center of the over-all peak is monotonicly increasing. This steady shift of the center of gravity of the peaks to smaller angles would at least appear to rule out the likelihood of instrumental sources for the predominance of $l=2$ type maxima. Taking these incongruencies into account, one cannot help but question the validity of labeling at least the upper two states (e,f) on the basis of their angular distribution. Further work is now underway to attempt to resolve the present confusion.

A level comparison between the two nuclides is made in Fig. 7. If one is willing to accept the comparison of Figs. 6(b) and 3(d) as a reasonable indication of a doublet, the K^{37} nucleus would appear to have a structure quite similar to that of A^{37}. Such a splitting, if of the order of 200 kev, could easily be hidden in the data. As may be seen in Fig. 4, group b is consistently broader than a, certainly suggestive of such structure. Further, if the two subgroups were of about equal intensity at zero degrees, as in the A^{37} case, this would only result in a systematic shift in the apparent Q for their center of gravity of half the amount of the separation. Unfortunately, the counting statistics were not sufficiently good to allow reliable detection of such a systematic variation.

From the difference of the ground-state Q values one obtains a value for the Coulomb energy difference at mass 37 of 6.87±0.15 Mev. This would compare quite favorably with the value of 6.95±0.07 Mev[15] extracted from the positron decay energetics of A^{37}.

ACKNOWLEDGMENTS

The authors would like to acknowledge the invaluable assistance of Miss E. McCauley and Miss M. Davis in the scanning of the nuclear emulsions. They would also like to thank Professor G. Breit for helpful discussions during the course of the work.

[15] R. Wallace and J. A. Welch, Phys. Rev. **117**, 1297 (1960).

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Measurement of the Annihilation-in-Flight Cross Section at 0° for 8.5-Mev Positrons*

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The differential cross section at 0° for the annihilation in flight of 8.5-Mev positrons has been measured. The positrons were created in a thick Ta target which was bombarded by 20-Mev electrons from a linear accelerator. They were directed onto a Be target where annihilation occurred, and the annihilation photons were measured by use of a thick-crystal spectrometer. The measured value for the cross section is 1.3±0.2 barns/steradian per electron, which is in agreement with theory.

INTRODUCTION

A HIGH-CURRENT 22-Mev electron linear accelerator has been used at the Lawrence Radiation Laboratory to produce beams of nearly monoenergetic photons in the approximate energy range of 5 to 15 Mev.[1] The photons were obtained from the annihilation-in-flight of fast positrons; thus, it was considered desirable to check the differential cross section for this process in the energy range of interest. Differential cross sections for annihilation in flight of positrons have been previously examined at 1.0, 2.2, and 3.3 Mev,[2] and total cross sections have been measured at 50, 100, and 200 Mev.[3] The measurements reported below are those for the differential cross section at 0° for 8.5-Mev positrons, and were obtained by the use of techniques quite different from those previously employed.

In this report only the two-quantum annihilation process will be considered, since the one- and three-quantum processes are negligible at this energy for

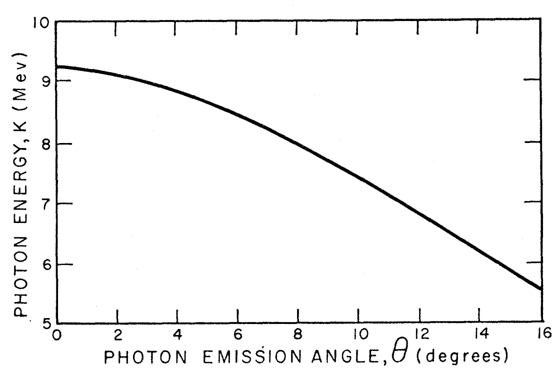

FIG. 1. Photon energy vs angle for annihilation in flight of 8.5-Mev kinetic energy positrons.

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] C. P. Jupiter, N. E. Hansen, R. E. Shafer, and S. C. Fultz, University of California Radiation Laboratory Report UCRL-6044, 1960 (to be published).

