2 through 20 and  $2 \pm 0.8 \text{ hr}^{-1}$  in channels 6 through 20—was obtained at the 90-Mev/c setting. This could be due to occasional (a few percent) emission of a high-energy gamma ray on pion capture,<sup>22</sup> the gamma ray then materializing to produce a 100-Mev electron. To investigate this possibility, we exposed Ilford G-5 nuclear plates in front of the target with the moderator at its normal thickness. An area scan showed 932 zero-prong meson endings, 18 one-prong endings, and two two-prong endings. Electron tracks were not recorded. This prong distribution agrees with that reported by Morinaga and Fry<sup>23</sup> for a similar type of exposure, and is markedly different from that observed for negative-pion endings, 49% of which have at least two prongs.<sup>24</sup> Moreover, the two-prong stars had the short prongs characteristic of muon stars. Thus a conservative upper limit on the fraction of pions stopping in the copper target at muon range is 0.4%. If about six times as many pions stop at  $\pi$  range as muons at  $\mu$  range, the upper limit on the pion effect at  $\mu$  range is

$$\frac{1}{6} \times 4 \times 10^{-3} \times (2 \pm 0.8) \times 95.2 = 0.13 \pm 0.05$$

events in channels 6 through 20. It is very unlikely that pion contamination accounts for the observed effect.

Large cosmic-ray air showers capable of being detected through the 239 g-cm<sup>-2</sup> iron shield above  $\beta_3$ ,  $\beta_4$ , and  $\beta_5$  but missing the  $\pi_1$  and  $\pi_2$  counters would produce real coincidences satisfying the selection criteria. The number of these expected in our sensitive time is  $\ll 1$ .<sup>25</sup>

No likely interpretation of the three recorded events is known to us. They represent either a combination of fluctuations of accidental and real background or the neutrinoless conversion of muons into electrons. Further experimentation is needed to decide between these alternatives.

## V. CONCLUSIONS

While we cannot draw a definite conclusion as to whether or not the hypothetical neutrinoless muon-electron conversion process has been detected, we can set an upper limit on the branching ratio,  $R$ , of this process relative to normal absorption. This is found by estimating  $R$  on the assumption that the events detected are sampled from reals plus accidentals in expected number  $0.23 \pm 0.04$ . Denoting the estimate of  $R$  by  $R^*$ , we have

$$(3 \pm \sqrt{3}) - (0.23 \pm 0.04) = 6.5 \times 10^5 R^*,$$

or

$$R^* = (4 \pm 3) \times 10^{-6}.$$

<sup>22</sup> K. Brueckner, R. Serber, and K. Watson, *Phys. Rev.* **84**, 258 (1951).

<sup>23</sup> H. Morinaga and W. F. Fry, *Nuovo cimento* **10**, 308 (1953).

<sup>24</sup> C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of Elementary Particles by the Photographic Method* (Pergamon Press, New York, 1959), p. 250, Fig. 8-3.

<sup>25</sup> B. Rossi, *Proceedings of the Moscow Cosmic-Ray Conference, July 6-11, 1959* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. II, p. 19.

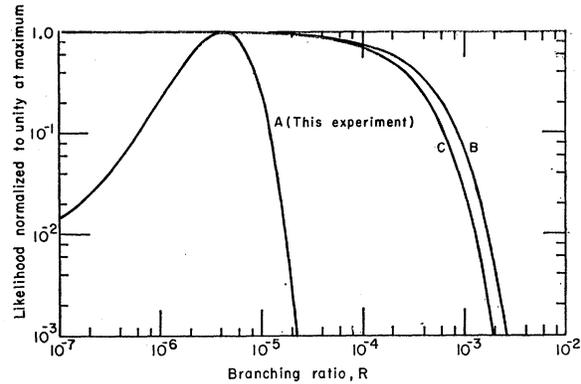


FIG. 4. Likelihood of the experimental result as a function of the true value of the branching ratio  $R$ . The curves are normalized to a maximum value of unity. (A) Curve for the present experiment. It is the probability of detecting three events when  $0.23 + 6.5 \times 10^5 R$  are expected. (B) Curve for experiment of reference 13. (C) Curve for experiment of reference 13, recalculated on the assumption that the conversion process is coherent rather than incoherent.

To express in quantitative terms the result of an experiment yielding no or, at best, a handful of events requires some convention about the statistical “confidence level” used. The result is best presented in the form of a curve showing the likelihood of the observed result as a function of the true value of the parameter being estimated. The absolute value of the likelihood is of no significance for estimating the parameter, but fractional changes are indicative of the relative likelihoods of the corresponding values of the parameter. To facilitate such comparison, we normalize the likelihood function to maximum value unity. In Fig. 4, curve *A*, is plotted the Poisson expression for the likelihood of obtaining three events when the expected number is  $0.23 + 6.5 \times 10^5 R$ . It has its maximum at  $R = 4.3 \times 10^{-6}$  and is reduced by a factor of  $\exp(-\frac{1}{2})$  at  $R = 7.3 \times 10^{-6}$ , in agreement with the estimate of the preceding paragraph. For comparison with the result of reference 13, we reproduce (curve *B*) the likelihood curve given there, as well as the corresponding function (curve *C*), calculated on the assumption that the conversion process is coherent rather than incoherent, so that the expected number of events is  $1.86 + 3.56 \times 10^5 R$  instead of  $1.86 + 2.62 \times 10^5 R$ . Evidently the present experiment reduces the upper limit on  $R$  by a factor of between 20 and 100, depending on the “confidence level” used.

## ACKNOWLEDGMENTS

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