[2] H. W. Kendall and M. Deutsch, Phys. Rev. **101**, 20 (1956).

[3] S. A. Colgate and F. C. Gilbert, Phys. Rev. **89**, 790 (1953).

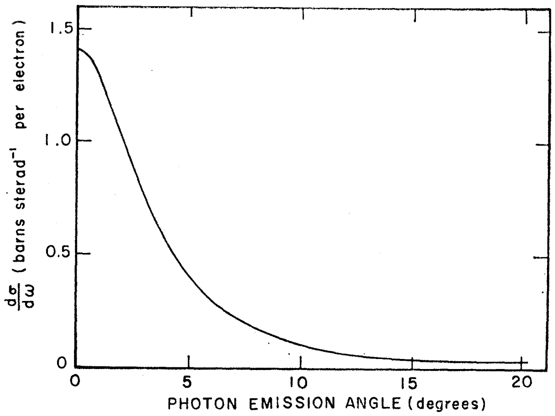

FIG. 2. Calculated differential cross section for annihilation in flight of 8.5-Mev kinetic energy positrons.

annihilation media of low atomic number.[4] Cross sections for the process have been derived and converted to the laboratory system.[2] If a positron of total energy E and momentum P annihilates with an electron at rest (having rest energy μ) to give two quanta of energies K_1 and K_2, where quantum K_1 makes an angle θ with the incident positron direction, the differential cross section for the process is given by

$$\frac{d\sigma}{d\omega}=\frac{e^4(E+\mu)}{P}\left[\frac{1}{(E+\mu-P\cos\theta)^2}-\frac{3\mu+E}{2\mu(E+\mu)(E-P\cos\theta)}+\frac{(e+\mu-P\cos\theta)^2}{2(E+\mu)^2(E-P\cos\theta)^2}\right],$$

$$K_1=\mu\left[1-\left(\frac{E-\mu}{E+\mu}\right)^{\frac{1}{2}}\cos\theta\right]^{-1},$$

$$K_2=E+\mu-K_1.$$

In Fig. 1 is shown the dependence of the energy K_1 on the angle θ for positrons with kinetic energy of 8.5 Mev. The variation of the differential cross section with angle θ is given in Fig. 2, while the dependence of the cross section at 0° on the positron kinetic energy is given in Fig. 3. The experimental points presented are from the works of Kendall and Deutsch[2] and the present authors.

APPARATUS

The arrangement of the experimental apparatus is shown in Fig. 4. The linear accelerator produces a pulsed beam of 20-Mev electrons. For this experiment its operating characteristics were: 100 pulses/sec, 2-μsec pulse duration, and a beam current of 0.15 ampere during the pulse. These electrons were incident upon a 0.25-in. Ta target placed just in front of the accelerator. A magnetic lens coil was placed in front of the Ta target in order to increase the number of positrons available in the experimental area.

Electrons, positrons, neutrons, and x rays were emitted from the Ta target. The x rays were stopped in a lead wall which was well shielded from the NaI(Tl) crystal of the gamma-ray spectrometer. The neutron background was reduced by the concrete walls of the experimental cells and by additional neutron shielding around the spectrometer. The first beam-bending magnet selected the charge and energy of the particles directed towards the experimental area. An adjustable aluminum slit, having vertical and horizontal jaws about 2 in. thick, determined the size of the positron beam entering the field of the second bending magnet. The slit aperture was set at 2 cm wide by 3 cm high for this experiment. When the current in the second bending magnet was off, the beam passed through an aluminum window, 0.016 in. thick, into a Faraday cup similar to that described by Brown and Tautfest.[5] With the second bending magnet on, the positron beam was deflected down a 16-ft extension of beam pipe and was focused with a pair of quadrupole magnets to a spot on the Be annihilation target. After passing through the Be target, the positrons were swept out of the line of sight between the gamma spectrometer and the Be target by another magnet having a horizontally directed field. The annihilation photons from the Be target passed through a final Al window, again 0.016 in. thick, then through a 1-in. diameter lead collimator towards the gamma-ray spectrometer. This

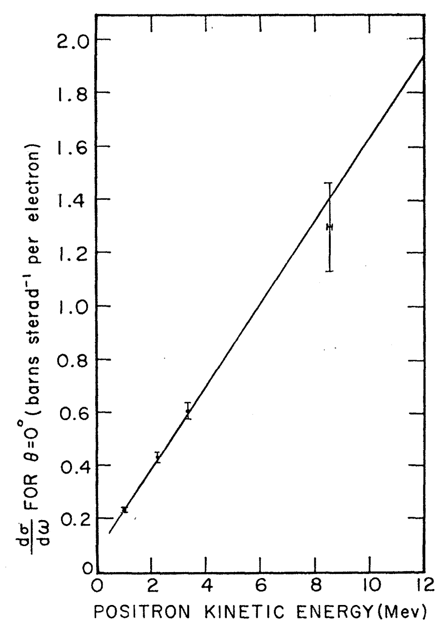

FIG. 3. Calculated cross section for positron annihilation in flight as a function of positron kinetic energy. The three points at low energies are from the experiment of Kendall and Deutsch. The point at 8.5 Mev is described in the text.

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed.

[5] K. L. Brown, G. W. Tautfest, Rev. Sci. Instr. **27**, 696 (1956).

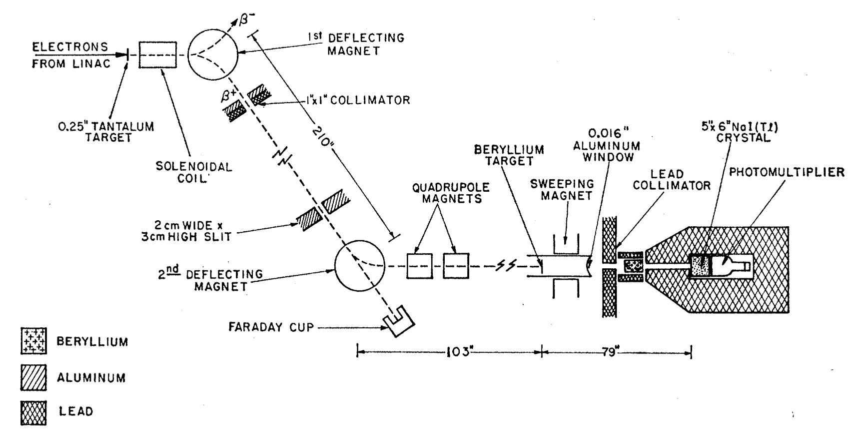

FIG. 4. Diagram of the experimental setup.

spectrometer consisted of a 6-in. by 5-in. diameter NaI(Tl) crystal coupled to a Dumont 6364 photomultiplier tube located behind an 18-in.-thick by 1-in.-diameter lead collimator. The crystal was shielded from background gamma rays and neutrons by 8 inches of lead and a 1-ft-thick layer of paraffin. The positron beam was sufficiently well focused so that very few annihilation photons were produced in the sections of the vacuum system wall or that part of the Al window seen by the gamma spectrometer. A Be beam hardener 1 ft thick was placed in front of the spectrometer to prevent pile-up from low-energy gamma rays in the NaI(Tl) crystal.

Pulses from the photomultiplier tube were amplified by conventional electronics, then passed into a RIDL 200-channel pulse-height analyzer. The analyzer was gated "on" only during the "on" time of the beam pulse. This gating reduced natural backgrounds in the NaI crystal by a factor of about 5×10^{-4}, causing them to be negligible throughout the experiment.

CROSS-SECTION MEASUREMENTS

To compute the positron annihilation-in-flight cross section it is necessary to know the number of electrons/cm^2 in the Be target, the effective solid angle subtended by the gamma-ray detector, the number of annihilation gamma rays entering the detector, and the number of positrons passing through that part of the Be target seen by the detector. The thickness of the Be target was easily measured to approximately one percent accuracy; hence, a calculation of the number of electrons/cm^2 in the target was straightforward.

The best available size of the beam spot on the Be target was larger than the aperture of the collimators in front of the detector; therefore, some vignetting of the photons from the Be target occurred at the detector, and the effective solid angle subtended by the crystal was smaller for the outer portions of the beam spot than for the central region. From photographic images of the beam spot a calculation was made of the effective solid angle subtended by the NaI crystal. This was found to be 0.85 ± 0.10 of the geometric solid angle.

Gamma rays created in the Be target were either bremsstrahlung gammas or nearly monoenergetic annihilation photons. A typical spectrum taken with the gamma detector is given in Fig. 5. The bremsstrahlung spectrum was largest at low energies (c), and the annihilation gamma rays appeared as a high-energy peak (a). Spectrum (d) is a background run taken with the Be target rotated out of the beam. This background came from a few positrons interacting with that part of the vacuum system seen by the spectrometer and from gamma rays and neutrons produced in the Ta target. The background was always negligible when computing the number of annihilation photons in the spectrum. The response of the detection system to monoenergetic gamma rays is shown by the solid line (b) in Fig. 5. This response function was taken from the work of Kockum and Starfelt,[6] and, although they used a different collimation diameter and size of NaI(Tl) crystal, their response functions were found to agree with spectra of monoenergetic fluorescent gamma

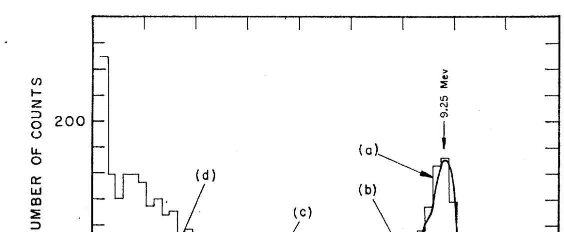

FIG. 5. Experimental pulse-height spectra from the NaI crystal. Parts (a) and (c) arise from gamma rays created when 8.5-Mev positrons were incident on a 0.030-in. thick Be target. (d) is the target-out background. (b) is the computed response of the NaI crystal to 9.3-Mev gamma rays.

[6] J. Kockum and N. Starfelt, Nuclear Instr. and Methods 4, 171 (1959).

rays taken with our spectrometer.[7] Since the Kockum and Starfelt response functions gave good fits to resonance fluorescence data, they were taken to represent the response functions of our system in this energy range. The number of annihilation gamma rays entering the spectrometer was computed by fitting a response function to the measured spectrum, as shown in Fig. 5, and then taking the area under this curve. The number of photons so obtained was then corrected for the efficiency of the NaI(Tl) crystal (computed from total absorption coefficients[8]) and the absorption in the Be beam hardener.

The spectrometer was calibrated several times during the experiment with 4.43-Mev gamma rays from a Po-Be source. Drifts were negligible, and the energy of the observed annihilation gamma rays was 9.3 ± 0.1 Mev. From this, conservation of momentum and energy gives the positron kinetic energy as 8.5 ± 0.1 Mev. By comparing the width of several measured spectra with the response function, it was possible to set an upper limit in the energy spread of the positron beam. In the present experiment this limit appeared to be ≤ 0.5 Mev.

The positron current was measured with the Faraday cup, which was placed as shown in Fig. 4. Since accelerator stability was best during continuous operation, the beam was switched into the Faraday cup, then on the Be target by means of the second bending magnet. The current was thus measured by making spot checks five or six times while taking a spectrum similar to that shown in Fig. 5. The positron current was approximately 10^{-13} ampere, and was measured by passing the current from the Faraday cup through a resistance of 2×10^{11} ohms and measuring the voltage drop across this resistance with a vibrating-reed electrometer.

The cross-sectional area and the position of the positron beam were determined photographically by passing the beam through a piece of polaroid film. Pictures taken in this manner gave assurance that the entire positron beam entered the aperture of the Faraday cup, and assisted in centering the beam on the Be target. The beam always returned to the same position when the second bending magnet was turned off and on again.

Because some positrons were lost in passing down the 16-ft beam tube to the Be target, if was necessary to measure the fraction transmitted down the beam pipe. This was done by using, as a positron detector, a 4-in. $\times$5-in.-diameter NaI(Tl) crystal bonded to a Dumont 6364 photomultiplier tube. The crystal was placed so that the beam, traveling parallel to the crystal axis, struck the center of the crystal. The crystal was large enough to absorb all the energy of 8.5-Mev positrons (or electrons), and it was necessary to use an optical attenuator between it and the photomultiplier in order to avoid saturating the latter. The average dc output of the photomultiplier tube was recorded, so the function of the detector was that of a current amplifier (with a gain of about 7×10^4). To measure the transmission of the beam down the pipes to the Be target, a section of the vacuum pipe near the second bending magnet was removed and the detector was placed just ahead of the quadrupoles. The positron beam was switched between the Faraday cup and the detector, and current measurements were made. In this manner the detector was calibrated. It was then moved to a position beyond the Be target and its response was again compared with current measured in the Faraday cup. The last measurement was repeated several times during the experiment, and yielded the value 0.80 ± 0.06 for the transmission down the beam pipe.

MULTIPLE SCATTERING

If there is appreciable multiple scattering of the positron beam in the Be target, the measured annihilation cross section will be in error. From Fig. 2 it can be seen that the cross section for annihilation decreases rapidly as the angle the gamma ray makes with the original positron direction increases. As Be target thickness increases, the average scattering angle of the positron beam increases because of multiple scattering; hence, the average measured cross section decreases. In order to determine the magnitude of multiple-scattering effects in the present experiment, cross-section measurements were made using Be targets of several thicknesses. The apparent decrease in the measured value of the cross section as the thickness of the Be target was increased is evident in Fig. 6. An approximate calculation of the multiple-scattering effect was therefore undertaken.

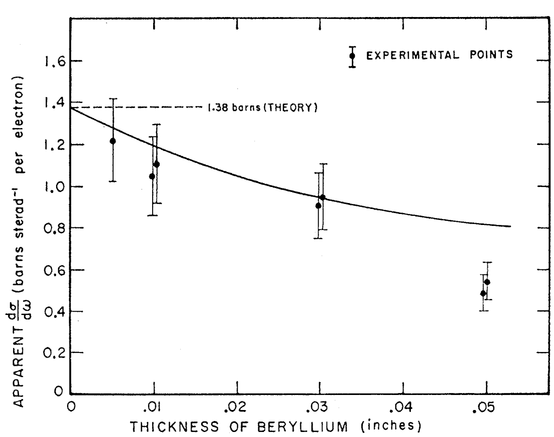

FIG. 6. Measured cross section at 0° for annihilation in flight of 8.5-Mev positrons as a function of annihilation-target thickness. The solid curve is the calculated effect of multiple scattering of positrons in the Be target.

[7] F. D. Seward, H. W. Koch, R. E. Shafer, and S. C. Fultz, Bull. Am. Phys. Soc. 5, 68 (1960).

[8] G. W. Grodstein, National Bureau of Standards Circular No. 583 (U. S. Government Printing Office, Washington, D. C., 1957).

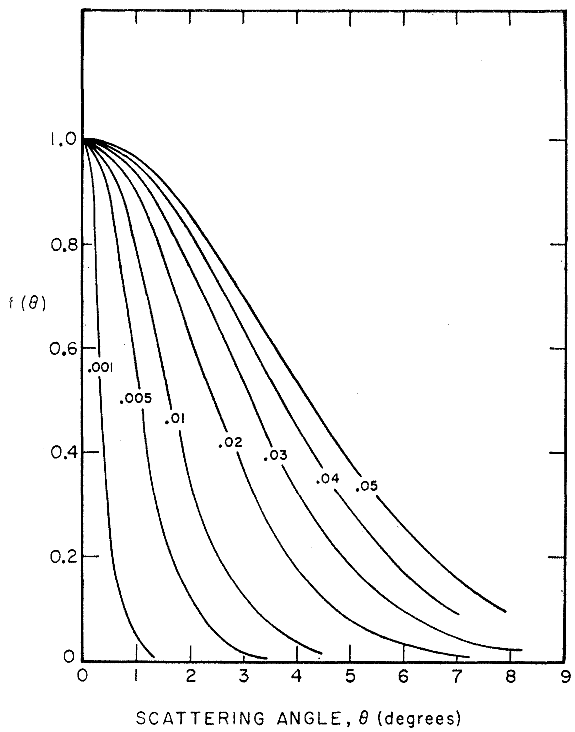

FIG. 7. Angular distribution of 8.5-Mev positrons after passing through various thicknesses of Be. Curves were calculated from theory of Molière and are normalized to 1.0 at 0°. The thickness of Be is given in inches for each curve.

Using the multiple-scattering theory of Molière,[9] the angular distribution $f(\theta)$ was calculated for positrons after they had passed through a layer of Be. In Fig. 7 are shown calculated angular distributions of positrons transmitted through targets of various thicknesses. The target was divided into a number of layers, and the contribution to the detector aperture of annihilation photons arising from each layer was calculated. This was done by combining the differential cross section $d\sigma(\theta)/d\omega$, shown in Fig. 2, with the angular distribution of the positrons. The total number "N_γ" of annihilation photons received by the detector was obtained by summing up the contributions from each layer.

$$N_\gamma = \text{const}\sum_i \int_0^\pi f_i(\theta)\frac{d\sigma(\theta)}{d\omega}\sin\theta d\theta \bigg/ \int_0^\pi f_i(\theta)\sin\theta d\theta.$$

The integral in the denominator normalizes the contribution of each layer. Almost the entire contribution of photons was found to come from $\theta<5°$. Over this range of angle the gamma-ray energy was essentially constant as far as the spectrometer was concerned. The result for the calculated effect of multiple scattering on the measured cross section is shown as a solid line in Fig. 6.

[9] G. Molière, Z. Naturforsch. **34**, 78 (1948).

RESULTS

In Fig. 6 are shown experimentally measured values of the cross section for various thicknesses of the Be target. Two measurements were made for each target thickness, with the exception of the 0.005-in. thick target. If multiple scattering in the Be target is neglected, the detector saw annihilation photons emitted at angles from 0° to 0.7°. The theoretical differential cross section shown is an average over this angular range and has the value of 1.38 barns. Error bars on each point include all uncertainties, including some possible systematic errors which are the same for each point. The differences between points for the same thickness of target are within the statistical uncertainties of the experiment, i.e., about 5%.

Estimates of the limits of possible experimental errors were made as follows: The uncertainty in the number of electrons/cm^2 in the Be targets was approximately 2%. The uncertainty in the solid angle $d\omega$ subtended by the spectrometer at the Be target was almost entirely in the estimation of the vignetting effect of the collimation system, which was about 10%. Measurement of the positron current was limited by measurement of the transmission of the beam through the quadrupoles to the Be target, and by intermittent measurement of the beam current. Uncertainty in the positron current was about 10%. The accuracy of the gamma-ray counting measurement was limited by correction for the tail of the crystal response function. The uncertainty in this tail was such that the accuracy of the gamma measurement was about 10%. The net uncertainty for each data point was therefore about 17%.

The measured differential cross section for the annihilation in flight of 8.5-Mev positrons is taken as 1.3 ± 0.2 barns. The results of the experiment therefore agree within the experimental uncertainty with the value 1.38 barns calculated from the theory of Dirac. They also agree with the calculated multiple-scattering effect for target thickness up to 0.030 in.

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